

An Iterative Algorithm for Noise Reduction in the System Identification of Shear Structures

¹Behnam Adhami, ²Karen Khanlari and ³Hamed Niroomand

^{1,2}Department of Civil Engineering, Central Tehran Branch, Islamic Azad University, Tehran, Iran.

³Department of Geotechnical Engineering, Faculty of Civil Engineering, Universiti Teknologi Malaysia, Malaysia.

Abstract:The objective of this work was to reduce the noise adverse effect on the "System Identification" (SI) of linear shear structures. Taking into account the fundamental and significant effect of noise attenuation in boosting the levels of precision and the correctness of SI, the proposed method facilitates direct noise attenuation in the domain of time in parallel with the identification of structural system. Since in such fields as "Damage Detection" in structures, identification of the system's characteristic matrices is of the same importance as the identification of the frequency characteristics, or even more, the proposed method tries to identify the matrices of mass, damping and stiffness of shear structures. Efficiency and precision of the method have been examined through application of "closed loop solution" to two structures with analytical model.

Key words: system identification; shear structure; noise reduction; average taking.

INTRODUCTION

"System Identification" (SI) is a process for determining the dynamic characteristics of a system/structure and it deals with building mathematical models from observations. These characteristics include both frequency characteristics (frequencies, mode shapes, and damping ratios) and the system's characteristic matrices (the matrices of mass, viscous damping, stiffness, Coulomb damping or coefficients of friction, and the Duffing stiffness). In such fields as "Damage Detection" (DD) in structures, by identifying the system's characteristic matrices, the intended goals in DD can be achieved. A review of past research work shows that discussions on SI usually involve methods for DD, too.

Among methods that use vibration data, modal analysis-based methods have attracted most attention, where the accuracy and capability of the structural evaluation mainly depends on the accuracy of the system identification technique which is used to estimate the modal properties on the basis of the measured data. Most system identification methods employ linear systems, although some research has been performed in the nonlinear field (Kerschen, G., 2006). As the modal parameters can describe the behavior of a linear system for any input, the linear system identification usually generates the modal properties (frequency characteristics). Some of the developed methods are the following: Eigen system realization algorithms (ERA) (Juang, J.S., 1985; He, X., 2004), the Peak Picking method (Bisht Saurabh, S., 2005), wavelet analyses (Daubechies, I., 1992; Graps, A., 1995; Hera, A., 2004; Zabel, V., 2005; Tsai, C.H., 1999; Basu, B., 2008), the Hilbert Huang transform (Huang, N.E., 1998), Parametric System Identification (Ljung, L., 1999), intelligent systems including Artificial Neural Networks (ANN), genetic algorithms, evolutionary strategies and fuzzy systems (Ghafory-Ashstiany, M., 2000; Marwala, T., 1999; Tang, H., 2008; Malinowski, P., 2009), vibration-based methods (Shye, K., 1987; Overbey, L.A., 2008; Zimin, V.D., 2009), and optimization algorithms (Gopaluni, R.B., 2009; Gopaluni, R.B., 2009). For each special case, one can select an appropriate system identification method, based on its reliability, accuracy and capability. In other words, due to the individual limitation of these methods, each can be applied to a special type of system, and the method which is applied can vary from one case to another case.

Despite their advantages, these methods cannot be used to determine with certainty whether any damage actually exists, since the frequency, and consequently the methods that measure its error, are not sensitive to slight or moderate structural damage. One of the main challenges in system identification is how to develop a method that could accurately defined the mass, stiffness and damping values of the system. Past studies have shown that changes in the structure's stiffness, as well as any changes in the damping, could be considered as good indicators in the damage detection of the linear system. On the other hand, in the inverse solution of the problem of identifying the system's characteristic matrices (M, C and K), one of the problems affecting the accuracy of the system's equation of motion is the existence of unknown and inevitable noise in the measured input and output data, which has an adverse effect on most of the existing identification methods. These are the main reasons for the development of the proposed method. Noise filtration in the frequency domain, although

Corresponding Author: Hamed Niroomand, Department of Geotechnical Engineering, Faculty of Civil Engineering, Universiti Teknologi Malaysia, Malaysia.
E-mail: niroomandh@gmail.com

leading to smooth measured data, may also alter the frequency content and reduce the levels of reliability of the identification process. The proposed method conducts the noise reduction process in parallel with the system identification. Considering that many building structures can be modeled as shear frames (stick models), this article tries to devise a method which could identify the system properties of shear structures with relatively good accuracy.

