Finite Element Analysis of Switched Reluctance Motor be Control of Firing Angles for Torque Ripple Minimization

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Abstract: In this paper, control of Switched Reluctance Motor by control of firing angles and the effect of variations of stator pole arc for Torque Ripple Reduction is reviewed. Steady state dynamic characterizations of basic switched reluctance motor 6/4 are calculated for three different sizes of stator pole arcs. The results of dynamic analysis indicates that steady state performance of SRM is improved with using of the control of firing angles and stator pole arc being increased by pole shoes. Two-Dimensional Finite Element Method (2D FEM) is utilized to model and control strategies of Switched Reluctance Motor.

Key words: Dynamic Characteristics, Firing Angles, Finite Element Method, Stator Pole Shoes, Switched Reluctance Motor, Voltage Control.

INTRODUCTION

Currently, wide investigation is done on Switched Reluctance Motors (SRM). Switched Reluctance Motors as compared with other Motors have such advantages that have caused, in ten recent years, them to be used increasingly in many industrial fields. Advantages are simple construction of Stator and Rotor, ability to operate over a wide speed range at constant power, high Torque to Weight Ratio, higher reliability and lower cost. Switched Reluctance Motor has salient poles on its Stator and Rotor that causes a large torque ripple and using SRM is limited due to the high torque ripple, acoustic noise and vibration.

The causes of torque ripple in SRM are mainly due to the switching of phase currents into their windings (phase commutation) and the highly nonlinear nature of phase inductance variation when the rotor rotates ([Srinivas and Arumugam, 2005]). Reasons for torque ripple in conventional SRM were reported in paper (Garip et al., 2002), due to saturation and phase commutation. Saturation reduces the torque since torque will not be a function of square of phase current. As far as the phase commutation is concerned, improper transfer of torque production duty between phases causes torque ripple. From the fundamental equation for the instantaneous torque production in SR motors (equation (1)), it is observed if improvements in torque profile are to be achieved, the excitation current i and/or the phase inductance \( L(i, \theta) \) are to be modified (Srinivas and Arumugam, 2003, 2005).

\[
T_{\text{dyn}} = \frac{1}{2} \frac{dL(i, \theta)}{d\theta}
\]  

(1)

Another possible way to suppress the torque ripple can be to control and shape waveforms of currents flowing in SRM’s windings to create instantaneous torque flat in waveforms (Ishikawa et al., 2002).

The torque ripple fundamentally depends upon geometric variables and stator pole arc and rotor pole arc are the most important of the geometric variables. An effective design of SRM requires determination of the set of geometrical parameters, which enable producing maximum electromagnetic torque. Influence of internal diameter of stator, stator’s and rotor’s pole breadth, rotor’s pole height and air gap on electromagnetic torque and torque ripple coefficient by using finite element method is presented in (Bienkowski et al., 2004). Set of motor’s parameters which allow producing high torque is not suitable in respect of torque ripple. Criterion of maximum efficiency is contradicted to criterion of minimum torque ripple. In (Sheth and Rajagopal, 2003) for a 4 phase 8/6 SRM, by keeping the stator pole arcs and excitation current waveform constant, the effect of variation of rotor pole arcs on static torque profile and torque ripple of motor are investigated.

New approaches to study the dynamic and static characteristics of SRM through finite element analysis are presented in (Srinivas and Arumugam, 2003, 2005) that improve torque profile. In the new geometry, pole shoes are affixed to the stator poles. The provision of stator pole shoes smoothen the torque profile in the conduction region, and reduced torque ripple. But there is a reduction in the average torque developed due to the provision of stator pole shoes. Also with proportional to pole shoe sizes, reduction in torque ripple will be changed. In (Srinivas and Arumugam, 2003, 2005) currents injected to stator windings are constant and phase excitations are in a given firing angle, exclusively.
As the application field of SRM is quite diversified, the torque performance and the acceptable range of torque ripple demanded in its practical use become different. For this reason, in addition to geometric variables, it is necessary to use the electric variables for minimizing torque ripple. As an example of electric variables that are effective in torque ripple reduction, are turning on and off the angles. One of the really complex aspects of controlling the SRM is the assignment of the commutation angles. Many key parameters like efficiency, components VA rating, torque ripple, acoustic noise are quite sensitive to commutation angles. Turning on and off the angles depend on speed and load of machine and effects on copper losses (Rasmussen, 2002).

