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Optimal Tuning of PID Controller Parameters for Set point Tracking Control of Nonlinear Systems

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Abstract: This paper presents a new approach for optimal tuning of PID controller parameters for the control of nonlinear systems. The design is based on optimal tracking of step response for nonlinear systems. The problem is first restated as a non linear optimal control infinite horizon problem, then This problem is transferred to measure space, and it is shown that an optimal measure must be determined which is equivalent to a linear programming problem. Then, optimal control law is determined as a piece wise constant function. Finally, PID controller parameters are determined using the optimal control law. Simulations are provided to show the effectiveness of the proposed methodology.

Key words: Measure theory, Optimal Control, PID Tuning, Nonlinear Systems, MIMO Systems.

INTRODUCTION

Measure theory is a powerful method, which has previously been applied to solve nonlinear optimal control problems. This include, optimal control of one and multidimensional diffusion equations (J.E. Robio, 1986) and (A.V. Kamiah, 1991), optimal control of wave and elliptic equations (J.E. Rubio, 1993; M.H. Faradic *et al.*, 1995) optimal Tuning of PID Parameters for single input-single output systems (A. Zare *et al.*, 2000) and also optimal control of HIV (Hassan Zarei *et al.*, 2010). In this paper a new method for optimal tuning of PID parameters is proposed using measure theory. Employing measure theory, the optimal control problem is reformulated as a linear programming problem, approximately. This problem is then solved. An outstanding property of the measure theory is its independence of the cost function convexity and its constraints. As PID is a conventional and widely used controller for different industrial applications, its optimal parameter settings are of vital practical importance. Simulation results are provided to show the effectiveness of the proposed method.

Optimal Control Problem and its Transformation to Measure Space:

The nonlinear optimal control problem with infinite horizon can be stated as follows (J.E. Robio, 1986; A.P.S age and C. White, 1977):

$$Min \quad I = \int_{t_a}^{\infty} f(x(t),u(t),t)dt \tag{1}$$

s.t

$$\frac{dx(t)}{dt} = g(x(t),u(t),t) \quad , \; x(t_a) = x_a = x_0 \tag{2}$$

Where *f(.)* and *g(.)* are nonlinear functions of state $x(t) \in A \subset R^n$, control signal $u(t) \in U \subset R^m$ and time $t \in [0,\infty)$, and $x_a = x_0$ is the initial state and the final state is free.

Define the following transformation (W. Rudin, 1966):

$$\Lambda(F) = \int_{t_a}^{t_b} F(x(t),u(t),t)dt \tag{3}$$

Where F is a continuous function ($F \in C(A,U,J)$), A and U are compact sets of state and control functions respectively. Also, $J = [t_a, t_b]$ and is a compact set. The transformation given by (3) is linear, continuous, positive and bounded. Hence, employing the Riesze representation theorem there exist a unique positive Radon measure μ that.

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$$\Lambda(F) = \int_J F(x(t),u(t),t)dt = \int_\Omega F d\mu \equiv \mu(F)$$

Where $\Omega = A \times U \times J$. Then using this measure, the optimal control problem is reformulated from classical control space to the measure space. The above linear operator has the following properties:

1. If $\phi(x,t) \in C(A \times J)$ (continuous function space on set $A \times J$) and $\phi^g(x,t)$ defined such as:

$$\phi^g(x,t) = \phi_x(x,t)g(x,u,t) + \phi_t(x,t) \tag{4}$$

Then:

$$\int_J \phi^g(x,t)dt = \phi(x(t_b),t_b) - \phi_t(x(t_a),t_a) \equiv \Delta\phi \tag{5}$$

