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Fuzzy and Fully Fuzzy Diagonal Matrices and Solving Related Systems

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Abstract: The simplest linear system in crisp case occur when the coefficient matrix is diagonal. But when system is fuzzy or fully fuzzy its solving will be more difficult. In this paper we present the straight solutions and properties of these systems.

Key words: Fuzzy matrix, LR Fuzzy number, Diagonal matrix, Fuzzy system, Fully fuzzy system.

INTRODUCTION

Fuzzy sets provide a widely appreciated tool to introduce uncertain parameters into engineering systems. In many positions, some parameters of the system should be represented by fuzzy numbers rather than crisp numbers. Hence, it is immensely important to develop crisp definitions in fuzzy definitions. Section 2 is some preliminaries about fully fuzzy linear systems and fuzzy systems. In section 3, we find the solution FFLS with nonnegative diagonal coefficient matrix and a theorem that clarifies its consistency. Finally, the solution of FLS and its kind are considered in section 4.

2 Preliminaries:

2.1 Fully Fuzzy Systems:

In this section, we recall the basic notions of fuzzy numbers.

***Definition 2.1*:**

A fuzzy number $\tilde{A}$ *is an upper semi continuous, normal and convex fuzzy subset of the real line* $\Re$ *so that* $\mu_{\tilde{A}} : \Re \longrightarrow [0,1]$ *where* $\mu_{\tilde{A}}(x)$ *is the membership function of* $\tilde{A}$*, i.e. there exists an* x *so that* $\mu_{\tilde{A}}(x) = 1$*, and* $\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq min\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\}$*, for* $\lambda \in [0,1]$.

In addition, a fuzzy number $\tilde{A}$ is called positive (negative), shown as $\tilde{A} > 0$ $(\tilde{A} < 0)$, if its membership function $\mu_{\tilde{A}}(x)$ satisfies $\mu_{\tilde{A}}(x) = 0,\ \forall x < 0\ \ (\forall x > 0)$.

***Definition 2.2*:**

(LR fuzzy number). A fuzzy number $\tilde{A}$ *is said to be an LR fuzzy number if*

$$\mu_{\tilde{A}}(x) = \begin{cases} L\left(\dfrac{a-x}{\alpha}\right), & x \leq a,\ \alpha > 0, \\ R\left(\dfrac{x-a}{\beta}\right), & x \geq a,\ \beta > 0, \end{cases}$$

where a is the mean value of $\tilde{A}$ and α and β are left and right spreads, respectively. The function L(.), which is called left shape function, satisfies:

(1) L(x)=L(-x),
(2) L(0)=1 and L(1)=0,
(3) L(x) is non-increasing on $[0,\infty)$.

The definition of a right shape function R(.) is usually similar to that of L(.).

The mean value, left and right spreads, and the shape function of an LR fuzzy number $\tilde{A}$ is symbolically shown as

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$\tilde{A} = (a, \alpha, \beta)_{LR}$.

Clearly, $\tilde{A} = (a, \alpha, \beta)_{LR}$ is positive, if and only if, $a - \alpha > 0$ (since L(1)=0).

Also, two LR fuzzy numbers $\tilde{A} = (a, \alpha, \beta)_{LR}$ and $\tilde{B} = (b, \gamma, \delta)_{LR}$ are said to be equal, if and only if $a = b,\ \alpha = \gamma,\ \beta = \delta.$

On the other hand, since each fuzzy number is a set, we can define its subset as follows:

An LR fuzzy number $\tilde{A} = (a, \alpha, \beta)_{LR}$ is said to be a subset of the LR fuzzy number $\tilde{B} = (b, \gamma, \delta)_{LR}$, if and only if $a - \alpha \geq b - \gamma$ and $a + \beta \leq b + \delta$.

During recent years the fuzzy arithmetic has been extended as an important tool of in fuzzy optimization and control theory. Based on the extension principle, Dubois and Prade designed the following exact formulas for adding LR fuzzy numbers and scalar multiplication. They also introduced an approximate formula for multiplying the LR fuzzy numbers (J.J. Buckley, 1991).