2. Proposed Methodology:

Figure (1) presents the proposed method for noise reduction in the system identification of shear structures. In the proposed method, the system identification is based on the inverse solution of the equation of motions of a Multi Degree of Freedom (MDOF) structure with a known response (the displacements $u(t)$, velocities $\dot{u}(t)$ and accelerations $\ddot{u}(t)$) corresponding to all known degrees of freedom due to a known input force with the same degrees of freedom. The equation of motion of an N -DOF linear elastic system for N_0 time steps, after the accumulation of individual response time vectors under $N \times N_0$ rectangular matrices ($[\ddot{u}(t)], [\dot{u}(t)]$ & $[u(t)]$), can be written as follows:

$$M_{N \times N} [\ddot{u}]_{N \times N_0} + C_{N \times N} [\dot{u}]_{N \times N_0} + K_{N \times N} [u]_{N \times N_0} = [f]_{N \times N_0} \Rightarrow PR = f \tag{1}$$

where the mass (M), damping (C) and stiffness (K) matrices are unknown but can be identified through matrix-assisted inverse solutions.

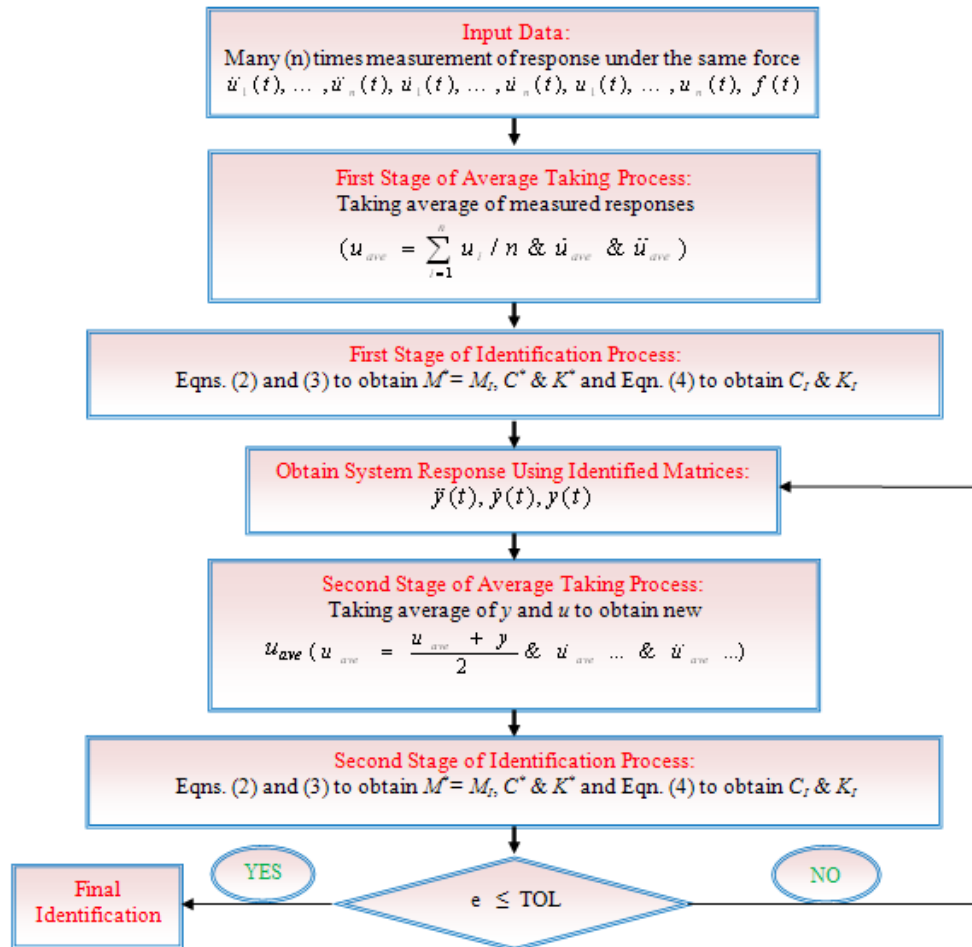


Fig. 1: The proposed noise reduction process.

Eqn. (1) can be written as $PR=f$, where P is $[M C K]$ and R is $[[\ddot{u}] \quad [\dot{u}] \quad [u]]^T$. Post-multiplying $PR=f$ by R^{-1} results in the unknown characteristic matrices: $P=f R^{-1}$. It has been shown that the system property (characteristic) matrices can be identified precisely when there are no noises in the input and output measurement. In practice the measured responses data are mixed with inevitable input and output random noises which cause the inequality in the equation of motion. This level of noise cause ill conditioning of the inverse problem. Matrix-assisted inverse solutions are highly sensitive to large noises which cause very large errors in identified system properties due to accumulation of errors in matrix inverse and multiplication operations.

In order to conduct the noise reduction process in the proposed method, an average taking technique has been used which has two stages, in the first stage, at the beginning of the algorithm, the system noisy responses are measured many times under the same random load $f(t)$ and the average of response is calculated separately (average of accelerations $\ddot{u}_{ave}(t)$, velocities $\dot{u}_{ave}(t)$ and displacements $u_{ave}(t)$).

The effect of number of the measurements on the final identification error has been assessed in a closed loop solution. As can be seen from Figure (2) as an example, the result of the identified stiffness and damping values of eighth story of an eight story shear structure with 5% noise level, shows that the error rate, decreases towards small percentages as the number of measurements increases. The argument behind the average taking is that the ensemble average of a large number of white noises inclines towards zero or base line. In this example, practically, 100 times measurement, seems to be acceptable.