2. If $\psi(t)$ is a function from $D(J^0)$ (infinity differentiable function space with compact support on $J^0 = (t_a, t_b)$), and $\psi_j(x,u,t)$ defined as:

$$\psi_j(x,u,t) = x_j \psi'(t) + g_j(x,u,t)\psi(t) \tag{6}$$

Where x_j is j'th element of x and g_j is j'th element of g, then the function satisfies (J.E. Robio, 1986):

$$\int_J \psi_j(x,u,t)dt = 0 \tag{7}$$

3. If $\theta(t)$ is a continuous function, then:

$$\int_J \theta(t)dt = a_\theta \tag{8}$$

Also it can be shown that using equations (5), (7) and (8), μ has the following properties:

$$\begin{aligned} &\mu(\phi^g) = \Delta\phi \\ &\mu(\psi_j) = 0 \qquad j = 1,2,3,...,n \\ &\mu(\theta) = a_\theta \end{aligned} \tag{9}$$

We shall consider the minimization of (1) over the set Q of all positive Radon measures on satisfying (9). This is an infinite dimensional linear programming problem, and all the functions in (9) are linear with respect to the positive Measure μ . Let $M^+(\Omega)$ be the set of all positive Radon measure on Ω . The functional $I : Q \to R$ defined by:

$$I(\mu) = \int_\Omega f d\mu \equiv \mu(f) \in R,\ \mu \in Q \tag{10}$$

Is the linear continuous functional on set Q with weak*_topology.

Approximation:

The optimal control is now estimated by a nearly optimal piecewise constant control. The functional: $I(\mu) = \int_\Omega f d\mu$ over a subset, this is defined by requiring only a finite number of the constraints in (9) to be satisfied (still infinite dimension). This will be achieved by choosing countable sets of functions, whose linear combinations are dense in the appropriate space. Then a finite number of these functions are selected. In the first step, an approximation to the optimal measure μ^* is obtained by a finite combination of atomic measures, that is from (J.E. Robio, 1986), μ^* has the form:

$$\mu^* = \sum_{i=1}^{N} \alpha_i^* \delta_{z_i^*} \quad \alpha_i^* \geq 0, \quad z_i^* \in Q \tag{11}$$

Here δ_z is the unitary atomic measure characterized by $\delta_z(F) = F(z)$. Then a piecewise constant control function corresponding to the finite dimensional problem is constructed. Therefore in infinite dimensional linear programming problem (10) with restrictions defined by (9) we shall consider only a finite number M_1 of functions ϕ, and only a finite number M_2 of functions ψ_j, also, only a finite number L of functions θ. Separating the sets $A \times U$ and J to s and d partitions respectively and F satisfying the Lipshitz condition (J.E. Robio, 1986), then Riesze theorem (J.E. Robio, 1986; W. Rudin, 1966) provides a suitable approximation of the measure $\mu(F) = \Lambda(F)$, expressed as follows (J.E. Robio, 1986; W. Rudin, 1966; H.L. Royden, 1963).

$$\mu(F) = \sum_{j=1}^{N} \alpha_j F(z_j) \tag{12}$$

Where N=s.d and z_j is the j'th separation sample and depends on F. The linear programming problem consists of minimizing the linear form:

$$\sum_{j=1}^{N} \alpha_j f(z_j) \tag{13}$$

Over the sets $\alpha_j \geq 0$, subject to

$$\sum_{j=1}^{N} \alpha_j \phi_i^g(z_j) = \Delta\phi_i \quad , i = 1,2,3,\ldots,M_1$$

$$\sum_{j=1}^{N} \alpha_j \psi_i(z_j) = 0 \quad , \quad i = 1,2,3,\ldots,M_2 \tag{14}$$

$$\sum_{j=1}^{N} \alpha_j \theta_s(z_j) = a_\theta \quad , \quad s = 1,2,3,\ldots,L$$

If the above conditions for the optimal control problem are satisfied, the problem can be transfer to the measure space. As the time horizon is infinite, J is not bounded, hence $\Omega = A \times U \times J$ is not a compact set. If define $\tau = \dfrac{t - t_a}{t+1}$ and functions $f_1(.)$, $g_1(.)$ as:

$$f_1(x(\tau), u(\tau), \tau) = \frac{f(x(\frac{\tau + t_a}{1-\tau}), u(\frac{\tau + t_a}{1-\tau}), \frac{\tau + t_a}{1-\tau})}{(1-\tau)^2} \tag{15}$$

$$g_1(x(\tau), u(\tau), \tau) = \frac{g(x(\frac{\tau + t_a}{1-\tau}), u(\frac{\tau + t_a}{1-\tau}), \frac{\tau + t_a}{1-\tau})}{(1-\tau)^2} \tag{16}$$

