The Efficiency Measurement of Bank Branches Using Two-Stage DEA Cooperation Model (Case Study: Guilan Saderat Bank Branches)

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Abstract: As banks are key institutions and fundamental pillars of any economy, and play a leading role in economic development and growth, their efficiency measurement has always been noted. Two-stage DEA model with shared flow inputs can measure DMU's overall efficiency and the connection between its stages, considering intermediate measures and shared inputs. In this research we study an approach in which shared flow input can flow freely between different stages, using an open theory in combining efficiencies of different stages, and optimizing overall efficiency in cooperation of different stages. By placing one of the efficiencies of stages as parameter, we change the non-linear models into linear programming models, and make use of them in efficiency measurement of bank branches. The results – using data gathered in 2010 from Saderat Bank branches in Guilan province – suggest that out of 20 branches studied only 3 are efficient whereas most of the branches have more deficiency in production stage than profitability stage.

Key words: Efficiency measurement, shared flow, intermediate measures, conventional DEA, two-stage DEA.

INTRODUCTION

Market globalization and financial services getting out of the hands government in many countries, while the competition gets harder, shows new opportunities as well (Soteriou and Zenios, 1999). In such a dynamic environment, few business units are found which are not hunting for opportunities and think of continuous improvement ways as unimportant. Efficiency measurement is one of the important continuous improvement tools for business units all around the world full of advanced technologies of computers and electrical communication tools, where competition is growing rapidly (Seiford and Zhu, 1999) and among various industries, banking industry, especially in recent times, is of particular importance. Increased competition in this field and presence of private banks has led the managers to choose an appropriate scale for banking services. Due to privatization of Saderat Bank of Iran and its presence in Stock market and increased level of competition in gaining more market share and profit, using a scientific method for efficiency measurement and structure reform of the branches seems to be self-evident.

A wide variety of methods in efficiency measurement can be used based on concepts of productivity and efficiency (Yu and Fan, 2009). In 1978, DEA was presented by Charnes, Cooper and Rhodes to assess efficiency of decision making units (DMUs). DEA is actually a mathematical programming approach which assesses the efficiency of a group of units in accordance with multiple performance measures - inputs and outputs (Chen et al., 2006). However, the main point here is that conventional DEA models can only measure a specific stage, even when a two-stage process is involved (Chen and Zhu, 2004). Also, conventional DEA models consider all DMU activities (like production project) as a black box in which inputs change into outputs (Chen et al., 2006) without keeping in mind the fact that outputs of the first stage are inputs for the second stage, and in fact, these models ignore intermediate measures (Kao and Hwang). Thus, we use the two-stage DEA model in this research, for this model is developed to include intermediate measures (Chen and Zhu, 2004). However, we should note that it's not possible for a DMU, in producing its final outputs, to use only the intermediate measures, without using any other inputs. Therefore, in some production scenarios inputs or outputs maybe shared between different activities. That is why we use shared flow. In literature of DEA, shared flow presents situations in which DMUs are divided into different components, which through the cooperation that exists between them need shared resources, products or services. Shared flow has been used to study a phenomenon like efficiency measurement and resource allocation, and presents conditions in which that each decision making unit consists of two sub units (Zha and Liang, 2010).

So, shared inputs between the two stages of DMUs show an obvious managerial value, because it can be a good suggestion for dividing resources between stages. Two-stage DEA model can assess DMU's overall efficiency and the connection between its stages very well, considering intermediate measures and shared inputs (Zha and Liang, 2010). In this study, we discuss a situation in which some inputs are used at the first and others
at the second stage, and also the outputs of the first stage are considered as inputs for the second stage. Furthermore, using this tool, managers can identify the potential candidates as reference units (Chen and Zhu, 2004). Thus, the aim of this research is answering these questions:

1. Which of the analyzed branches work efficiently and which work inefficiently?
2. What are the reference units of each of the inefficient units?
3. How can we logically set goals for inefficient units?

And finally, in order to answer these questions, we measure branch efficiency, determine reference units for inefficient branches, and make suggestion for proper resource allocation. Therefore, we divide each bank branch (DMU) into two sub-units, and making use of the two-stage DEA model, measure efficiency of the whole branch and its sub-units. This is how we separate efficient and inefficient units try to identify reference units, and finally suggest the proper way of allocating resources. In the next section we'll have a review of the past studies. In section 3 we introduce the research methodology and in section 4 a practical example is presented. Sections 5 and 6 deal with results and conclusions, respectively.