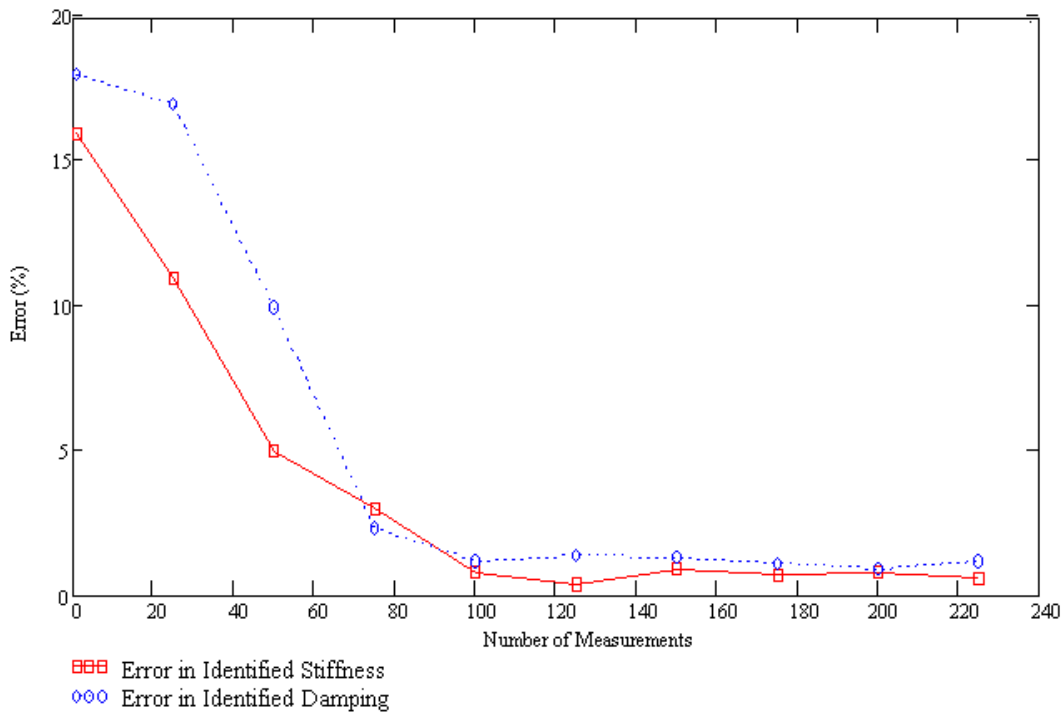


Fig. 2: Error variation in the identified stiffness and damping with the number of measurements at the eighth story of an eight story shear structure with 5% noise level.

After the first stage of average taking, by $\ddot{u}_{ave}(t), \dot{u}_{ave}(t) \& u_{ave}(t)$ (instead of $\ddot{u}(t), \dot{u}(t) \& u(t)$), and by the same $f(t)$, the equation $P=f R^{-1}$ is formed and Pre-multiplied by the unit vector $\langle 1 \dots 1 \rangle_{1 \times N}$ that results in the following expression:

$$\{1\}^T P = \{1\}^T f R^{-1} = \langle m_1 \quad m_2 \quad \dots \quad m_N \quad c_1 \quad c_2 \quad \dots \quad c_N \quad k_1 \quad k_2 \quad \dots \quad k_N \rangle_{1 \times 3N} \quad (2)$$

Using the first, second and third N-terms of Eqn. (2), respectively, results in the diagonalized matrices of M^*, C^* and K^* with these values along their diagonals (Eqn. 3):

$$M^* = \begin{bmatrix} m_1 & 0 & \dots & 0 \\ 0 & m_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & m_N \end{bmatrix}_{N \times N} \quad C^* = \begin{bmatrix} c_1 & 0 & \dots & 0 \\ 0 & c_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & c_N \end{bmatrix}_{N \times N} \quad K^* = \begin{bmatrix} k_1 & 0 & \dots & 0 \\ 0 & k_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & k_N \end{bmatrix}_{N \times N} \quad (3)$$

The matrices of C^* and K^* then become symmetric and trigonal through Eqn. (4):

$$C_I = A^T C^* A \quad \& \quad K_I = A^T K^* A \quad (4)$$

where