Therefore, the infinite optimal control problem can be stated as follows:

$$Min \quad I = \int_0^1 f_1(x(\tau), u(\tau), \tau) d\tau$$

s.t (17)

$$\frac{dx}{d\tau} = g_1(x(\tau), u(\tau), \tau) \quad , x(0) = x_a$$

It has been shown that the integral (cost function) can be computed on the set $[0,1-\varepsilon]$, where ε is a positive small number (S. Effati, 2000). It is seen that, the optimal control problem with infinite horizon is transferred to an optimal control problem with finite horizon and free final state. Equations (9), (10) and (11) indicated the optimal control problem with infinite horizon are restated as a linear programming problem with a suitable approximation given by minimizing (13) subjected to (14) with $\alpha_j \geq 0$. As x_f is free, the right hand side of the second equation (14) is unknown, and is therefore solved as follows. Note that the optimal control problem has a solution and therefore the objective value *I* in (1) is a real number, which gives:

$$\lim_{t\to\infty} f(x(t), u(t), t) = 0 \tag{18}$$

And because state function is differentiable and since its limit when time tends to infinity is x_f, so *x(t)* in the limit is a constant, hence:

$$\lim_{t\to\infty} \frac{dx(t)}{dt} = 0 \tag{19}$$

Then in the limiting case:

$$\begin{cases} f(x_f, u_f, t_b) = 0 \\ g(x_f, u_f, t_b) = 0 \end{cases} \tag{20}$$

Where: $x_f = \lim_{t\to\infty} x(t)$ and $u_f = \lim_{t\to\infty} u(t)$. Because suppose limit of state and control functions are exist and t_b positive large real number. Then:

$$\lim_{t\to\infty} x(t) = x_f \Rightarrow \forall \delta > 0 \ \exists \varepsilon > 0 \, s.t. \forall t \geq t_b : \left| x(t_b) - x_f \right| < \delta \tag{21}$$

Now we obtain optimal x_f and u_f as:

- The system equations (20) has a unique solution and for x_f and u_f.
- The system equations (20) have multiple solutions, so final state and control can be determined such that the energy in the steady state case is minimized:

$$Min \ u_f^2$$

s.t

$$f(x_f, u_f, t_b) = 0 \\ g(x_f, u_f, t_b) = 0 \tag{22}$$

With solution linear programming problem given by equations (14) and determine α_j, hence the optimal control function is determined as a piecewise constant function.

Optimal PID Parameter Tuning for Nonlinear SISO Systems:

Consider the nonlinear closed loop system shown in Figure (1), where *r(t)* is the step input, *e(t)* is the error function, *u(t)* is the control function, *x(t)* is the state function and *y(t)* is the system output. The nonlinear dynamical system is characterized by the following nonlinear equations:

$$\frac{dx(t)}{dt} = g(x(t), u(t)) \tag{23}$$

$$y(t) = h(x(t), u(t) \tag{24}$$

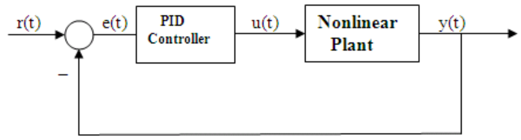

Fig. 1: Closed loop system.