**Review of literature:**

Much research has been done on the two-stage DEA that show its importance in efficiency measurement of decision making units in various industries. In 2004, Castelli discussed 3 different approaches to efficiency assessment of a two-stage DMU and suggested that cooperation between the sub-units might be recommended for explaining significant efficiency assessments of a two-stage structure. By the same token, Chen (2004) suggested an IT investment assessment, and Liang (2006) measured supply chain efficiency. Furthermore, Castelli developed the main idea of using production in describing the overall efficiency of a two-stage production process in 2004, which was used by Kao and Hwang in 2008 for exemplifying two-stage production processes assessments in situations like a life insurance company in Taiwan. Zha and Liang (2010) used two-stage DEA model with shared flow inputs and developed Castelli's research (2004) and considered production in describing cooperation between different stages efficiency and DMU's overall assessment. various methods had been suggested for better use of shared flow model, such as weighted restrictions (Beasley, 1995), various returns to scale (Molinero and Tsai, 1997; Tsai and Molinero, 2002), different weights on shared inputs (Cook et al., 2000), additive objective function (Cook and Hababou, 2001), overlapping outputs (Cook and Green, 2004) and panel data and non-discretionary inputs (Jahanshahloo et al., 2004a,b) (see Castelli et al., (2010) for more details). Analysis of the literature showed that for efficiency measurement the two-stage Data Envelopment Analysis model has been used frequently in different industries, especially banking. And it's because of the capabilities of this model, like its more accurate measurement compared to conventional models, helping managers in making decisions about defining improvement projects, optimized resource allocation, and many others.

**Methodology:**

The managers of the Saderat Bank of Iran are among those who seek achieving such capabilities. That's why we've chosen the branches of this bank as samples of our study, ignoring distinguished and first-rate branches which are few. Moreover, the number of second-rate branches is in a way that does not follow the rule of "3 (number of inputs + number of outputs) ≤ number of assessed units". On the other hand, inputs and outputs index values in fourth and fifth rate branches are considerably different from their values in third-rate branches. Thus, in this research we only study and measure efficiency of the third-rate branches using two-stage DEA model – presented in 2010 by Zha and Liang. Except that we analyze only one intermediate output variable. Using this method the outputs of the first stage are used as the inputs for the second stage; however, these are not the only inputs of the second stage: each DMU's inputs are divided freely between different stages.

In figure (1) we can see the model presented by Zha and Liang that, according to the indices of our research, the number of personnel, assets and expenses are the shared inputs of both stages, amount of which is determined by \( a_i x_{ij} \) in the first stage, and by \((1 - a_i) x_{ij}\) in the second stage. The \( a \) is a factor which is used as a decision variable in distributing inputs among different stages. In other words, some of the personnel, assets and expenses flow to the second stage and together with the intermediate outputs act as inputs for the second stage. Income is considered as the only intermediate output (\( Z_j \)) – output of first stage and input of second –, and facilities and equipment are considered as the final outputs (\( Y_{ij} \)).

![Fig. 1: Two-stage DEA model with input shared flow – Zha and Liang (2010).](image-url)
We said that conventional DEA model which is referred to in model (1) measures performance supposing production as a black box. The inputs which are sent to this box change into outputs. The actual conversion process is usually not modeled accurately; however, it is clear what enters the box and what gets out. In fact, one advantage of DEA is that it reveals the conversion process structure faster than imposition (Fre and Grosskopfb, 2000).

\[
\begin{align*}
\text{Max} & \quad Z_0 = \sum_{r=1}^{p} u_r y_{r0} \\
\text{s.t.} & \quad \sum_{r=1}^{p} u_r y_{rj} - \sum_{j=1}^{m} v_i x_{ij} \leq 0, \quad j = 1, 2, \ldots, n \\
& \quad \sum_{i=1}^{m} v_i x_{i0} = 1, \\
& \quad u_r, v_i \geq 0
\end{align*}
\]

(1)

\(y_{rj}\): the final outputs of each DMU, \(y_{r0}\): final output of the analyzed unit, \(u_r\): output values, \(x_{ij}\): the inputs of each DMU, \(x_{i0}\): input of the analyzed unit, \(v_i\): input values, \(Z_0\): optimal efficiency.