$$A = \begin{bmatrix} 1 & 0 & \dots & 0 & 0 \\ -1 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 1 & 0 \\ 0 & 0 & \dots & -1 & 1 \end{bmatrix} \quad (5)$$

Using the identified matrices $M^* = M_I$, C_I and K_I under the same $f(t)$, the system response is obtained as $y(t)$, $\dot{y}(t)$ and $\ddot{y}(t)$ according to Eqn. (6):

$$M_I \cdot \ddot{y}(t) + C_I \cdot \dot{y}(t) + K_I \cdot y(t) = f(t) \quad (6)$$

Then in the second stage of average taking process, by taking average of $\ddot{u}_{ave}(t)$, $\dot{u}_{ave}(t)$ & $u_{ave}(t)$ and the corresponded values of $\ddot{y}(t)$, $\dot{y}(t)$ & $y(t)$ ($\frac{u_{ave}(t) + y(t)}{2}, \dots$), the main noise reduces more. A few

Iterations of this process that is a renewed identification of characteristic matrices of system by the average responses and renewed average taking, attenuate the noisy signals without filtration in the frequency domain. With an adequate repetition number based on an appropriate mathematical criterion, like Eqn. (7), after the last iteration of identification, the characteristic matrices of system (M, C and K) are finalized.

$$e = \text{Max} \left(\left| 1 - \left(\frac{|a^{<n>}|}{|a^{<n-1>}|} \right) \right| \right) \leq TOL \quad (7)$$

In Eqn. (7), $a^{<n-1>}$ and $a^{<n>}$, are each identified mass, stiffness and damping values from Eqn. (2) in two consecutive cycle of n and $n-1$, while TOL stands for the tolerance to control the precision of the calculations and e is the maximum of (3×NDOF) quantities of $\left| 1 - \left(\frac{|a^{<n>}|}{|a^{<n-1>}|} \right) \right|$.

