**Static Characteristics of Switched Reluctance Motor 6/4 By Finite Element Analysis**

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**Abstract:** In this paper, processing the static modeling of Switched Reluctance Motor 6/4 with three different stator pole arcs sizes are described. The result of static analysis, including inductance, flux linkage, and static torque vs. rotor position for various current excitations by considering saturation effect in three states of motor with and without stator pole shoes are investigated. The effect of the increase in stator pole arcs is presented for improving SRM performance. Finally, two existing methods of inductance calculation in the finite element analysis package of ANSYS are reviewed and their results are compared.

**Key words:** Static Characteristics, Finite Element Method, Stator Pole Shoes, Switched Reluctance Motor.

**INTRODUCTION**

Determination of the magnetic characteristics is a key point to the optimizing design and / or control strategy evaluation in a switched reluctance machine (SRM). The SRM's drive performance is strongly dependent on its design and control. Therefore, the motor’s mathematical model and its accuracy are important. A key factor for all SRM models is constituted by the flux linkage versus current characteristics calculation, which can be done analytically, via numerical field analysis by employing usually the finite element method (FEM), or by combining the analytic and numeric calculation (Pa duraria et al., 2010).

Analytical calculation of the non-linear $\Psi$-i characteristic of SRM is needed on the one hand to control the machine and on the other hand to make torque estimation in the process of modeling or designing a new SRM (Schinnerl and Gerling et al., 2008). The magnetization property of independent phases together with motor $\Theta$-i characteristics imposed some torque ripple during the performance. Thus, the geometric variables of the construction of SRM are used in order to improve in torque profile. The effects of change in stator pole arcs by affixing stator pole shoes to basic SRM in the performance and magnetic parameters of SRM are presented in (Srinivas and Arumugam et al., 2003; 2005). Also, by keeping the stator pole arcs, the effect of change in rotor pole arcs to SRM parameters is evaluated in (Sheth and Rajagopal et al., 2003).

In particular, the SRM phase magnetization characteristics vary strongly as a function of the excitation current and rotor position. Therefore, numerical methods must be used for the calculation of the magnetic field and for the prediction of the SRM magnetization characteristics. The finite element method is used most often because of its ability to take magnetic saturation into account (Omekanda et al., 1997).

Finite element method analysis was used by Dawson to determine the magnetic characteristics of an SRM. But, a precise computation of the field is difficult, owing to the intricate geometry of the SRM. The finite elements located in the machine gap posed more problems. Many numerical algorithms for FEM analysis of SR machines have been done in the last several years. These methods were found to be quite laborious (Parreira et al., 2005; Kumar and Isha et al., 2008). Although the FEM was a time-consuming and difficult method in the past, with contemporary high speed computers with large memory, the method can be used properly in analysis of electric machines. In addition, various commercial finite-element packages such as ANSYS have been so far developed for this purpose (Faiz et al., 2006).

Using finite element analysis is used to obtain magnetic characteristics of machines by considering the saturation effect and by affixing the stator pole shoes. Method of modeling SRM is described and static analysis results are presented for SRM with three different stator pole arc sizes. Finally, the methods of inductance calculations in ANSYS are discussed.

**Fe modeling of Srm:**

**Modeling Procedure:**

Main dimension and geometry of basic switched reluctance motor under study with the rated specifications of motor which is the primary model of SRM’s (Srinivas and Arumugam, 2004), is shown in Fig. 1.

The intricate geometry of SRM has, however, posed precision problems in its field computation. Mainly, for those finite elements located at the machine gap. Air gap has a higher degree of changing the magnetic quantities. So, six layer elements are used to mesh these regions. The regions of stator and rotor are connected.
with constrain equations in the middle of the air gap with equal divisions of meshing lines. Let $\beta_s$ be the stator pole arc and $\beta_r$ the rotor pole arc. For sensitivity study of the stator pole arc on SRM inductance and static torque by keeping other design parameters, stator pole arc sizes are chosen in three values i.e $\beta_s = 29^\circ$, $\beta_s = 32^\circ$ and $\beta_s = 35^\circ$ (according to Fig. 2).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated torque</td>
<td>3.6 N.m</td>
</tr>
<tr>
<td>Rated current</td>
<td>8 A</td>
</tr>
<tr>
<td>Rated speed</td>
<td>3000 RPM</td>
</tr>
<tr>
<td>Air gap</td>
<td>0.25 mm</td>
</tr>
<tr>
<td>Stack length</td>
<td>90 mm</td>
</tr>
<tr>
<td>Width of stator pole core</td>
<td>21 mm</td>
</tr>
<tr>
<td>Stator pole arc</td>
<td>29$^\circ$</td>
</tr>
<tr>
<td>Rotor pole arc</td>
<td>32$^\circ$</td>
</tr>
<tr>
<td>Number of stator coil turns / ph</td>
<td>110</td>
</tr>
</tbody>
</table>