![Fig. 1: phase energizing](Image)

The optimal turn on and off of the angles, to provide a maximum torque at a given speed or a maximum efficiency for the required torque at a given speed, can be found by numerical analysis. Different turn on and turn off angles, with and without pulse width modulated (PWM) voltage control, are investigated in (Wu et al., 2003) to evaluate the behavior of parameters of average torque and efficiency versus speed variations and various load conditions.

The effect of firing angles on torque production is in this way that, when the voltage is applied to the stator phase, the motor creates torque in the direction of increasing inductance. For a constant phase voltage, the phase current has its maximum in the position when the inductance starts to increase. This corresponds to the position when the rotor and the stator poles start to overlap. After the phase is turned off, phase current falls to zero. The phase current, present at the region of decreasing inductance, generates negative torque (Fig. 1). The torque, generated by the motor, is controlled by the applied phase voltage and by the appropriate definition of switching turn on and turn off angles (Visinka, 2005).

An extensive review of the origin of torque ripple and the approaches adopted over the past decade to minimize the torque ripple presented in (Husain, 2002). The purpose of (Zhang et al., 2010) is to propose an accurate method to model the SRM magnetization characteristic, representing the accurate inductance profile, in order to achieve higher control performance.

In this paper, the effect of variation of stator pole arc with affixed to pole shoes, is under study on the case that external electric circuit coupled to 2-D finite element model is used and it is approximation of 3 phase SRM connected to DC link, that through six switch conventional converter is in the single pulse voltage control. This nonlinear modeling includes saturation effects and multi phase excitation. However, 3-D end effects on phase inductance due to the end windings and the axial fringing field are neglected. Also, the fraction torque and iron loss of stator and rotor laminations in under study SRM is ignored in the calculation.

### Table 1: Respective dimensions of SRM

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated torque</td>
<td>3.6 N.m.</td>
</tr>
<tr>
<td>Rated current</td>
<td>8 A</td>
</tr>
<tr>
<td>Rated speed</td>
<td>3000 RPM</td>
</tr>
<tr>
<td>Air gap</td>
<td>0.25 mm</td>
</tr>
<tr>
<td>Stack length</td>
<td>90 mm</td>
</tr>
<tr>
<td>Width of stator pole core</td>
<td>21 mm</td>
</tr>
<tr>
<td>Stator pole arc</td>
<td>29</td>
</tr>
<tr>
<td>Rotor pole arc</td>
<td>32</td>
</tr>
<tr>
<td>Number of stator coil turns / ph</td>
<td>110</td>
</tr>
</tbody>
</table>
Fig. 2: Central section geometry of the 6/4 switched reluctance motor and respective dimensions

Fig. 3: Meshed model of air gap and meshing of poles $\beta_a = 29^\circ$, $\beta_s = 32^\circ$, $\beta_s = 35^\circ$

Fig. 4: Meshed model of SRM with surroundings air modeled by INFIN110

**Femodeling of SRM:**

**Modeling procedure:**

Main dimension and geometry of basic switched reluctance motor under study with the rated specifications of motor are shown in Fig. 2 and TABLE I, which are the primary models of SRM (Srinivas and Arumugam, 2004).
Let $\beta_s$ be the stator pole arc and $\beta_r$ the rotor pole arc. For sensitivity study of the stator pole arc on SRM torque ripple, stator pole arc sizes are chosen in three values i.e. $\beta_s=29^\circ$, $\beta_s=32^\circ$ and $\beta_s=35^\circ$ (according to Fig.3).

In the modeling of SRM for dynamic studies, the region of air gap, stator and rotor laminations are modeled with PLANES5 elements. For stator and rotor laminations, B-H characteristics of M27 are assigned. The winding is magnetically modeled as air. Outer surrounding air of stator is modeled by INFIN110 elements so as to presents real imagine of free space (Fig. 4).