4. Numerical Results:

The proposed method has been applied to the identification of a eight and twelve storey linear shear building with the properties that are shown in Table (1). The following assumptions were made in the numerical studies a) A random excitation force is applied at each floor; b) the response is measured at each floor for a hundred times; c) a time step of $\Delta t = 0.02$ sec is assumed; d) 2, 5 and 10 % RMS ambient noise is taken into consideration, and added to the response as the measured input and output signals. Tables 2-a, 2-b and 2-c show errors of the identified system properties of the eight storey building, respectively, for the case of 2%, 5% and 10% noise levels, while Tables 3-a to 3-c show the same system properties' errors for the twelve storey building. The mass, stiffness, and damping matrices were used to calculate the frequencies. Tables (2) and (3) shows that the identified stiffness and damping of the shear buildings are in excellent agreement with the actual values, especially for the first few stories and the mass and frequencies of the identified system are almost equal to the actual values.

Table 1: Models' properties.

Story	m (kgs ² /m)		c (kgs/m)		k (kg/m)		Load Amplitude (kg)	
	8 St. model	12 St. model	8 St. model	12 St. model	8 St. model	12 St. model	8 St. model	12 St. model
1	17	17	210	200	6250	6250	80	100
2	16	17	200	200	6000	6250	70	100
3	15	17	170	200	5750	6000	85	100
4	14	17	160	200	5500	6000	95	100
5	13	16	150	200	5250	6000	35	100
6	12	16	130	200	5000	6000	50	100
7	11	16	110	200	4750	5750	45	100
8	10	16	100	200	4500	5750	65	100
9	-	15	-	200	-	5750	-	100
10	-	15	-	200	-	5750	-	100
11	-	15	-	200	-	5500	-	100
12	-	15	-	200	-	5500	-	100

As can be seen from Tables (2) and (3), using the proposed method, all of the modal frequencies can be identified with an error of less than 0.35% for 8 storey model and 0.3% for 12 storey model. The method can also succeed in identifying mass, stiffness and damping values, respectively with maximum errors of 0.242%, 2.217% and 1.404% for 8 storey model and 0.146%, 2.518% and 2.74% for 12 storey model. It can be concluded that the errors of the frequency and mass of the 12 storey model can be identified more accurately than the of the 8 storey model, while this conclusion is contrary to the stiffness and damping.

Table (2-a): Errors of the identified properties of the eight storey shear building for the case of 2% RMS ambient noise.

Story for m,c,k / Mode for frequency	Mass (kg.s ² /cm)		Stiffness (kg/cm)		Damping (kg.s/cm)		Frequency (rad/s)	
	Identified	Error (%)	Identified	Error (%)	Identified	Error (%)	Identified	Error (%)
1	16.9994	0.0034	6243	0.107	210.036	0.0171	4.02810	0.0036
2	16.0022	0.0142	6008	0.145	200.032	0.0159	10.9482	0.0113
3	14.9995	0.0035	5757	0.1204	170.489	0.2879	17.6764	0.0133
4	14.0005	0.0038	5489	0.1929	160.057	0.0358	23.8381	0.0257
5	12.9983	0.0134	5254	0.0808	149.139	0.5734 ⁺	29.1751	0.0364 ⁺
6	11.9992	0.0069	4992	0.1511	129.881	0.0917	33.5479	0.0291
7	11.0033	0.0297 ⁺	4769	0.4084 ⁺	110.0349	0.0318	36.7412	0.0315
8	10.0003	0.0035	4497	0.0669	99.77551	0.2245	39.0314	0.0275

*Maximum Error

Table (2-b): Errors of the identified properties of the eight storey shear building for the case of 5% RMS ambient noise.