And the PID controller is given as:

$$u(t) = k_p e(t) + k_d \frac{de(t)}{dt} + k_i \int_0^t e(t)dt \tag{25}$$

The optimal tracking problem is to tune the controller parameters (determine k_p, k_d, k_i parameters) such that the following cost function is minimized:

$$Min\ I = \int_0^\infty t^i e(t)^2 dt \tag{26}$$

s.t

$$\frac{dx(t)}{dt} = g(x(t), u(t)) \tag{27}$$

$$y(t) = h(x(t), u(t) \tag{28}$$

$$e(t) = r(t) - y(t) \tag{29}$$

Where *i* is used to control the speed of tracking and different cost function such as: ISE (Integral Square Error), IAE (Integral Absolute Error), ITAE (Integral Time Absolute Error), can be obtained with varying *i*. Consider x_0 as the initial state of the nonlinear system. Then the optimization problem is to transfer the state to that is free, such that the cost function given by (26) is minimized. The optimal tracking problem can be transferred to a nonlinear optimal control problem with infinite horizon:

$$Min\ I = \int_0^\infty t^i (r(t) - h(x(t), u(t))^2 dt \tag{30}$$

s.t

$$\frac{dx(t)}{dt} = g(x(t), u(t)) \tag{31}$$

Hence, assuming that the optimal solution be exists, it is readily seen that:

$$\lim_{t\to\infty} t^i e(t)^2 = \lim_{t\to\infty} t^i (r(t) - h(x(t), u(t))^2 = 0 \tag{32}$$

And the necessary condition for convergence is $\lim_{t\to\infty} e(t) = 0$. On the other hand:

$$\int_0^\infty \frac{de(t)}{dt} dt = e(\infty) - e(0) < \infty \tag{33}$$

Since *e(t)* is a differentiable function, then $\lim_{t\to\infty} \dot{e}(t) = 0$. From (19) we have:

$$\lim_{t\to\infty}\frac{dx(t)}{dt}=\lim_{t\to\infty}g(x(t),u(t))=0 \tag{34}$$

Therefore, with $r(t)=1-e^{-\alpha t}$ $\alpha\geq 0$, the following nonlinear equation in the limiting case is obtained:

$$\begin{cases} f(x_f,u_f)=0 \\ g(x_f,u_f)=0 \end{cases} \tag{35}$$

Where, $x_f=\lim_{t\to\infty}x(t)$ and $u_f=\lim_{t\to\infty}u(t)$ and $\lim_{t\to\infty}r(t)=1$. The optimal final state can be determined using the above nonlinear system equations. If the solution is not unique then the condition below can be imposed to conclude the unique solution:

$$Min\ u_f^2$$

s.t

$$g(x_f,u_f)=0 \tag{36}$$

$$h(x_f,u_f)=1$$

Note that we have a nonlinear optimal control and as the new variable is $\tau=\frac{t-t_a}{t+1}$, then time horizon is also bounded. This problem has been solved in section (1) and optimal control function can thus be determined as a piecewise constant function. Using the system equations, the optimal state and error can be determined. Hence, the input (*e(t)*) and the output (*u(t)*) of the PID controller are determined. A problem that remains is that of the differentiability of *e(t)* which is tackled as follows. If *u(t)* is discontinuous at $t=t_0$, then define *u(t)* at $N(t_0,\delta)$ such as:

$$u(t)=\frac{b-a}{4\delta}[\frac{(t-t_0)^3}{\delta^2}-3(t-t_0)]+\frac{b+a}{2} \tag{37}$$

Where $t\in[t-\delta,t+\delta], b=u(t_0-\delta)$ and $a=u(t_0+\delta)$, then $\dot{u}(t_0-\delta)=\dot{u}(t_0+\delta)=0$. Therefore *u(t)* and *e(t)* are now differentiable functions of time. Let the equation governing the PID controller be as given below:

$$u(t)=k_p e(t)+k_d\frac{de(t)}{dt}+\dot{k}_i\int_0^t e(t)dt \tag{38}$$

Lemma 1:

Assuming *u(t)* and *e(t)* as bounded functions and *e(t)* integrable, then the optimal integral gain is given as:

$$k_i=\frac{\lim_{t\to\infty}u(t)}{\int_0^\infty e(t)dt} \tag{39}$$

Proof:

As *e(t)* and $e(t)'$ approach zero for $t\to\infty$, the proof is trivial.