Recently, DEA has been developed to measure two-stage process efficiency, where all the outputs from the first stage are intermediate measures which are considered as inputs for the second stage. The results from the two-stage DEA model not only give overall efficiency score for each stage, but it also gives an efficiency score for each specific stage. Because of intermediate measures the conventional DEA approach will not necessarily determine the border points for the inefficient units (Zha and Liang, 2010). Moreover, because of the existence of intermediate measures, conventional DEA cannot be used directly to measure efficiency of each unit and its sub-units (Liang et al., 2006). The two-stage model can be analyzed from two perspectives: cooperation, and non-cooperation.

**Non-Cooperation:**

The non-cooperation two-stage model can be examined in two forms: first, the sub-unit might be dominant on the system in the first stage, and subordinate in the second stage. And second, it might be dominant in the second stage, and subordinate in the first (Zha and Liang, 2010). However, in a specific case that the intermediate output is single, the optimized efficiency of the first stage of the second type is equal to the optimized efficiency of the second stage of the first type. Model (2) mentioned below is also known as leader-follower model. In leader-follower structure, first the leader is measured and then the follower is assessed using the information from the leader's efficiency. In other words, the dominant sub-unit is measured first and the subordinate sub-unit second.

In model (2) we can see a CCR model which can measure the efficiency of the sub-unit in stage 1.

\[
\begin{align*}
\text{Max} & \quad \frac{wz_0}{\sum_{i=1}^{m} v_i \alpha_i x_{i0}} = \epsilon_{11} \\
\text{s.t.} & \quad \frac{\sum_{j=1}^{n} w_d d_{dj}}{\sum_{j=1}^{n} v_i \alpha_i x_{ij}} = 1, \quad j = 1, 2, \ldots, n \\
& \quad w \geq 0, \quad v_i \geq 0, \quad i = 1, 2, \ldots, m; \quad \alpha_i \geq 0, \quad i = 1, 2, \ldots, m.
\end{align*}
\]

(2)

\(w\): the value given to intermediate output, \(v_i\): the value given to the \(i^{th}\) input, \(z_j\): intermediate output value for the \(j^{th}\) unit, \(x_{ij}\): input value for the \(j^{th}\) unit, \(x_{i0}\): intermediate output value for the analyzed unit, \(x_{i0}\): input value for the analyzed unit, \(\alpha_i\): input distribution coefficient, \(\epsilon_{11}\): optimized efficiency of stage-1 sub-unit

Model (2) can be turned into model (3) by these changes: \(t = \frac{1}{\sum_{i=1}^{m} v_i \alpha_i x_{i0}}, tv_i = v_i, tw_d = w_d\).

\[
\begin{align*}
\text{Max} & \quad \omega z_0 = \epsilon_{11} \\
\text{s.t.} & \quad \sum_{j=1}^{n} v_i \alpha_i x_{ij} \omega z_j \geq 0, \quad j = 1, 2, \ldots, n \\
& \quad \sum_{i=1}^{m} v_i \alpha_i x_{i0} = 1 \\
& \quad \omega \geq 0, \quad v_i \geq 0, \quad i = 1, 2, \ldots, m; \quad \alpha_i \geq 0, \quad i = 1, 2, \ldots, m.
\end{align*}
\]

(3)

By replacing \(v_i, \alpha_i = \pi_{i}^{\dagger}\), model (3) which is a non-linear programming, is changed into model (4).

\[
\begin{align*}
\text{Max} & \quad \omega z_0 = \epsilon_{11} \\
\text{s.t.} & \quad \sum_{j=1}^{n} \pi_{i}^{\dagger} x_{ij} - \omega z_j \geq 0, \quad j = 1, 2, \ldots, n \\
& \quad \sum_{i=1}^{m} \pi_{i}^{\dagger} x_{i0} = 1.
\end{align*}
\]

(4)
ω ≥ 0, π^1_i ≥ 0, i = 1, ... , m.

The corresponding efficiency of the sub-unit in second stage is measured by model (5).