The region of the stator winding is modeled using PLANES5 with the stranded coil option coupled to external circuit. With assignment resistivity $1.72\times10^{-8}$ $\Omega$.m to each region of coils, the value of resistivity of windings of each phase is 0.193 $\Omega$. The external circuit is created by using CIRCUIT24 and CIRCUIT25 elements. Thus, electric circuit model coupled to magnetic circuit is the approximation of 3 phases SRM connected to DC link, which through six switch conventional converter is in single pulse voltage control(Fig. 5).

The turn on voltage source, $V_1$ and the turn off voltage source, $V_2$, which are the inverted DC link voltage, used to simulate the voltage control. The voltage applied to each phase is controlled, in accordance with the rotor positions.

**Circuit analysis:**

**Governing equations:**

By considering the following fundamental equations, circuit analysis is presented (Rajapakse et al., 2004):

\[
\lambda(\theta,i) = L(\theta,i) i
\]

\[
v = Ri + \frac{d\lambda(\theta,i)}{di} \frac{di}{dt} + \frac{d\lambda(\theta,i)}{d\theta} \frac{d\theta}{dt}
\]

\[
v = Ri + L(\theta,i) \frac{di}{dt} + K(\theta,i) \cdot \omega
\]

$L(\theta,i)$: Phase inductance by considering saturation (incremental)

$K(\theta,i)$: Instantaneous back emf coefficient

$\omega$: Angular speed

**Single pulse voltage control method:**

A simpler control of the converter is voltage control. The positive voltage mode is applied between the Turn on and Turn off angles. After the Turn-off, the negative voltage mode is applied to remove the current. Equations for magnetic circuit of SRM for single pulse voltage control can be investigated in two Modes, according to Fig. 5.

**Mode1**: This mode is named positive voltage mode. Both $S_1$ and $S_2$ are on and the voltage across the stator winding of phase A being equal to dc link voltage ($V_{dc}$), and the stator current in the winding phase A rises rapidly. This mode corresponds to powering mode of the voltage source $V_1$ and is indispensable for the converter.
Fig. 5: Current flow in converter (a) Mode I, phase A turn on, (b) Mode II, phase A turn off and phase B turn on.

According to Fig. 5-a, when the current flows in one phase (phase A turn on) circular current equation $I_a$ can be obtained by:

$$V_i = R_a I_a + L_a \left( \frac{dI_a}{dt} \right) + E_a$$ \hspace{1cm} (5)

Where, $R_a$ phase resistance, $L_a$ phase inductance, $E_a$ back emf of phase winding and $V_i$ is phase voltage.

**Mode 2:** This mode is named negative voltage mode. Both $S_1$ and $S_4$ are off and energy is returned to $V_2$ via freewheel diodes $D_1$ and $D_4$. The voltage across the winding phase A is equal to $-V_{dc}$. This mode is also indispensable to remove the stator current through the phase winding, which gradually reduces to zero.

When current followed in two phases (i.e. phase A turn off and phase B turn on) for providing the continuous torque, drive circuit should do commutation to turn off the switches of phase A ($S_1,S_4$) and turn on the switches of phase B ($S_3,S_6$). According to Fig. 5-b, equations of currents for two phases can be written by:

$$-V_{i} = R_b I_b + L_b \left( \frac{dI_b}{dt} \right) + E_b - V_d$$ \hspace{1cm} (6)

$$V_{i} = R_a I_a + L_a \left( \frac{dI_a}{dt} \right) + E_a$$ \hspace{1cm} (7)

$V_d$: Voltage drop in the route of freewheel diodes.

These modes can be repeated according to the rotor position for other phases.