To determine k_p and k_d equation (38) gives:

$$k_p e(t)+k_d\frac{de(t)}{dt}=u(t)-\dot{k}_i\int_0^t e(t)dt\overset{\Delta}{=}L(t) \tag{40}$$

L(t) is known, and the minimization problem below, determines the proportional and derivative gains:

$$\underset{k_p,k_d}{Min} \quad J_a = \int_0^\infty (k_p e(t) + k_d \frac{de(t)}{dt} - L(t))^2 \, dt$$

Hence:

$$\frac{\partial J_a}{\partial k_p} = 2\int_0^\infty e(t)(k_p e(t) + k_d \frac{de(t)}{dt} - L(t))dt = 0 \quad (41)$$

$$\frac{\partial J_a}{\partial k_d} = 2\int_0^\infty \frac{de(t)}{dt}(k_p e(t) + k_d \frac{de(t)}{dt} - L(t))dt = 0 \quad (42)$$

Solving above linear equations, k_p and k_d parameters are determined.

Optimal PID Parameter Tuning for Nonlinear Multivariable Systems:

Consider the nonlinear closed loop system shown in Figure (1). Where: *R(t)* is the step input vector, *E(t)* is the error function vector, *U(t)* is the control function vector, *X(t)* is the state function vector and *Y(t)* is the system output vector.

The nonlinear dynamical system is characterized by the following nonlinear equations:

$$\frac{dX(t)}{dt} = G(X(t), U(t)) \quad (43)$$

$$Y(t) = H(X(t), U(t) \quad (44)$$

Where G and H are continuous vector functions and the PID controller is given as:

$$U(t) = K_p E(t) + K_d \frac{dE(t)}{dt} + K_i \int_0^t E(t)dt \quad (45)$$

Where $E(t) = [e_1(t),...,e_l(t)]^T$, $R(t) = [r_1(t),...,r_l(t)]^T$, $Y(t) = [y_1(t),...,y_l(t)]^T$, $U(t) = [u_1(t),...,u_m(t)]^T$ And $X(t) = [x_1(t),...,x_n(t)]^T$.

The optimal tracking problem is to tune the controller Matrices (determine K_p, K_d, K_i Matrices) such that the following cost function is minimized:

$$Min \ I = \int_0^\infty \sum_{j=1}^{l} t^{i_j} e_j(t)^2 dt$$

s.t

$$\frac{dX(t)}{dt} = G(X(t), U(t)) \quad (46)$$

$$Y(t) = H(X(t), U(t)$$

$$E(t) = R(t) - Y(t)$$

Where i_j is used to control the speed of tracking in every output. Consider X_0 as the initial state of the nonlinear Multivariable system. Then the optimization problem is to transfer the state from X_0 to X_f that is free, such that the cost function given by problem (46) is minimized. The optimal tracking problem can be transferred to a nonlinear optimal control problem with infinite horizon:

$$\underset{U(t)}{Min} \ I = \int_0^\infty t^i \| R(t) - H(X(t), U(t)) \|^2 \, dt$$

s.t (47)

$$\frac{dX(t)}{dt} = G(X(t), U(t))$$

Hence, assuming that the optimal solution exists, it is readily seen that:

$$\lim_{t\to\infty} t^{i_j} e_j(t)^2 = \lim_{t\to\infty} t^{i_j}(r_j(t) - h_j(X(t), U(t))^2 = 0 \quad (48)$$

And the necessary condition for convergence is: $\lim_{t\to\infty} e_j(t) = 0 \Rightarrow E(t) = 0$.

On the other hand:

$$\int_0^\infty \frac{de_j(t)}{dt} dt = e_j(\infty) - e_j(0) < \infty \quad (49)$$

Since $e_j(t)$ is a differentiable function, then: $\lim_{t\to\infty} \dot{e}_j(t) = 0$ implies $\frac{dE(t)}{dt} = 0$.