Max \[ \sum_{j=1}^{m} v_j(1-\alpha)x_{i0} + \omega \pi^0_j = e_{12} \]

s.t. \[ \sum_{j=1}^{m} v_{ij} \alpha x_{i0} + \omega \pi^0_j \leq 1, \quad j=1,2,...,n \]
\[ \sum_{j=1}^{m} u_{ij} \gamma_{ij} \leq 1, \quad j=1,2,...,n \]
\[ \sum_{j=1}^{m} v_j(1-\alpha)x_{i0} + \omega \pi^0_j \leq 1, \quad j=1,2,...,n \]
\[ w_i \geq 0; \quad v_{ij} \geq 0, \quad i=1,2,...,m; \quad u_r \geq 0, \quad r=1,2,...,s; \quad 1 \geq \alpha_i \geq 0, \quad i=1,2,...,m. \]

Keep in mind that \( e_{11}^* \) is the optimized efficiency of the first sub-unit of the analyzed unit, \( y_0 \) is the output amount of the jth unit, and \( u_r \) is the value given to the ith output.

Consider the following variable changes:

\[ t = \frac{1}{\sum_{j=1}^{m} v_j \alpha x_{i0}}, \quad t' = \frac{1}{\sum_{j=1}^{m} v_j(1-\alpha)x_{i0} + \omega \pi^0_j} \]
\[ v_{ij} = v_{ij}, \quad t w_{d} = \omega \omega_d, \quad t' v_{ij} = v_{ij}', \quad t' w_{d} = \omega \omega_d', \quad t' u_r = \mu_r, \]
\[ v_{ij} \alpha = \pi^1_i, \quad v_{ij}' = \pi^1_i' \]

Note that \( \delta = \frac{\omega_d}{\omega_d'} = \frac{v_{ij}}{v_{ij}'} = \frac{t}{t'} \), and because \( \delta e_{11}^* \leq 1, 0 \leq \delta \leq \frac{1}{e_{11}^*} \). By replacing \( \pi^1_i = \alpha_i \pi^2_i' \), model (5) is changed into:

Max \[ \sum_{r=1}^{s} \mu_r y_r = e_{12} \]

s.t. \[ \omega = \omega^* \]
\[ \delta [ \sum_{i=1}^{m} (v_{ij} - \pi^1_i) x_{i0} + \omega \pi^0_j - \sum_{r=1}^{s} \mu_r y_r ] \geq 0, \quad j=1,2,...,n \]
\[ \delta [ \sum_{i=1}^{m} (v_{ij} - \pi^1_i) x_{i0} + \omega \pi^0_j - \sum_{r=1}^{s} \mu_r y_r ] = 1, \quad \pi_{ij} \geq \pi^1_i \geq 0, \quad i=1,2,...,m; \quad \omega \geq 0; \quad \mu_r \geq 0, \quad r=1,2,...,s. \]

Considering \( \delta e_{11}^* = \rho \), and \( \delta (\pi_{ij} - \pi^1_i) = \pi_{ij} \), model (7) for measuring second stage efficiency is as follows.

Max \[ \sum_{r=1}^{s} \mu_r y_r = e_{12} \]

s.t. \[ \sum_{i=1}^{m} \pi_{ij} x_{i0} + \rho z_j - \sum_{r=1}^{s} \mu_r y_r \geq 0, \quad j=1,2,...,n \]
\[ \sum_{i=1}^{m} \pi_{ij} x_{i0} + \rho z_j - \sum_{r=1}^{s} \mu_r y_r = 1, \quad \pi_{ij} \geq 0, \quad \rho \geq 0; \quad \mu_r \geq 0, \quad r=1,2,...,s. \]

It's obvious that in order to measure a case in which the sub-unit in second stage is dominant on the system, the steps above are reversed; in fact \( e_{12}^* = e_{22}^* \)

**Cooperation:**

Up to now, we've been considering situation in which the sub-units were dominant on the system, however in bank branches there are times when the sub-units are not dominant on the system and having the same amount of power cooperate together. In cooperation structure the efficiency is measured in this way: the means of efficiency scores for the first sub-unit and the second sub-unit are maximized, and the sub-units are assessed simultaneously.