**Copper losses:**

The copper losses are due to the resistive component of the windings, which also depend on the motor load and commutation angles of motor. In electrical motors, the copper losses are generally calculated using the root mean square (RMS) value of the current in each phase, which represents the equivalent DC value of the current in the phase. The RMS value of the current in one phase is (Raulin et al., 2004):

$$I_{\text{RMS}} = \sqrt{\frac{1}{T} \int_{0}^{T} I_{ph}^2 \, dt}$$ \hspace{1cm} (8)

Where, $I_{\text{RMS}}$ is the RMS value of the phase current and $I_{ph}$ is the total current flowing in the phase and $T$ is the time period of the current flowing in each phase.

The copper losses in each phase are:

$$P_{\text{copper}} = R_{ph} I_{\text{RMS}}^2$$ \hspace{1cm} (9)
Where, $R_{ph}$ is the winding resistance of each phase.

**Basic speed of motor and time steps:**

For doing the procedure of finite element analysis, speed of motor in each degree should be determined vs. time step. So time profile ($t$) must be converted to rotor angular displacement profile ($\theta_r$) by using the following relation:

$$t = \frac{\theta}{6 \times n}$$  \hspace{1cm} (10)

Where, $t$ is time in seconds corresponding to rotor position $\theta$ in mech. degrees and $n$ is the speed of rotor in revolutions per minute.

With attention to this relation, each degree of revolution of rotor for basic speed 3000 rpm is correspondent with the time step of 55.56 $\mu$sec.

The phase switching frequency is determined by the number of rotor poles (Hassanin et al., 2001):

$$f_{ph} = \frac{N_r \times n}{60}, \quad \tau_{ph} = \frac{1}{f_{ph}}$$  \hspace{1cm} (11)

$N_r$: Number of rotor poles

Also, for an $m$ phase machine, the phases are excited successively with under switching frequency:

$$f_s = m \times f_{ph}$$  \hspace{1cm} (12)

The rotor position angle in electrical degree is $\theta_e$ and its relationship with the rotor position angle in mechanical degree $\theta_r$ is (Wu et al., 2003):

$$\theta_e = \frac{N_r}{m} \times \theta_r$$  \hspace{1cm} (13)

Because of the symmetric rotor structure, the mechanical angle an over one electrical cycle for the three phases 6/4 SRM is given by (Ye et al., 2000):

$$\alpha = \frac{360^\circ}{N_r} = 90^\circ \text{ mech.}$$  \hspace{1cm} (14)

The mechanical angle $\beta$ between two adjacent pluses is obtained by:

$$\beta = \frac{360^\circ}{m \times N_r} = 30^\circ \text{ mech.}$$  \hspace{1cm} (15)

The total instantaneous electromagnetic torque, $T$, produced by the SRM is the sum of the instantaneous torque developed by each phase:

$$T = \sum_{j=1}^{n} T_j$$  \hspace{1cm} (16)

The average torque of SRM, $T_{avg}$ can be evaluated by integrating the instantaneous torque values over a repetitive excitation cycle as the following:

$$T_{avg} = \frac{1}{\alpha} \int_T T d\theta = \frac{1}{\alpha} \int_{\theta_{on}}^{\theta_{off}} \sum_{j=1}^{n} T d\theta$$

$$-45^\circ \leq \theta_{on} \leq -30^\circ, \quad \theta_{on} \leq \theta \leq \theta_{off} \leq 0^\circ$$  \hspace{1cm} (17)
With rotor speed is assumed to be reasonably constant \((\text{d}t/\text{d}t=\text{const.})\) during one electrical cycle, the following expressions for turn on and turn off angles for 3 phases 6/4 SRM are satisfied:

\[
\begin{align*}
\theta_{j-on} &= \theta_{j-on} - (j-1)\beta \\
\theta_{j-off} &= \theta_{j-off} - (j-1)\beta 
\end{align*}
\]  

(18)  

(19)

Where \(j=1, 2, 3\) are related to phases A, B, C.

**Dynamic characteristic calculations:**

Zero rotor position (\(\theta=0^\circ \text{ mech.}\)) is defined at the aligned position when one of the stator teeth is aligned with one of the rotor teeth. At a given constant speed, electromagnetic transient analysis in order to change firing angles is repeated for each of the two or three degrees of rotation from angle \(\theta=4^\circ\) (i.e. unaligned position) to \(\theta=-30^\circ\). In each stage, rotor rotates along \(120^\circ\) mechanical degree so that torque and other electric and magnetic parameters can be calculated. Thus, phase currents and the output torque of SRM are determined by firing angle control.