**Fig. 1:** Central section geometry of the 6/4 switched reluctance motor and respective dimensions.

In the modeling of SRM for static studies, the region of air gap, stator and rotor laminations are modeled with PLANE53 elements. For stator and rotor laminations, B-H characteristics of M27 are assigned. The winding is magnetically modeled as air. Meshing of SRM with equal pole arcs of stator and rotor is shown in Fig. 3. The region of the stator winding is modeled using PLANE53 with the AZ degree of freedom with assignment resistivity $1.72 \times 10^{-8}$ Ω.m to each region of coils. The value of resistivity of windings of each phase is calculated by 0.193 Ω.

In static analysis, current density ($J_s$) is injected to each of the coil regions of one phase inside direction (-$J_s$) and outside direction (+$J_s$) by the following relation:

$$J_s = \frac{N_p \cdot i}{S}$$  \hspace{1cm} (1)

Where $N_p = 55$ is the number of turns in each pole and $S$ is the cross section of coil on each pole and $i$ is the applied current to motor.

Static analysis will do after the modeling and applying of boundary conditions and loading for three states of motor. So one of the stator phases for example phase A is excited for different values of currents. Zero rotor position ($\theta = 0^\circ$ mech.) is defined at the aligned position when one of the stator teeth is aligned with one of the rotor teeth.

Static analysis for each one degree of rotation from unaligned position ($\theta = -45^\circ$) to aligned position ($\theta = 0^\circ$) repeated. Because keeping the value of applied current to phase A is constant all over the period of rotation, calculation of self inductance can be obtained by the following relation:

$$L(\theta) = \frac{\mathcal{L}(\theta)}{i}, \quad i = \text{const.}$$  \hspace{1cm} (2)

Also, average torque can be calculated by:

$$T_{\text{avg}} = \frac{1}{\tau} \int_{\theta_0}^{\theta_\pi} T(\theta) \ d\theta$$  \hspace{1cm} (3)

Where, $\tau = \pi/4$ for 6/4 SRM and $T(\theta)$ is the torque at position $\theta$.

Equation (3) in discrete format can be written as:

$$T_{\text{avg}} = \frac{1}{n} \sum_{j=1}^{n} T(j)$$  \hspace{1cm} (4)

Where $n$ is the number of steps that rotor rotates from unaligned to aligned position and $T(j)$ is the calculated torque in $j$th stage.

**Magnetization Curves:**

In order to explain the energy-conversion principles for the Switched Reluctance Motors, the magnetization curve are widely used. The magnetization curves tell us how much current or MMF is required to reach a certain flux in different rotor-positions.
Fig. 2: Meshed model of air gap and meshing of poles $\beta_s = 29^\circ$, $\beta_s = 32^\circ$, $\beta_s = 35^\circ$.

Fig. 3: Meshing of SRM.

**Static Characteristics of SRM:**

The result of static analysis includes flux linkage, inductance, static torque, and co-energy stored energy of magnetic fields.

Furthermore, by using magnetization curves and with a given excitation current, inductance can be obtained in each rotor position. Also, static torque profiles can be reached by using magnetization curve data. Magnetization curves for three stator pole geometry of SRM are shown and compared in Fig. 4. In unaligned position air gap then reluctance is maximum and saturation don’t occur, so flux linkage is linear function of current. Saturation in magnetization curves happen in two states; first, when the overlap between the corners of stator and rotor poles is very negligible; therefore, the focusing of flux in the corners causes saturation (Fig. 5) and the linear relation between the current and flux linkage to be destroyed, which is true even for low excitation currents. Second, whenever the overlapped poles are closer to align position, yokes are saturated at high currents and have tendency toward flux linkage to be limited in a maximum limit.

Fig. 4: Magnetization curves of SRM ($\beta_s = 29^\circ$, $\beta_s = 32^\circ$, $\beta_s = 35^\circ$).

By comparing the characteristic curves in Fig. 4 it is observed that by affixing the pole shoes to stator poles, flux linkage has increased where this increase is more notable in regions between unaligned to aligned positions and it’s in aligned position negligible, but pole shoes approximately don’t have effect on unaligned position which is due to high reluctance between stator and rotor under this condition.
The reasons for the increase in flux linkage, by affixing pole shoes, can be accounted for by shortening the flux route between stator and rotor poles, and because of this, route reluctance decreases (specially from unaligned to a position that stator and rotor start to overlap).