From (43) we have:

$$\lim_{t\to\infty} \frac{dX(t)}{dt} = \lim_{t\to\infty} G(X(t), U(t),) = 0 \quad (50)$$

Therefore, with $r_j(t) = 1 - e^{-\alpha_j t}$ $\alpha_j \geq 0$, the following nonlinear equations in the limiting case are obtained:

$$\begin{cases} G(X_f, U_f) = 0 \\ H(X_f, U_f) = [1,1,1,...,1]^T \end{cases} \quad (51)$$

Where, $X_f = \lim_{t\to\infty} X(t)$ and $U_f = \lim_{t\to\infty} U(t)$ and $\lim_{t\to\infty} R(t) = [1,1,1,...,1]^T$. The optimal final state vector can be determined using the above nonlinear equations. If it is not unique then the condition below can be imposed to conclude the unique solution:

$$Min \ \|U_f\|^2$$

s.t (52)

$$G(X_f, U_f) = 0$$
$$H(X_f, U_f) = [1,1,1,...,1]^T$$

Note that we have a nonlinear optimal control and as the new variable is $\tau = \frac{t - t_a}{t+1}$, then time horizon is also bounded. This problem has been solved in section (1) and the optimal control function vector can thus be determined as a piecewise constant functions. Using the system equations, the optimal state and error vectors can be determined. Hence, the input (*E(t)*) and the output (*U(t)*) of the PID controller are determined. As in the SISO case, a problem that remains is that of the differentiability of *E(t)* which is solved similarly by choosing $u_j(t)$ at $N(t_0, \delta)$ such as:

$$u_j(t) = \frac{b-a}{4\delta}\left[\frac{(t-t_0)^3}{\delta^2} - 3(t-t_0)\right] + \frac{b+a}{2}, \quad t \in [t-\delta, t+\delta] \quad (53)$$

Where $b = u_j(t_0 - \delta)$ and $a = u_j(t_0 + \delta)$, then $\dot{u}(t_0 - \delta) = \dot{u}(t_0 + \delta) = 0$. Therefore $u_j(t)$ and $e_j(t)$ are now differentiable functions of time. Let the equation governing the PID controller be as given below:

$$U(t) = K_p E(t) + K_d \frac{d\dot{E}(t)}{dt} + K_i \int_0^t E(t)dt \tag{54}$$

Lemma 2:

Assuming *U(t)* and *E(t)* as bounded vector functions and *E(t)* integrable, then the optimal integral gain Matrices satisfy the following equation:

$$K_i . \int_0^\infty E(t)dt = \lim_{t \to \infty} U(t) \tag{55}$$

Proof:

As *E(t)* and $\frac{dE(t)}{dt}$ Approach to zero for $t \to \infty$, the proof is trivial using (54).

If $\|K_i\|$ is minimized then $\|U_f\|$ will be minimized, because from (55):

$$\|K_i\| . \left\| \int_0^\infty E(t)dt \right\| \leq \|U_f\| .$$

It is Important that minimize the amplitude of control signals and possible $\int_0^\infty E(t)dt$ is not invertible, Let K_i be determine such

$$Min \ \|K_i\|^2$$

s.t (56)

$$K_i . \int_0^\infty E(t)dt = U_f$$

To determine K_d and K_p equation (54) gives:

$$K_p E(t) + K_d \frac{dE(t)}{dt} = \dot{U}(t) - K_i \int_0^t E(t)dt \overset{\Delta}{=} L(t)$$

L(t) is known, and the minimization problem below, determines the proportional and derivative gains matrices:

$$\underset{K_p, K_d}{Min} \quad J_a = \int_0^\infty \left\| K_p E(t) + K_d \frac{dE(t)}{dt} - L(t) \right\|^2 dt \tag{57}$$

This cost function minimizes the differences between the real optimal control and the control law that is determined by measure theory.

Hence:

$$\frac{\partial J_a}{\partial [K_p]_{i,j}} = 0, \ i = 1,2,...,m, \ j = 1,2,...,l$$

$$\frac{\partial J_a}{\partial [K_d]_{i,j}} = 0, \ i = 1,2,...,m, \ j = 1,2,...,l$$

Then, solving the above linear system K_d and K_p matrices are determined

Simulations Results:

To show the effectiveness of the proposed methodology, nonlinear multivariable systems with no convex cost functions are controlled with the optimal PID controller tuned using the measure theory.