**Torque ripple calculation:**

Torque ripple is an undesirable factor in switched reluctance motors which depends on overlapping of stator and rotor poles, pole shape, material property, number of poles, and number of phases. Also, it’s a very important parameter to assess the construction. During steady state operation, torque ripple is defined as relation (20) (Matveev, 2006; Husbain, 2002), (Mademlis and Kioskeridis, 2003):

\[
T_{\text{ripple}} = \frac{T_{\text{max}}(\text{max}) - T_{\text{min}}(\text{max})}{T_{\text{avg}}} \times 100\% 
\]  

(20)

Where, \(T_{\text{max}}(\text{max})\), \(T_{\text{min}}(\text{min})\) and \(T_{\text{avg}}\) are the maximum and minimum instantaneous torque and average torque, respectively, during steady state operation.

**Analysis Results:**

In this section, the results obtained from finite element analysis are presented for SRM with three sizes of stator pole arcs. Applied voltage waveform and obtained current in one phase of SRM at a given firing angle with developed torque profile by the same phase is shown in Fig. 6.

By attention to Fig. 6, it’s observed that in the rotor position of \(35^\circ\), sign of torque is changed to minus. It is due to changing in the sign of inductance slope (from positive to negative) at the same phase that current still flow. So, because of changing in the sign of inductance slope, after the critical angle (\(35^\circ\)), negative torque is produced.

**Fig. 6:** (a) voltage and current profile of phase B. (b) phase current and torque profile of SRM by considering inductance profile when its slope has changed \(\theta_{\text{on}}=-35^\circ\), \(\theta_{\text{off}}=-5^\circ\)
observed that its torque ripple values are lower as compared with correspondence values in the state of angles, smaller than So, in this case, we can choose the greatest efficiency; however values related to economic. Also, the type of application of motor is a determiner in choosing the firing angles. In numerical efficiency become in balance limit. It means purely attention to one parameter exclusively cannot be desired and firing angles should be chosen under the conditions that parameter of average torque, torque ripple and efficiency. By attention to TABLE 2, we can choose the optimal firing angles. The optimal presented in TABLE 2. The principle cases considered in table are effective current of phase, average torque, Numerical results obtained from steady state dynamic analysis of SRM for three sizes of stator pole arcs are
to torque production is also higher, and it causes copper losses to increase, and consequently, efficiency become low (Fig. 7-d). Therefore, if the desired state of SRM becomes operated with low ripple, according to Fig. 7-c, we can define the angle of $\theta_{on}=\beta_{on}$ as optimal angle because this angle has relatively low torque ripple with acceptable average torque and efficiency. Also, from saturation point of view it is suitable because with this turn-on angle, the RMS value of phase current of motor is in a limit that motor can’t be in saturation state.

Between the firing angles placed in the limitations of $\theta_{on}=35^o$ to $\theta_{on}=40^o$, if we compare from high average point of view, we can choose $\theta_{on}=39^o$ because average torque produced by this turn-on angle is approximately equal to the rated torque of under-study basic SRM.

In the numerical values related to $\beta_{on}>\beta_{on}$, the turn-on angle $\theta_{on}=\beta_{on}$ has higher efficiency, but its torque ripple is greater as compared with $\theta_{on}=35^o$. By attention to torque ripple values in the state of $\beta_{on}>\beta_{on}$, it is observed that its torque ripple values are lower as compared with correspondence values in the state of $\beta_{on}>\beta_{on}$. So, in this case, we can choose the $\theta_{on}=37^o$ as optimal angle because it has the highest efficiency, and its average torque is acceptable.

Turn-on angle of $\theta_{on}=35^o$ has the lowest torque ripple; however, because of low average torque, only when the required value of output average torque is small, we can use this as optimal firing angle.