Consider two LR fuzzy numbers $\tilde{A} = (a, \alpha, \beta)_{LR}$ and $\tilde{B} = (b, \gamma, \delta)_{LR}$.

- Addition

$$(a, \alpha, \beta)_{LR} \oplus (b, \gamma, \delta)_{LR} = (a + b, \alpha + \gamma, \beta + \delta)_{LR}. \tag{1}$$

- Multiplication

$\tilde{A} \tilde{\otimes} \tilde{B},\quad \tilde{A} \otimes \tilde{B},$

the approximate formulas for the extended multiplication of two LR fuzzy numbers can be summarized as follows:

i) $(a, \alpha, \beta)_{LR} \tilde{\otimes} (b, \gamma, \delta)_{LR} = (ab, a\gamma + b\alpha, a\delta + b\beta)_{LR}.$

ii) If $\tilde{A} > 0$ and $\tilde{B} > 0$, then

$$(a, \alpha, \beta)_{LR} \otimes (b, \gamma, \delta)_{LR} \cong (ab, a\gamma + b\alpha, a\delta + b\beta)_{LR}. \tag{2}$$

If $\tilde{A} < 0$ and $\tilde{B} > 0$, then

$$(a, \alpha, \beta)_{RL} \otimes (b, \gamma, \delta)_{LR} \cong (ab, b\alpha - a\delta, b\beta - a\gamma)_{RL}. \tag{3}$$

- Scalar multiplication $\lambda \tilde{\otimes} \tilde{A}, \lambda \otimes \tilde{A}$

i) $\lambda \tilde{\otimes} (a, \alpha, \beta)_{LR} = (\lambda a, \lambda\alpha, \lambda\beta)_{LR},$

ii) $$\lambda \otimes (a, \alpha, \beta)_{LR} = \begin{cases} (\lambda a, \lambda\alpha, \lambda\beta)_{LR}, & \lambda > 0, \\ (\lambda a, -\lambda\beta, -\lambda\alpha)_{RL}, & \lambda < 0. \end{cases} \tag{4}$$

Definition 2.3:

A crisp matrix A is called inverse-nonnegative if $A \geq 0$ and $A^{-1} \geq 0$.

2.2 Fully Fuzzy Linear Systems:

Definition 2.4:

A matrix $\tilde{A} = (\tilde{a}_{ij})$ is called a fuzzy matrix if each element of $\tilde{A}$ is a fuzzy number.

The fuzzy matrix $\tilde{A}$ will be positive (negative) and is shown by $\tilde{A} > 0$ ($\tilde{A} < 0$) if each element of $\tilde{A}$ be a positive (negative) number. The nonnegative and nonpositive fuzzy matrices may be defined similarly.

We may denote $\tilde{A} = (\tilde{a}_{ij})$ that $\tilde{a}_{ij} = (a_{ij}, \alpha_{ij}, \beta_{ij})_{LR}$, with new notation $\tilde{A} = (A, M, N)$, where A, M and N are three crisp matrices, with the same size of A, so that $A = (a_{ij}),\ M = (\alpha_{ij})$ and $N = (\beta_{ij})$, are called the $center$ and the $right$ and the $leftspreadmatrices$, respectively.

Definition 2.5:

A square fuzzy matrix $\tilde{A} = (\tilde{a}_{ij})$ *is an upper (lower) triangular fuzzy matrix if*

$\tilde{a}_{ij} = \tilde{0} = (0,0,0),\ \forall i > j\ (\forall i < j).$

Definition 2.6:

Let $\tilde{A} = (\tilde{a}_{ij})$ *and* $\tilde{B} = (\tilde{b}_{ij})$ *be two* $m \times n$ *and* $n \times p$ *fuzzy matrices respectively. We define* $\tilde{A} \otimes \tilde{B} = \tilde{C} = (\tilde{c}_{ij})$ *which is an* $m \times p$ *matrix where*

$$\tilde{c}_{ij} = \sum_{k=1,\ldots,n}^{\oplus} \tilde{a}_{ik} \otimes \tilde{b}_{kj}.$$