Max \[ \frac{\omega \pi^0_j}{\sum_{i=1}^{m} v_j \alpha x_{i0} + \omega \pi^0_j} \times \frac{\omega \pi^0_j}{\sum_{i=1}^{m} v_j(1-\alpha)x_{i0} + \omega \pi^0_j} = e_{12} \]

s.t. \[ \sum_{j=1}^{m} v_{ij} \alpha x_{i0} + \omega \pi^0_j \leq 1, \quad j=1,2,...,n \]

2033
\[
\sum_{j=1}^{n} \omega_j y_{ij} \leq 1, \quad j = 1, 2, \ldots, n \\
\sum_{j=1}^{n} \nu_j (1-\alpha_j) x_{ij} + w_{\pi} \leq 1
\]  
(8)

\[
\sum_{i=1}^{m} \nu_i = 1, \quad d = 1, 2, \ldots, D; \quad \nu_i \geq 0, \quad i = 1, 2, \ldots, m; \quad \omega_j \geq 0, \quad r = 1, 2, \ldots, s; \\
1 \geq \alpha_i \geq 0, \quad i = 1, 2, \ldots, m.
\]

Let
\[
t = \frac{1}{\sum_{i=1}^{m} \nu_i \alpha_i x_{i0}}, \quad t' = \frac{1}{\sum_{i=1}^{m} \nu_i (1-\alpha_i) x_{i0} + w_{\pi}}
\]

\[
t = v_i, \quad t' = w, \quad t'' = v_i, \quad t'' = w', \quad t' u_r = \mu_r, \quad \nu_i \alpha_i = \pi_i.
\]

Keep in mind that by the replacement, \( \frac{\omega}{v_i} = \frac{v_i}{t'} = \frac{\omega}{t} \), model (8) can be turned into model (9).

\[
\begin{align*}
\text{Max} & \quad \omega z_0 \times \sum_{r=1}^{s} \mu_r y_{r0} \\
\text{s.t.} & \quad \delta \left[ \sum_{i=1}^{m} (v_i - \pi_i^2) x_{i0} \right] = 1, \\
& \quad \delta \left[ \sum_{i=1}^{m} (v_i - \pi_i^2) x_{i0} \right] - \sum_{r=1}^{s} \mu_r y_{rj} \geq 0, \quad j = 1, 2, \ldots, n \\
& \quad \nu_i \geq 0, \quad i = 1, 2, \ldots, m; \quad \omega \geq 0, \quad d = 1, 2, \ldots, D; \quad \mu_r \geq 0, \quad r = 1, 2, \ldots, s.
\end{align*}
\]  
(9)

By replacing \( \delta \omega = 0, \quad \delta (v_i - \pi_i^2) = 0, \quad k = \omega \cdot z_0 = e_{11}^s \), we'll have:

\[
\begin{align*}
\text{Max} & \quad k \times \sum_{r=1}^{s} \mu_r y_{r0} \\
\text{s.t.} & \quad \sum_{i=1}^{m} (v_i - \pi_i^2) x_{i0} + \rho z_j - \sum_{r=1}^{s} \mu_r y_{rj} \geq 0, \quad j = 1, 2, \ldots, n \\
& \quad \sum_{i=1}^{m} (v_i - \pi_i^2) x_{i0} + \rho z_j = 1, \\
& \quad \pi_i \geq 0, \quad i = 1, 2, \ldots, m; \quad \rho \geq 0; \quad \mu_r \geq 0, \quad r = 1, 2, \ldots, s.
\end{align*}
\]  
(10)

We can also consider a situation in which \( k = \sum_{r=1}^{s} \mu_r y_{r0} = e_{12}^s \), and then measure the cooperation model. However, because measuring \( \delta \) is more complex in this case, we prefer model (4).

**Practical Example:**

Table (1) is related to the indexes of inputs, intermediate outputs, and outputs. This statistical data which is related to the Saderat Bank branches in Guilan province in 1389 has been measured using non-cooperation and cooperation models and the results are given in tables (2) and (3).