Example:

Consider the following nonlinear multi variable system:

$$\frac{dx_1(t)}{dt} = -\sin(x_1(t) + x_2(t) - u_2(t)) + \sin(u_1(t))$$

$$\frac{dx_2(t)}{dt} = -\sin(x_1(t).x_2(t) + u_1(t)) + u_2(t)$$

$$y_1(t) = x_1(t)^2 + x_1(t).x_2(t)$$

$$y_2(t) = x_2(t)^2 + 2\sin(x_1(t).x_2(t))$$

This system is nonlinear and exists interaction with inputs and out puts. It is desired to tune the PID controller matrices to achieve set-point tracking for every output with control functions in the interval [0 15]. Using the proposed methodology results in a tuning of $K_p = \begin{bmatrix} -6.8 & -8.5 \\ 1.7 & 2.3 \end{bmatrix} \times 10^{-3}, K_i = \begin{bmatrix} -0.37558 & 0.6678 \\ 0.6678 & 0.1810 \end{bmatrix}$ and $K_d = \begin{bmatrix} -4.5210 & 5.0553 \\ 2.6269 & -2.0165 \end{bmatrix}$ the value of cost function is I=0.8880 Figures 1,2,3 and 4, show the control and state functions, and Figures 5 and 6 shows the set-points tracking behavior. Because existence interaction with inputs and out puts, Matrixes K_p, K_i and K_d are not diagonal. Rise time, Overshot and steady state error of step response of system are suitable. Control functions are pieces wise constant, system states and outputs are differentiable functions.

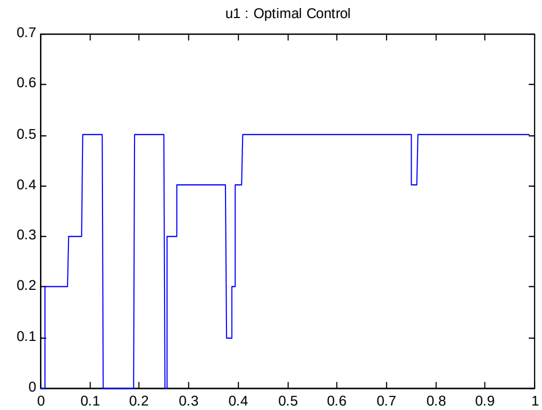

Fig. 1: First Control input.

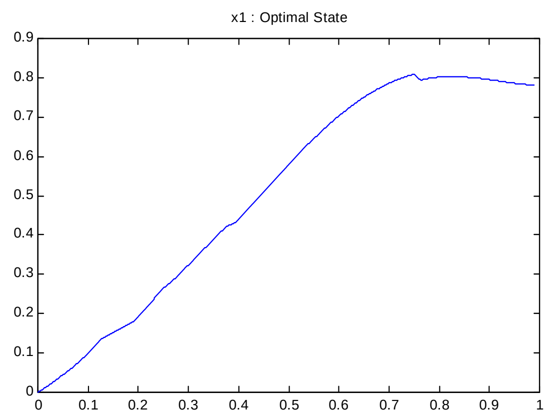

Fig. 2: First state variable.

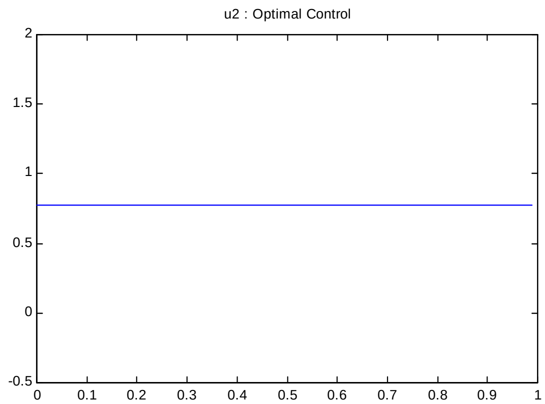

Fig. 3: Second control input.