Story for m,c,k / Mode for frequency	Mass (kg.s ² /cm)		Stiffness (kg/cm)		Damping (kg.s/cm)		Frequency (rad/s)	
	Identified	Error (%)	Identified	Error (%)	Identified	Error (%)	Identified	Error (%)
1	17.014	0.08 ⁺	6238	0.185	209.268	0.348	4.028	0.006
2	15.999	0.006	6023	0.378	200.054	0.027	10.945	0.016
3	14.996	0.029	5705	0.779	169.807	0.113	17.677	0.017
4	13.996	0.029	5533	0.595	159.926	0.047	23.813	0.077 ⁺
5	13.008	0.065	5274	0.473	149.656	0.230	29.169	0.056
6	11.995	0.038	4956	0.883 ⁺	129.602	0.307	33.546	0.024
7	11.004	0.039	4774	0.506	109.614	0.351 ⁺	36.733	0.010
8	9.996	0.044	4487	0.281	99.941	0.060	39.017	0.010

* Maximum Error

Table (2-c): Errors of the identified properties of the eight storey shear building for the case of 10% RMS ambient noise.

Story for m,c,k / Mode for frequency	Mass (kg.s ² /cm)		Stiffness (kg/cm)		Damping (kg.s/cm)		Frequency (rad/s)	
	Identified	Error (%)	Identified	Error (%)	Identified	Error (%)	Identified	Error (%)
1	16.981	0.111	6258	0.125	210.300	0.143	4.029	0.035
2	15.995	0.033	6030	0.500	198.884	0.558	10.947	0.005
3	14.995	0.032	5730	0.340	167.614	1.404 ⁺	17.687	0.074
4	14.011	0.076	5508	0.152	161.044	0.653	23.803	0.124
5	13.019	0.151	5258	0.159	152.079	1.386	29.090	0.330 ⁺
6	11.971	0.242 ⁺	4916	1.681	130.087	0.067	33.511	0.081
7	11.006	0.057	4853	2.217 ⁺	108.559	1.311	36.753	0.065
8	9.9930	0.065	4409	2.014	99.301	0.697	39.008	0.033

* Maximum Error

Table (3-a): Errors of the identified properties of the twelve storey shear building for the case of 2% RMS ambient noise.

Story for m,c,k / Mode for frequency	Mass (kg.s ² /cm)		Stiffness (kg/cm)		Damping (kg.s/cm)		Frequency (rad/s)	
	Identified	Error (%)	Identified	Error (%)	Identified	Error (%)	Identified	Error (%)
1	17.002	.012	6261	0.174	199.223	0.388	2.477	0.003
2	17.000	0	6222	0.443	200.398	0.2	7.178	0.005
3	16.999	.004	6001	0.024	200.130	0.065	11.801	0.014
4	16.999	.006	6004	0.058	200.733	0.366	16.285	0.028
5	16.000	.002	6029	0.491	200.978	0.189	20.461	0.018
6	15.995	.033 ⁺	5964	0.597 ⁺	199.075	0.463 ⁺	24.338	0.08
7	15.997	.017	5772	0.390	200.049	0.025	27.942	0.03
8	16.003	.017	5746	0.069	199.755	0.122	30.934	0.028
9	14.997	.02	5759	0.161	200.038	0.019	33.476	0.029
10	15.000	.002	5751	0.019	200.288	0.145	35.473	0.049 ⁺
11	14.997	.021	5477	0.424	200.872	0.436	37.049	0.004
12	15.000	.002	5509	0.160	200.267	0.133	37.988	0.009

* Maximum Error

Table (3-b): Errors of the identified properties of the twelve storey shear building for the case of 5% RMS ambient noise.