Flux linkage of phase A vs. rotor position \( (\beta_s = 29^\circ) \) for different excitation currents is shown in Fig. 6. Other phases will have similar curves. For state of \( \beta_s = 29^\circ (\beta_s > \beta_r) \) with an increase in current excitation, flux linkage of unaligned position has negligible increase. However flux linkage by closing the aligned position has a notable increase. Also, because of more increase in current limit, saturation effect limiting the flux linkage in environs of aligned position is seen well as in Fig. 6.

**Inductance Profiles:**

In this paper with ANSYS static analysis, inductance is calculated based on following ways:

1. **LMATRIX command of ANSYS** \( (L_{ANSYS}) \)
2. **Flux linkage divided by current** \( (L_{\lambda/i}) \)
3. **Magnetic field co-energy** \( (L_{Co-energy}) \)
4. **Magnetic field stored energy** \( (L_{Stored energy}) \)

Where, the right way is item b (proved in last section).

Inductance profiles of SRM with \( \beta_r > \beta_s, \beta_r = \beta_s, \beta_r < \beta_s \) are shown in Fig. 7 for deferent excitation current values. With attention to each of the inductance profiles, it’s completely clear that for different excitation currents at closing the unaligned position (until \( \theta = -29^\circ \) for \( \beta_s > \beta_r \) and until \( \theta = -27^\circ \) for \( \beta_s = \beta_r \) and \( \theta = -26^\circ \) for \( \beta_s < \beta_r \)), inductance has equal values, approximately.

**Fig. 6:** Flux linkage vs. rotor position for \( \beta_s = 29^\circ (\beta_s < \beta_r) \).

**Fig. 5:** Permeability of basic SRM at the beginning of overlapping between stator and rotor poles (I=12A).
Also, inductance profiles have nearly linear part for low values of excitation currents and in case of increasing the excitation current (by starting the overlap between stator and rotor and by reasons of saturation) the value of aligned inductance ($L_{\text{max}}$) has reduced. To take into consideration the inductance profiles of Fig. 8, it seems clear that pole shoes affect inductance profiles and cause them to increase a little. Also, in the regions that overlap is started between stator and rotor (with a few degrees of difference between three states of motor), rising slope of inductance is considerable because these regions have much effect on production of flat torque (low torque ripple).

In TABLE I maximum inductance per minimum inductance ratio value for different current values is presented for SRM with $\beta_r > \beta_s$, $\beta_r = \beta_s$ and $\beta_r < \beta_s$, which explains how much the static analysis for a given current the ratio of $L_{\text{max}} / L_{\text{min}}$ in three different stator pole sizes has near values toward each other.

Modifications should be undertaken to maintain this almost the same ratio to avoid any reduction in the static torque per ampere. According to TABLE I, it seems that the values of $L_{\text{max}} / L_{\text{min}}$ ratio are reduced by increasing the $\beta_s$ for higher current values. So an upper limit should come into account. If for value of $\beta_s$ the increase continues, this will lead to a larger decrease in the static torque per ampere, at the cost of which, torque ripple minimization should not be encouraged (Srinivas and Arumugam, 2005).

Table 1: RATIO OF $L_{\text{max}} / L_{\text{min}}$ FOR VARIOUS CURRENTS

<table>
<thead>
<tr>
<th>Excitation current</th>
<th>$\beta_r &gt; \beta_s$</th>
<th>$\beta_r = \beta_s$</th>
<th>$\beta_r &lt; \beta_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/5 A</td>
<td>20.783</td>
<td>21.248</td>
<td>20.430</td>
</tr>
<tr>
<td>3 A</td>
<td>20.657</td>
<td>21.032</td>
<td>20.193</td>
</tr>
<tr>
<td>4/5 A</td>
<td>19.621</td>
<td>19.545</td>
<td>18.656</td>
</tr>
<tr>
<td>6 A</td>
<td>17.291</td>
<td>16.800</td>
<td>15.936</td>
</tr>
<tr>
<td>9 A</td>
<td>12.879</td>
<td>12.384</td>
<td>11.714</td>
</tr>
<tr>
<td>12 A</td>
<td>10.201</td>
<td>9.780</td>
<td>9.246</td>
</tr>
</tbody>
</table>

Fig. 7: Inductance vs. rotor position for (a) $\beta_s = 29^\circ$ ($\beta_s < \beta_r$) (b) $\beta_s = 32^\circ$ ($\beta_s = \beta_r$) (c) $\beta_s = 35^\circ$ ($\beta_s > \beta_r$).

Fig. 8: Comparing Inductance profiles for three states of SRM with excitation currents of (a) $I=3A$. (b) $I=12A$.

Static torque Profiles:

From the foundation reluctance torque, positive torque is generated when each phase is excited during the rising of inductance, and vice versa (negative torque is produced by falling of inductance). Static torque relevant to stator poles of $\beta_s > \beta_r$, $\beta_s = \beta_r$, $\beta_s < \beta_r$ for different current excitation is shown in Fig. 9.
Fig. 9: Static torque vs. rotor position for (a) $\beta_s = 29^\circ$ ($\beta_s < \beta_r$) (b) $\beta_s = 32^\circ$ ($\beta_s = \beta_r$) , (c) $\beta_s = 35^\circ$ ($\beta_s > \beta_r$).