***Definition 2.7*:**

Consider the $n \times n$ *linear system of equations*

$$\begin{cases} (\tilde{a}_{11} \otimes \tilde{x}_1) \oplus (\tilde{a}_{12} \otimes \tilde{x}_2) \oplus \cdots \oplus (\tilde{a}_{1n} \otimes \tilde{x}_n) = \tilde{b}_1, \\ (\tilde{a}_{21} \otimes \tilde{x}_1) \oplus (\tilde{a}_{22} \otimes \tilde{x}_2) \oplus \cdots \oplus (\tilde{a}_{2n} \otimes \tilde{x}_n) = \tilde{b}_2, \\ \qquad\qquad \vdots \\ (\tilde{a}_{n1} \otimes \tilde{x}_1) \oplus (\tilde{a}_{n2} \otimes \tilde{x}_2) \oplus \cdots \oplus (\tilde{a}_{nn} \otimes \tilde{x}_n) = \tilde{b}_n. \end{cases} \tag{5}$$

The matrix form of the above equations is

$$\tilde{A} \otimes \tilde{x} = \tilde{b}, \tag{6}$$

or simply $\tilde{A}\tilde{x} = \tilde{b}$ where the coefficient matrix

$$\tilde{A} = (\tilde{a}_{ij}) = (a_{ij}, \alpha_{ij}, \beta_{ij})_{LR} = (A, M, N), 1 \le i, j \le n$$

is a fuzzy matrix and

$\tilde{x} = (\tilde{x}_i) = (x_i, y_i, z_i)_{LR} = (x, y, z),\ \tilde{b} = (\tilde{b}_i) = (b_i, g_i, h_i)_{LR} = (b, g, h), \quad 1 \le i \le n.$

This system is called a fully fuzzy linear system (FFLS). Also, if each element of $\tilde{A}$ and $\tilde{b}$ is a nonnegative LR fuzzy number, we call the system (6) a nonnegative FFLS.

Definition 2.8:

The fuzzy vector $\tilde{x} = (x, y, z)$ *is said to be a consistent solution of nonnegative FFLS, if* $y > 0,\ z > 0$ *and* $x - y \ge 0$.

2.3 Fuzzy Numbers And Fuzzy Linear Systems:

At first we recall the basic notions of fuzzy numbers arithmetic and fuzzy linear system.

Definition 2.9:

A fuzzy number in parametric form is an ordered pair of functions $(\underline{u}(r), \overline{u}(r)),\ 0 \le r \le 1$ *which satisfies the following requirements:*

1. $\underline{u}(r)$ is a bounded left continuous nondecreasing function on [0,1].
2. $\overline{u}(r)$ is a bounded left continuous nonincreasing function on [0,1].
3. $\underline{u}(r) \le \overline{u}(r),\ 0 \le r \le 1.$

The addition and the scalar multiplication for arbitrary fuzzy numbers, $u = (\underline{u}, \overline{u})$ and $v = (\underline{v}, \overline{v})$ and $\lambda \in \Re^1$ are defined by

$$\begin{cases} (\underline{u+v})(r) = \underline{u}(r) + \underline{v}(r), \\ (\overline{u+v})(r) = \overline{u}(r) + \overline{v}(r), \\ (\underline{\lambda u})(r) = \lambda \underline{u}(r), \quad (\overline{\lambda u})(r) = \lambda \overline{u}(r), \quad \lambda \ge 0, \\ (\underline{\lambda u})(r) = \lambda \overline{u}(r), \quad (\overline{\lambda u})(r) = \lambda \underline{u}(r), \quad \lambda \le 0. \end{cases} \tag{7}$$

A general fuzzy linear system is as follows:

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= \tilde{y}_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= \tilde{y}_2 \\ &\vdots \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n &= \tilde{y}_n, \end{aligned} \tag{8}$$

where the coefficient matrix $A = (a_{ij}),\ 1 \le i, j \le n$ is a crisp $n \times n$ matrix and $\tilde{y} = (\underline{y}_i, \overline{y}_i)$, $1 \le i \le n$ is a fuzzy vector. This system is called a fuzzy linear system (FLS).