Obtained numerical values for state of $\beta_{on}<\beta_{on}$, by referring to TABLE II, indicate that $\theta_{on}=37^o$ has the greatest efficiency; however $\theta_{on}=35^o$ produce lowest torque ripple with very low value of average torque. So for these reasons, we can choose the $\theta_{on}=37^o$ as the optimal turn on angle for this state of stator. It is very important that by increasing stator pole arcs (by pole shoes), torque ripple decreases considerably. Therefore, the choosing of optimal firing angles introduced is based on a trade-off between torque ripple, efficiency and produced average torque. From the viewpoint of comparing the three states of motor reviewed, performance of SRM with three different sizes of stator pole arcs are shown in Fig. 8.

<table>
<thead>
<tr>
<th>$\theta_{on}$</th>
<th>$\theta_{off}$</th>
<th>$I_{phase}$ (A)</th>
<th>$T_{avg}$ (N.m)</th>
<th>$T_{rmp}$ (%)</th>
<th>Effic. (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-30</td>
<td>0</td>
<td>$\beta_{on}&gt;\beta_{on}$</td>
<td>3.43</td>
<td>0.059</td>
<td>2337.00</td>
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<tr>
<td></td>
<td></td>
<td>$\beta_{on}=\beta_{on}$</td>
<td>3.20</td>
<td>0.051</td>
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<td></td>
<td></td>
<td>$\beta_{on}&lt;\beta_{on}$</td>
<td>3.03</td>
<td>0.037</td>
<td>3861.65</td>
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<tr>
<td>-33</td>
<td>-3</td>
<td>$\beta_{on}&gt;\beta_{on}$</td>
<td>3.67</td>
<td>0.793</td>
<td>110.77</td>
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<tr>
<td></td>
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<td>$\beta_{on}=\beta_{on}$</td>
<td>3.38</td>
<td>0.626</td>
<td>174.29</td>
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<tr>
<td></td>
<td></td>
<td>$\beta_{on}&lt;\beta_{on}$</td>
<td>3.17</td>
<td>0.518</td>
<td>219.99</td>
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<tr>
<td>-35</td>
<td>-5</td>
<td>$\beta_{on}&gt;\beta_{on}$</td>
<td>4.20</td>
<td>1.474</td>
<td>164.98</td>
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<tr>
<td></td>
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<td>1.175</td>
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<td>3.46</td>
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<td>95.75</td>
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<td>-7</td>
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<td>2.448</td>
<td>198.86</td>
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<td></td>
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<td>4.61</td>
<td>2.026</td>
<td>168.21</td>
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<td>4.14</td>
<td>1.703</td>
<td>108.51</td>
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<tr>
<td>-39</td>
<td>-9</td>
<td>$\beta_{on}&gt;\beta_{on}$</td>
<td>6.88</td>
<td>3.753</td>
<td>203.96</td>
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<td></td>
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<td>$\beta_{on}=\beta_{on}$</td>
<td>6.07</td>
<td>3.231</td>
<td>179.39</td>
</tr>
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<td></td>
<td>$\beta_{on}&lt;\beta_{on}$</td>
<td>5.45</td>
<td>2.826</td>
<td>134.14</td>
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<tr>
<td>-40</td>
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<td>$\beta_{on}&gt;\beta_{on}$</td>
<td>7.86</td>
<td>4.532</td>
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<td>$\beta_{on}=\beta_{on}$</td>
<td>6.99</td>
<td>3.966</td>
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<tr>
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<td></td>
<td>$\beta_{on}&lt;\beta_{on}$</td>
<td>6.31</td>
<td>3.516</td>
<td>137.36</td>
</tr>
<tr>
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<td>$\beta_{on}&gt;\beta_{on}$</td>
<td>10.16</td>
<td>6.319</td>
<td>193.77</td>
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<td></td>
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<td>$\beta_{on}=\beta_{on}$</td>
<td>9.18</td>
<td>5.673</td>
<td>172.30</td>
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<td></td>
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<td>8.37</td>
<td>5.130</td>
<td>146.15</td>
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<td>-15</td>
<td>$\beta_{on}&gt;\beta_{on}$</td>
<td>14.25</td>
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<td>8.710</td>
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<td></td>
<td></td>
<td>$\beta_{on}&lt;\beta_{on}$</td>
<td>12.17</td>
<td>8.032</td>
<td>149.78</td>
</tr>
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</table>
**Discussion:**