Story for m,c,k / Mode for frequency	Mass (kg.s ² /cm)		Stiffness (kg/cm)		Damping (kg.s/cm)		Frequency (rad/s)	
	Identified	Error (%)	Identified	Error (%)	Identified	Error (%)	Identified	Error (%)
1	16.995	0.029	6275	0.403	203.522	1.774	2.477	0.024
2	17.001	0.005	6264	0.226	197.016	1.487	7.178	0.024
3	17.000	0	5937	1.042	198.883	0.561	11.800	0.064
4	17.009	0.051	6066	1.084	198.992	0.501	16.285	0.155
5	15.996	0.024	6000	0.015	200.746	0.373	20.461	0.230 ⁺
6	15.992	0.052	5874	2.095 ⁺	194.984	2.508 ⁺	24.338	0.064
7	15.993	0.041	5793	0.742	198.190	0.906	27.943	0.133
8	16.001	0.007	5794	0.752	201.722	0.859	30.934	0.113
9	15.000	0.003	5684	1.13	198.155	0.919	33.476	0.082
10	15.006	0.038	5820	1.196	201.462	0.735	35.473	0.043
11	14.996	0.028	5450	0.891	200.107	0.060	37.049	0.061
12	15.010	0.066 ⁺	5540	0.67	200.776	0.391	37.988	0.032

* Maximum Error

Table (3-c): Errors of the identified properties of the twelve storey shear building for the case of 10% RMS ambient noise.

Story for m,c,k / Mode for frequency	Mass (kg.s ² /cm)		Stiffness (kg/cm)		Damping (kg.s/cm)		Frequency (rad/s)	
	Identified	Error (%)	Identified	Error (%)	Identified	Error (%)	Identified	Error (%)
1	16.975	0.146 ⁺	6274	0.374	199.477	0.304	2.478	0.047
2	16.982	0.104	6208	0.635	198.959	0.546	7.176	0.028
3	16.981	0.110	5998	0.087	198.006	1.005	11.809	0.048
4	16.999	0.0006	5967	0.535	199.097	0.467	16.270	0.066
5	15.994	0.038	6151	2.518 ⁺	197.118	1.453	20.475	0.050
6	16.012	0.074	5937	1.030	202.278	1.149	24.385	0.273 ⁺
7	16.012	0.073	5791	0.675	199.458	0.278	27.883	0.241
8	15.990	0.061	5656	1.614	202.413	1.215	30.976	0.165
9	15.003	0.019	5881	2.272	199.632	0.194	33.504	0.055
10	14.997	0.019	5687	1.109	197.944	1.048	35.426	0.180
11	14.978	0.145	5439	0.940	205.499	2.740 ⁺	37.076	0.066
12	14.997	0.023	5515	0.052	196.803	1.603	37.996	0.031

* Maximum Error

5. Conclusions:

The proposed method by which the instrumental noises reduce, can be used to identify the individual properties of shear structures in terms of their mass, stiffness, damping matrices and frequencies using the data from the measured input and output response. However it is necessary to determine the overall topology of any given structure, as well as the number and location of its degrees of freedom. Taking into account this topology, the method of installation of excitatory-sensor equipment is designed. The obtained results show that the presented method can be effective, and can achieve precise results, given the existence of large systemic noises. The results of the studies have shown that mass matrices can be identified more accurately than stiffness matrices, whereas stiffness matrices can be identified more accurately than damping matrices. At increased

noise levels, identification errors increase and by increasing structures stories, maximum errors of frequency and mass decrease while maximum errors of stiffness and damping increase.