Definition 2.10:

Let $X = \left\{ (\underline{x}_i(r), \overline{x}_i(r));\ 1 \le i \le n \right\}$ denotes a solution of $A\tilde{x} = \tilde{y}.$ Then the fuzzy vector $U = \left\{ (\underline{u}_i(r), \overline{u}_i(r));\ 1 \le i \le n \right\}$ defined by

$$\begin{aligned} \underline{u}_i(r) &= min\left\{ \underline{x}_i(r), \overline{x}_i(r), \underline{x}_i(1), \overline{x}_i(1) \right\} \\ \overline{u}_i(r) &= max\left\{ \underline{x}_i(r), \overline{x}_i(r), \underline{x}_i(1), \overline{x}_i(1) \right\} \end{aligned} \tag{9}$$

is called a fuzzy solution of $A\tilde{x} = \tilde{y}.$ If $(\underline{x}_i(r), \overline{x}_i(r));\ 1 \le i \le n$ are all fuzzy numbers and $\underline{u}_i(r) = \underline{x}_i(r), \overline{u}_i(r) = \overline{x}_i(r),\ 1 \le i \le n$ then U is called a strong fuzzy solution. Otherwise, U is a weak fuzzy solution.

3 Fully Fuzzy System With Nonnegative Diagonal Coefficient Matrix:

In this system we analyze the fully fuzzy linear systems(FFLS) $\tilde{A} \otimes \tilde{x} = \tilde{b}$ with nonnegative diagonal coefficient matrix i.e.

$$\begin{pmatrix} (a_1,\alpha_1,\beta_1) & (0,0,0) & \cdots & (0,0,0) \\ (0,0,0) & (a_2,\alpha_2,\beta_2) & \cdots & (0,0,0) \\ \vdots & \vdots & \ddots & \vdots \\ (0,0,0) & (0,0,0) & \cdots & (a_n,\alpha_n,\beta_n) \end{pmatrix} \otimes \begin{pmatrix} (x_1,y_1,z_1) \\ (x_2,y_2,z_2) \\ \vdots \\ (x_n,y_n,z_n) \end{pmatrix} = \begin{pmatrix} (b_1,g_1,h_1) \\ (b_2,g_2,h_2) \\ \vdots \\ (b_n,g_n,h_n) \end{pmatrix}.$$

Let

$$E_1 = \begin{pmatrix} (1,-\frac{\alpha_1}{a_1},-\frac{\beta_1}{a_1}) & (0,0,0) & \cdots & (0,0,0) \\ (0,0,0) & (1,0,0) & \cdots & (0,0,0) \\ \vdots & \vdots & \ddots & \vdots \\ (0,0,0) & (0,0,0) & \cdots & (1,0,0) \end{pmatrix}$$

then

$$A_1 = E_1 \otimes A = \begin{pmatrix} (a_1,0,0) & (0,0,0) & \cdots & (0,0,0) \\ (0,0,0) & (a_2,\alpha_2,\beta_2) & \cdots & (0,0,0) \\ \vdots & \vdots & \ddots & \vdots \\ (0,0,0) & (0,0,0) & \cdots & (a_n,\alpha_n,\beta_n) \end{pmatrix},$$

similarly, let

$$E_2 = \begin{pmatrix} (1,0,0) & (0,0,0) & \cdots & (0,0,0) \\ (0,0,0) & (1,-\frac{\alpha_2}{a_2},-\frac{\beta_2}{a_2}) & \cdots & (0,0,0) \\ \vdots & \vdots & \ddots & \vdots \\ (0,0,0) & (0,0,0) & \cdots & (1,0,0) \end{pmatrix},$$