<table>
<thead>
<tr>
<th>Table 1: Data of Guilan Saderat Bank Branches.</th>
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RESULTS AND DISCUSSIONS

In the literature of DEA, two-stage cooperation with shared input flow has been used in order for intermediate measures to be included and inputs to flow freely between different stages. Zhu and Chen (2004), and Liang et al., (2006) presented an important effort in measuring efficiency in a two-stage network, which measured DMU's and its stages' efficiency. The subject of this research studies an approach in which the shared flow can flow freely between different stages.

According to table (2), the column related to model (1) shows conventional model of efficiency measurement. In this column the units 5, 6, 9, 14, 16, 17, 18 and 20 have become efficient. However, the defect of this model is not considering intermediate measures. It is obvious that for this reason the amount of efficiency in conventional model (CCR, output-oriented) in most cases is more than the corresponding amounts of the two-stage models. As in the first DMU whose efficiency is 0.96 in model (1) conventional efficiency –; however, the corresponding amount in the non-cooperation first stage is 0.36 and in non-cooperation second stage is 0.71. Furthermore, some units are inefficient in model (1), but are efficient in other models; for example units 8 and 13. This is because model (1) operates like a black box.

Table 2: Efficiency of Traditional and Non-Cooperation Models

| Bank | Model 1 | Situation | Reference Units | Model 2 | Situation | Reference Units | Model 3 | Situation | Reference Units | Model 4 | Situation | Reference Units | Model 5 | Situation | Reference Units | Model 6 | Situation | Reference Units | Model 7 | Situation | Reference Units |
|------|---------|-----------|----------------|---------|-----------|----------------|---------|-----------|----------------|---------|-----------|----------------|---------|-----------|----------------|---------|-----------|----------------|---------|-----------|----------------|---------|-----------|----------------|
| 1    | 0.96    | inefficient | 5,18,20        | 0.36    | inefficient | 5,18          | 0.71    | inefficient | 13             | 2      | 0.67      | inefficient | 5,18,20 | 0.44    | inefficient | 5,18,20 | 0.53      | inefficient | 13             | 3      | 0.63      | inefficient | 14,18,20 | 0.34    | inefficient | 5,18,20 | 0.49      | inefficient | 13             | 4      | 0.88      | inefficient | 5,18,18 | 0.72    | inefficient | 5,18,18 | 0.79      | inefficient | 13             | 5      | 1         | efficient  | -         | 1         | efficient  | -         | -         | -         | -         | 13             | 6      | 1         | efficient  | -         | 0.47      | inefficient | 5,18,14 | 0.85      | inefficient | 13             | 7      | 0.73      | inefficient | 17,14,5  | 0.43    | inefficient | 5,18,18 | 0.55      | inefficient | 13,8           | 8      | 0.89      | inefficient | 15,5      | 1         | efficient  | -         | 1         | efficient  | -         | -         | -         | -         | 13             | 9      | 1         | efficient  | -         | 0.28      | inefficient | 5,18,14 | 0.61      | inefficient | 13             | 10     | 0.91      | inefficient | 5,15,20  | 0.59    | inefficient | 5,18,14 | 0.96      | inefficient | 13             | 11     | 0.89      | inefficient | 5,18,20  | 0.38    | inefficient | 5,18,18 | 0.68      | inefficient | 13             | 12     | 0.96      | inefficient | 20,15    | 0.93    | inefficient | 5,14,18 | 1         | efficient  | -         | -         | -         | -         | -         | -         | -         | 13             | 13     | 0.48      | inefficient | 5,18,20  | 1         | efficient  | -         | 1         | efficient  | -         | -         | -         | -         | -         | -         | -         | 14             | 15     | 0.4       | inefficient | 15,20    | 0.41    | inefficient | 5,14,18 | 0.9       | inefficient | 13,8           | 16     | 1         | efficient  | -         | 0.16      | inefficient | 5,18,14 | 0.3       | inefficient | 13             | 17     | 1         | efficient  | -         | 0.32      | inefficient | 5       | 0.46      | inefficient | 13             | 18     | 1         | efficient  | -         | 1         | efficient  | -         | 1         | efficient  | -         | -         | -         | -         | -         | 13             | 19     | 0.92      | inefficient | 5,15     | 0.21    | inefficient | 5,14,18 | 0.57      | inefficient | 13             | 20     | 1         | efficient  | -         | 0.43      | inefficient | 5,18,18 | 0.88      | inefficient | 13             |