Comparison of static torques for current values of 3A and 6A for three stator pole arcs are shown clearly in Fig. 10 with great value of stator pole arc size, torque profile is improved. So the maximum torque is allowed to remain the same for more positions of rotor. On the other hand, by increasing stator pole arc, inductance profile is modified and therefore with attention to torque profile in Fig. 10, position and inductance rising slope is not similar to the three states and by the reasons of its role on production of torque, rising slope of $\beta_s < \beta_r$ is desired than other states.

By affixing pole shoes to stator poles, the possibility of using firing angles (turn on angle) near the unaligned position increases. Pole arc sizes for stator (following it, pole shoes) depend on some parameters like the value of torque ripple and requested average torque which has an upper limit because of leakage reasons.

Fig. 10: Static torque and Inductance vs. rotor position for (a) I=3A. (b) I=6A.

Comparison of Inductance Calculation Methods in ANSYS:

In order to inductance calculation, static analysis is done by the methods of constant current density injection ($J_s$) to elements of coil region of one phase in three states of motor. But with attention to inductance profiles obtained by the above mentioned methods, different results are obtained. In basic SRM, results of four methods for current values of 3A, 6A, 12A are compared in Fig. 11.
Fig. 11: Inductance profiles based on 4 methods of current density injection for three various excitation currents ($\beta_s < \beta_r$) (a) $I=3A$. (b) $I=6A$. (c) $I=12A$.

Fig. 12: (a) Model of basic SRM coupled to external circuit for verifying the inductance calculations, (b) Two positions of SRM, non-saturation and full saturation, $I=12A$.

Fig. 13: Correct Inductance profile ($L_{NMISC(Str. Coil)}$) ($I=12A$ & $\beta_s < \beta_r$).
The method of inductance calculation by external circuit coupled to FE is used to determining, comparing and verifying the obtained inductance by static analysis. For this reason, basic SRM modeled and coupled to external circuit (Fig. 12-a) which one phase of motor is modeled by stranded coil in order for inductance calculation. Voltage sources and external resistor values are chosen so that in steady state, current value of 12A flows in modeled phase. (in case complete saturation occurs in basic SRM Fig. 12-b). By doing the static analysis of circuit coupled to FE domain and for every five mechanical degrees of rotation from unaligned to aligned position, inductance profiles of $L_{NMISC9(Str. Coil)}$ are obtained, which are shown in Fig. 13.

With attention to Fig. 14 and comparison of inductances in states of (a) means inductance obtained by external circuit coupled to FE (b) means analysis by the methods of constant current density injection to coil regions for a given current value of 12A, it’s concluded that inductance $L_{NMISC9(Str. Coil)}$ of (a) and inductance $L_{\lambda/i}$ of (b) have approximately equal value and profiles.

By comparison of obtained results of inductance calculation methods in static analysis and circuit coupled to FE, it can be understood that:

First, applied current is constant and co-energy and stored energy of magnetic fields are proportional by Current Square (equation (5)) and they don’t give right values in the saturation state of motor.

$$W_{\text{in, stored energy}} = \frac{1}{2} L(\theta) \cdot i^2 , \quad i = \text{const.}$$

(5)

Second, static analysis of ANSYS by using LMATRIX command and current density injection to coil regions without coupling of external circuit, can not give correct answer in saturation states of SRM. Therefore, by noting the equation

$$\lambda(\theta, i) = L(\theta) \cdot i , \quad i = \text{const.}$$

(6)

And with the assumption that current applied to a phase of SRM is constant, it can be the result that:

$$\lambda_{zD}(\theta) = L_{zD}(\theta) \cdot i, \quad i = \text{const.}$$

(7)

Finally, calculated inductance based on flux linkage, divided in to current in equation (8) can be used for 2D state and by neglecting the end effects of coils. This method is mentioned in [Matveev, 2006].

**Conclusion:**

By using static analysis of magnetic field in SRM, it can be concluded that by increasing stator poles and by affixing pole shoes, inductance profile is modified, and hence static torque profile is improved, which as a result, torque ripple decreases. Also, by comparing the two modeling processes of inductance calculation in ANSYS, it seems that static analysis relevant to inductance calculation by the injection of constant current density to coil regions of one phase of SRM (i.e. LMATRIX command), whenever motor is under the saturation, won’t give a correct answer. However, calculated inductance based on flux linkage divided in to current (constant applied current) with calculated inductance by external circuit coupled to FE, presents inductance values exactly.
REFERENCES