then

$$A_2 = E_2 \otimes A_1 = \begin{pmatrix} (a_1,0,0) & (0,0,0) & \cdots & (0,0,0) \\ (0,0,0) & (a_2,0,0) & \cdots & (0,0,0) \\ \vdots & \vdots & \ddots & \vdots \\ (0,0,0) & (0,0,0) & \cdots & (a_n,\alpha_n,\beta_n) \end{pmatrix},$$

and finally letting

$$\tilde{E}_n = \begin{pmatrix} (1,0,0) & (0,0,0) & \cdots & (0,0,0) \\ (0,0,0) & (1,0,0) & \cdots & (0,0,0) \\ \vdots & \vdots & \ddots & \vdots \\ (0,0,0) & (0,0,0) & \cdots & (1,-\frac{\alpha_n}{a_n},-\frac{\beta_n}{a_n}) \end{pmatrix},$$

we have

$$\tilde{A}_n = \tilde{E}_n \tilde{\otimes} \tilde{A}_{n-1} = \begin{pmatrix} (a_1,0,0) & (0,0,0) & \cdots & (0,0,0) \\ (0,0,0) & (a_2,0,0) & \cdots & (0,0,0) \\ \vdots & \vdots & \ddots & \vdots \\ (0,0,0) & (0,0,0) & \cdots & (a_n,0,0) \end{pmatrix}.$$

$$\tilde{b}_1 = \tilde{E}_1 \tilde{\otimes} \tilde{b} = \begin{pmatrix} (b_1, g_1 - b_1\frac{\alpha_1}{a_1}, h_1 - b_1\frac{\beta_1}{a_1}) \\ (b_2, g_2, h_2) \\ \vdots \\ (b_n, g_n, h_n) \end{pmatrix},$$

$$\tilde{b}_2 = \tilde{E}_2 \tilde{\otimes} \tilde{b}_1 = \begin{pmatrix} (b_1, g_1 - b_1\frac{\alpha_1}{a_1}, h_1 - b_1\frac{\beta_1}{a_1}) \\ (b_2, g_2 - b_2\frac{\alpha_2}{a_2}, h_2 - b_2\frac{\beta_2}{a_2}) \\ \vdots \\ (b_n, g_n, h_n) \end{pmatrix},$$

$$\tilde{b}_n = \tilde{E}_n \tilde{\otimes} \tilde{b}_{n-1} = \begin{pmatrix} (b_1, g_1 - b_1\frac{\alpha_1}{a_1}, h_1 - b_1\frac{\beta_1}{a_1}) \\ (b_2, g_2 - b_2\frac{\alpha_2}{a_2}, h_2 - b_2\frac{\beta_2}{a_2}) \\ \vdots \\ (b_n, g_n - b_n\frac{\alpha_n}{a_n}, h_n - b_n\frac{\beta_n}{a_n}) \end{pmatrix}.$$

We know that

$$\hat{I} \otimes \tilde{A} = diag[(1,0,0),(1,0,0),\ldots,(1,0,0)] \otimes \tilde{A} = \tilde{A}, \tag{10}$$

and

$$diag[(a_1,0,0),(a_2,0,0),\ldots,(a_n,0,0)] \otimes diag[(\frac{1}{a_1},0,0),(\frac{1}{a_2},0,0),\ldots,(\frac{1}{a_n},0,0)] = \hat{I} \tag{11}$$

thus if we set

$$\tilde{A}_n^{-1} = diag[(\frac{1}{a_1},0,0),(\frac{1}{a_2},0,0),\ldots,(\frac{1}{a_n},0,0)] \tag{12}$$

then we have from (10), (11), (12),

$$\tilde{x} = \tilde{A}_n^{-1}\tilde{b}_n = \begin{pmatrix} \frac{1}{a_1}(b_1, g_1 - b_1\frac{\alpha_1}{a_1}, h_1 - b_1\frac{\beta_1}{a_1}) \\ \vdots \\ \frac{1}{a_n}(b_n, g_n - b_n\frac{\alpha_n}{a_n}, h_n - b_n\frac{\beta_n}{a_n}) \end{pmatrix}. \tag{13}$$

Theorem 3.1:

If $a_i \neq o, i = 1,\ldots,n$ *then (13) is the solution of fully fuzzy system with nonnegative diagonal fuzzy matrix.*

An important discussion in fuzzy systems is the consistency of solutions which will be established by the following.