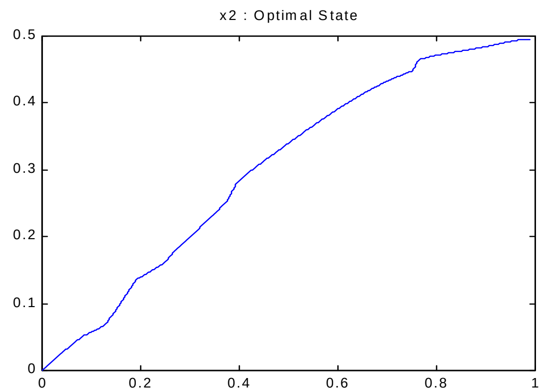

Fig. 4: Second state variable.

Note that: control and state functions are plotted respect to with $\tau = \frac{t - t_a}{t+1}$.

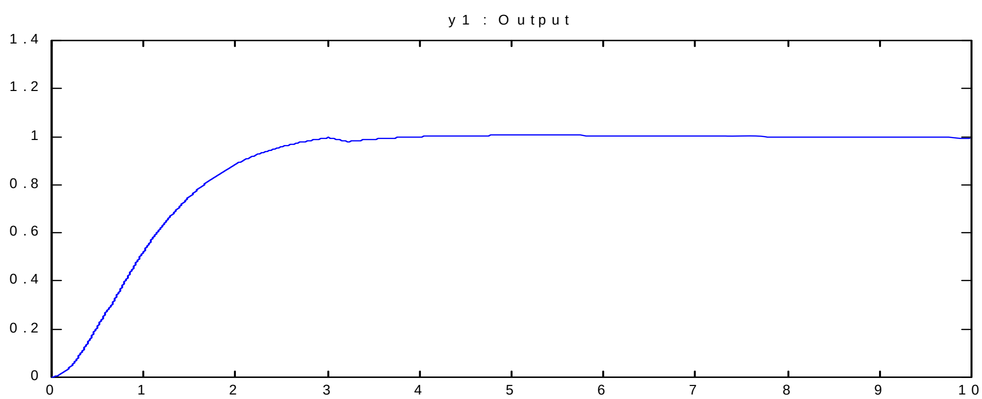

Fig. 5: First output.

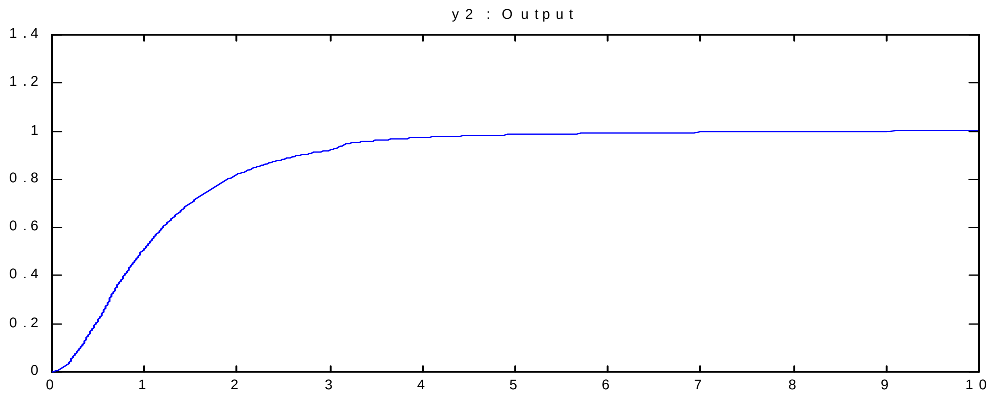

Fig. 6: Second output.

Conclusion:

In this paper, measure theory is used for the optimal tuning of PID controller parameters both in single input single output and multivariable nonlinear systems. The problem is restated as a nonlinear optimal control with infinite time horizon, and a suitable change of variable changes the time horizon to the set [0,1). Then the time horizon is transferred to [0, 1-ε], with an acceptable approximation. Solving an optimization problem with free end point, and using its solution another optimization problems are solved which gives the parameters or matrices of the PID controller. Finally, simulation results are used to show the effectiveness of the proposed methodology.

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