The numerical results of TABLE II and Fig. 9, provide possibility to choose the optimal firing angles. By attention to the presented values in the table it’s clear when firing angles approach to the unaligned positions, the RMS value of phase current increases because the amount of overlap between stator and rotor poles is limited. Thus, with the increasing of phase currents, average torque is increased. Also, the difference between maximum and minimum instantaneous torque will be great and therefore high torque ripple will produce. Copper losses due to an increase in phase currents, too, will increase that will lead to a decrease in efficiency for firing angles near the unaligned positions (Fig. 10).

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**Fig. 7:** Phase Currents, average and instantaneous torque profile as obtained by FEM for SRM \( \beta_i > \beta_s \). (a) \( \theta_{on} = -30^\circ, \theta_{off} = 0^\circ \). (b) \( \theta_{on} = -33^\circ, \theta_{off} = -3^\circ \). (c) \( \theta_{on} = -35^\circ, \theta_{off} = -5^\circ \). (d) \( \theta_{on} = -42^\circ, \theta_{off} = -12^\circ \).
For firing angles near the align position, in addition to average torque decrease, the values of current and efficiency are low. Torque ripple in $\theta_{on} = -30^\circ$ has the highest value because this angle is a critical angle and for turn-on angles even less than this value, negative torque will produce in motor. So the optimal firing angle for $\beta_i > \beta_i$ could be $\theta_{on} = -35^\circ$. 

(a) 

(b) 

(c) 

(d)
Fig. 8: Phase’s Currents, average and instantaneous torque profile for SRM ($\beta_r > \beta_s, \beta_i = \beta_s, \beta_i < \beta_s$) (a) $\theta_{on} = -30^\circ, \theta_{off} = 0^\circ$, (b) $\theta_{on} = -33^\circ, \theta_{off} = 3^\circ$, (c) $\theta_{on} = -35^\circ, \theta_{off} = 5^\circ$, (d) $\theta_{on} = -37^\circ, \theta_{off} = 7^\circ$, (e) $\theta_{on} = -40^\circ, \theta_{off} = -10^\circ$, (f) $\theta_{on} = -45^\circ, \theta_{off} = -15^\circ$.

By increasing stator pole arcs, torque ripple will decrease; thus, by attention to Fig. 9, the turn-on angle of $\theta_{on} = 37^\circ$ in the case of motor with $\beta_i = \beta_s, \beta_i < \beta_s$ (by affixing pole shoes) will be desired. Torque ripple and average torque for each three state of motor are shown in Fig. 10, which indicates reduction in torque ripple and average torque when we increase the stator pole arcs.

Fig. 9: Choosing of optimal firing angles from efficiency, average torque and torque ripple profiles vs. turn on angles for SRM ($\beta_r > \beta_s, \beta_i = \beta_s, \beta_i < \beta_s$).

Fig. 10: Comparisons of efficiency, average and torque ripple profiles vs. turn on angles for SRM ($\beta_r > \beta_s, \beta_i = \beta_s, \beta_i < \beta_s$).
Conclusion:

One of the methods in order to predict the steady state performance of switched reluctance motor by using single pulse voltage control for minimizing torque ripple is reviewed. Torque ripple is produced during the phase commutation, so simulation of torque profile for three phase excitation indicates that by affixing stator pole shoes, torque ripple is lower without this, but instead of this, average torque produced (in equal firing angle for three sizes of stator pole arcs) will decrease, which is contradictory to the reduction of torque ripple. Also, by changing firing angles from aligned position to unaligned position, the optimal firing angles are determined based on the desired criterions. Finally, in order to correct the choosing of optimal firing angles by attention to the type of application of SRM, balance should be established between average torque, efficiency and torque ripple because SRM performance becomes effective and economic.

REFERENCES