REFERENCES

- Basu, B., S. Nagarajaiah, A. Chakraborty, 2008. Online Identification of Linear Time-varying Stiffness of Structural Systems by Wavelet Analysis. *Structural Health Monitoring*, 7: 21-36.
- Bisht Saurabh, S., 2005. Methods for Structural Health Monitoring and Damage Detection of Civil and Mechanical Systems. Thesis submitted to the Faculty of the Virginia Polytechnic Institute and State University: Blacksburg, Virginia.
- Daubechies, I., 1992. Ten lectures on wavelets. CBMS-NSF Regional Conference Series in Applied Mathematics, Society for Industrial and Applied Mathematics: Philadelphia, PA.
- Ghafory-Ashtiany, M., K. Saberi-Haghighi, 2000. Damage Detection in Damaged Structures Using Neural Network and Genetic Algorithm (In Persian). International Institute of Earthquake Engineering and Seismology (IIIES): Tehran, Iran, Publication No: 79-2000-2.
- Gopaluni, R.B., 2009. Comparison of Expectation-Maximization based parameter estimation using Particle Filter, Unscented and Extended Kalman Filtering techniques. Proceedings of IFAC Symposium on System Identification: St. Malo, France.
- Gopaluni, R.B., T. Schon, A.G. Wills, 2009. Particle Filter Approach to Nonlinear System Identification under Missing Observations with a Real Application. Proceedings of IFAC Symposium on System Identification: St. Malo, France.
- Graps, A., 1995. An introduction to wavelets. *IEEE Computational Science and Engineering*, 2(2): 50-61.
- He, X., B. Moaveni, J.P. Conte, A. Elgamal, S. Masri, 2004. System identification of Vincent Thomas Bridge using simulated wind response data. Second International Conference on Bridge Maintenance, Safety and Management (IABMAS'04): Kyoto, Japan, 19-22.
- Hera, A., Z. Hou, 2004. Application of wavelet approach for ASCE structural health monitoring benchmark studies. *Journal of Engineering Mechanics*, 130(1): 96-104.
- Huang, N.E., Z. Shen, S.R. Long, M.C. Wu, H.H. Shih, Q. Zheng, N.C.Yen, C.C. Tung, H.H. Liu, 1998. The empirical model decomposition and the Hilbert spectrum for nonlinear and non-stationary time series analysis. Proceedings of the Royal Society of London, Series A, Mathematical, Physical and Engineering Sciences, 454: 903-995.
- Juang, J.S., R.S. Pappa, 1985. An eigensystem realization algorithm for modal parameter identification and model reduction. *AIAA Journal of Guidance, Control and Dynamics*, 12: 620-627.
- Kerschen, G., K. Wordenb, A.F. Vakakis, J.C. Golinval, 2006. Past, present and future of nonlinear system identification in structural dynamics. *Mechanical Systems and Signal Processing*, 20: 505-592.
- Ljung, L., 1999. *System Identification: Theory for the User*. Prentice Hall PTR: Upper Saddle River, NJ.
- Malinowski, P., T. Wandowski, I. Trendafilova, W. Ostachowicz, 2009. A Phased Array-based Method for Damage Detection and Localization in Thin Plates. *Structural Health Monitoring*, 8: 5-15.
- Marwala, T., H.E.M. Hunt, 1999. Fault Identification Using Finite Element Models and Neural Networks. *Mechanical Systems and Signal Processing*, 13(3): 475-490.
- Overbey, L.A., M.D. Todd, 2008. Damage Assessment using Generalized State-Space Correlation Features. *Structural Health Monitoring*, 7: 347-363.
- Shye, K., M. Richardson, 1987. Mass, Stiffness, and Damping Matrix Estimates from Structural Measurements. Structural Measurement Systems, Inc: San Jose, California.
- Tang, H., S. Xue, C. Fan, 2008. Differential evolution strategy for structural system identification. *Journal of Computers and Structures*, 86(21-22): 2004-2012. ISSN:0045-7949.
- Tsai, C.H., D.S. Hsu, 1999. Damage Diagnosis of Existing Reinforced Concrete Structures. Civil-Comp Ltd.: Edinburgh, Scotland.
- Zabel, V., 2005. An Application of Discrete Wavelet Analysis and Connection Coefficients to Parametric System Identification. *Structural Health Monitoring*, 4: 5-18.
- Zimin, V.D., D.C. Zimmerman, 2009. Structural Damage Detection Using Time Domain Periodogram Analysis. *Structural Health Monitoring*, 8: 125-135.