Theorem 3.2:

The fully fuzzy system (6) with nonnegative diagonal coefficient matrix has a consistent solution iff

$$\begin{cases} g_i a_i > b_i \alpha_i \\ h_i a_i > b_i \beta_i \\ b_i(1+\frac{\alpha_i}{a_i}) \geq g_i \end{cases}, \quad i = 1,\ldots,n.$$

Proof:

From the theorem, the solution of FFLS with nonnegative diagonal fuzzy matrix is as $\tilde{x} = (x, y, z)$ where

$$x = (\frac{b_1}{a_1}, \frac{b_2}{a_2}, \ldots, \frac{b_n}{a_n})^T,$$

$$y = \left(\frac{1}{a_1}(g_1 - b_1\frac{\alpha_1}{a_1}), \frac{1}{a_2}(g_2 - b_2\frac{\alpha_2}{a_2}), \ldots, \frac{1}{a_n}(g_n - b_n\frac{\alpha_n}{a_n})\right)^T,$$

and

$$z = \left(\frac{1}{a_1}(h_1 - b_1\frac{\beta_1}{a_1}), \frac{1}{a_2}(h_2 - b_2\frac{\beta_2}{a_2}), \ldots, \frac{1}{a_n}(h_n - b_n\frac{\beta_n}{a_n})\right)^T,$$

from hypothesis we have $g_i a_i > b_i \alpha_i$ thus implies $y_i = \frac{1}{a_i}(g_i - b_i \frac{\alpha_i}{a_i}) > 0$ and so $y > 0$. Also $h_i a_i > b_i \beta_i$ yields $z_i = \frac{1}{a_i}(h_i - b_i \frac{\beta_i}{a_i})$ thus $z > 0$. Finally, from $b_i(1 + \frac{\alpha_i}{a_i}) \geq g_i$ we have $\frac{b_i}{a_i} \geq \frac{1}{a_i}(g_i - b_i \frac{\alpha_i}{a_i})$, or equivalently $x - y \geq 0$.

4 Fuzzy System:

In this section we have some theorems that analyze the fuzzy systems with diagonal matrix, i.e.

$$A\tilde{x} = \tilde{y} \tag{14}$$

where

$$A = diag(a_1, \ldots, a_n), \quad \tilde{y} = \left((\underline{y}_1, \overline{y}_1), \ldots, (\underline{y}_n, \overline{y}_n)\right)^T.$$

Let

$$I = \{i : a_i > 0, i = 1, \ldots, n\}, \; J = \{i : a_i < 0, i = 1, \ldots, n\}, \tag{15}$$

then

$$I \cup J = \{1, \ldots, n\}, \; I \cap J = \varnothing$$

if A^{-1} exists.

Theorem 4.1:

Let A be a nonsingular diagonal matrix, then (14) has a solution

$$\tilde{x} = \left((\underline{x}_1, \overline{x}_1), \ldots, (\underline{x}_n, \overline{x}_n)\right)^T$$

with

$$\underline{x}_i = \begin{cases} \frac{\underline{y}_i}{a_i}, & i \in I \\ \frac{\overline{y}_i}{a_i}, & i \in J \end{cases}, \quad \overline{x}_i = \begin{cases} \frac{\overline{y}_i}{a_i}, & i \in I \\ \frac{\underline{y}_i}{a_i}, & i \in J \end{cases}, \quad i = 1, \ldots, n.$$

Proof:

Without loss of generality, suppose

$$I = \{1, \ldots, n\}, \; J = \{k+1, \ldots, n\}$$

from hypothesis we have

$$\underline{x}_i = \begin{cases} \frac{\underline{y}_i}{a_i}, & i = 1, \ldots, k \\ \frac{\overline{y}_i}{a_i}, & i = k+1, \ldots, n, \end{cases}$$

and

$$\bar{x}_i = \begin{cases} \dfrac{\bar{y}_i}{a_i}, & i = 1,\ldots,k \\ \dfrac{\underline{y}_i}{a_i}, & i = k+1,\ldots,n \end{cases}.$$

Therefore

$$diag(a_1,\ldots,a_n)\times\begin{pmatrix} (\underline{x}_1,\bar{x}_1) \\ \vdots \\ (\underline{x}_k,\bar{x}_k) \\ (\underline{x}_{k+1},\bar{x}\ k+1) \\ \vdots \\ (\underline{x}_n,\bar{x}_n) \end{pmatrix} = \begin{pmatrix} a_1(\underline{x}_1,\bar{x}_1) \\ \vdots \\ a_k(\underline{x}_k,\bar{x}_k) \\ a_{k+1}(\underline{x}_{k+1},\bar{x}\ k+1) \\ \vdots \\ a_n(\underline{x}_n,\bar{x}_n) \end{pmatrix}$$

$$= \begin{pmatrix} (a_1\underline{x}_1, a_1\bar{x}_1) \\ \vdots \\ (a_k\underline{x}_k, a_k\bar{x}_k) \\ (a_{k+1}\bar{x}_{k+1}, a_{k+1}\underline{x}_{k+1}) \\ \vdots \\ (a_n\bar{x}_n, a_n\underline{x}_n) \end{pmatrix} = \begin{pmatrix} (\underline{y}_1,\bar{y}_1) \\ \vdots \\ (\underline{y}_k,\bar{y}_k) \\ (\underline{y}_{k+1},\bar{y}\ k+1) \\ \vdots \\ (\underline{y}_n,\bar{y}_n) \end{pmatrix}.$$

Now we present a theorem which tells us whether the solution is strong or weak.

Theorem 4.2:

If a solution of fuzzy linear system with diagonal coefficient matrix exists, then it solution is a strongly fuzzy solution.

Proof: Let

$$I = \{i_1, i_2, \ldots, i_k\},\ \ J = \{i_{k+1},\ldots,i_n\}.$$

Since $a_{i_j} > 0$ for $j = 1,\ldots,k$, and $\underline{y}_{i_j} \le \bar{y}_{i_j}$ we have

$$\underline{x}_{i_j} = \frac{\underline{y}_{i_j}}{a_{i_j}} \le \frac{\bar{y}_{i_j}}{a_{i_j}} = \bar{x}_{i_j}, = 1,\ldots,k, \tag{16}$$

and since $\underline{y}_{i_j}$ is nondecreasing and $\bar{y}_{i_j}$ is nonincreasing, thus $\underline{x}_{i_j} = \dfrac{\underline{y}_{i_j}}{a_{i_j}}$ and $\bar{x}_{i_j} = \dfrac{\bar{y}_{i_j}}{a_{i_j}}$ are non decreasing and nonincreasing respectively.

For $j = k+1,\ldots,n$ we have $a_{i_j} < 0$ and $\underline{y}_{i_j} \le \bar{y}_{i_j}$ concludes that

$$\underline{x}_{i_j} = \frac{\overline{y}_{i_j}}{a_{i_j}} \leq \frac{\underline{y}_{i_j}}{a_{i_j}} = \overline{x}_{i_j}, = k+1, \ldots, n \tag{17}$$

Similarly, because $\underline{y}_{i_j}$ is nondecreasing and $\overline{y}_{i_j}$ is non increasing, thus $\underline{x}_{i_j} = \frac{\overline{y}_{i_j}}{a_{i_j}}$ and $\overline{x}_{i_j} = \frac{\underline{y}_{i_j}}{a_{i_j}}$ are non decreasing and non increasing respectively.

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