The efficiency of most units in model 4 (non-cooperation first stage model) is less than their efficiency in model 7 (non-cooperation second stage model), and this shows the weakness of units in production. But, units 5 and 14 which are efficient in production stage (first stage) and inefficient in profitability stage (second stage), are exceptions. Therefore, by dividing the DMUs into two sub-DMUs the weakness of the bank branches is better shown. Non-cooperation model has a leader-follower structure, so that the leader is assessed using conventional DEA model 1, and then the efficiency of the follower is measured using the optimized efficiency of the leader. The main problem with non-cooperation models is that the intermediate output is considered as the output of the first stage and input of the second, so its increase for the first stage (leader) and its decrease for the second stage (follower) is desired. Table (2) shows that only units 8, 13, and 18 have been able to preserve efficiency in both stages. We used cooperation models in order to solve the problem with non-cooperation models, the results of which are shown in table (3).

The cooperation model has been used to maximize the shared efficiency of two sub-units, and that's why in most cases the efficiency of units in this model is less than their corresponding non-cooperation stages efficiency. Although this model is non-linear programming, we showed how it can be solved as a linear parametric programming problem. In order to change non-linear programming into linear programming, we put \( \omega z_0 + \sum_{r=1}^{\mu} \eta r y_r = 0 \) equal to \( K \), separately. The results show that the cooperation model when assuming \( K = e_{11}^1 \) to be constant gives lower results than when assuming \( K = e_{12}^1 \) to be constant. This is because the results form the objective function (as we said) because of weakness in production is lower than the corresponding amount of \( e_{12}^1 \). As in non-cooperation models' results, the units 8, 13, and 18 are efficient in cooperation models too. Thus, it can be said that according to the results only these three branches work efficiently; although in conventional DEA model the units 8 and 13 have been shown to be inefficient, they were identified as efficient using two-stage model with shared flow input. These results are in line with similar research on two-stage models, like Zha and Liang (2010). According to tables 2 and 3, we can see that all models have introduced...
some units as reference units, but what are more important in this research are the two-stage models. As is seen in non-cooperation model in the first stage mostly units 5 and 8 and sometimes 14 and 18 are identified as reference units, and in non-cooperation model in the second stage and also in cooperation model unit 13 has been identified as the reference unit for most inefficient units.

Table 3: Efficiency of Cooperation Model.

<table>
<thead>
<tr>
<th>Bank</th>
<th>$K = \omega z^*_t$</th>
<th>$K = \sum_{r=1}^{s} \mu_r r^*_r$</th>
<th>Efficiency</th>
<th>Situation</th>
<th>Reference Units</th>
<th>Efficiency</th>
<th>Situation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.36</td>
<td>0.71</td>
<td>0.25</td>
<td>inefficient</td>
<td>13</td>
<td>0.5</td>
<td>inefficient</td>
</tr>
<tr>
<td>2</td>
<td>0.44</td>
<td>0.53</td>
<td>0.23</td>
<td>inefficient</td>
<td>13</td>
<td>0.28</td>
<td>inefficient</td>
</tr>
<tr>
<td>3</td>
<td>0.34</td>
<td>0.49</td>
<td>0.17</td>
<td>inefficient</td>
<td>13</td>
<td>0.24</td>
<td>inefficient</td>
</tr>
<tr>
<td>4</td>
<td>0.72</td>
<td>0.79</td>
<td>0.27</td>
<td>inefficient</td>
<td>13</td>
<td>0.62</td>
<td>inefficient</td>
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<tr>
<td>5</td>
<td>1</td>
<td>0.65</td>
<td>0.38</td>
<td>inefficient</td>
<td>13</td>
<td>0.42</td>
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<tr>
<td>6</td>
<td>0.47</td>
<td>0.85</td>
<td>0.4</td>
<td>inefficient</td>
<td>13</td>
<td>0.72</td>
<td>inefficient</td>
</tr>
<tr>
<td>7</td>
<td>0.43</td>
<td>0.55</td>
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<td>inefficient</td>
<td>13</td>
<td>0.3</td>
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<tr>
<td>8</td>
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<td>1</td>
<td>1</td>
<td>efficient</td>
<td>13</td>
<td>1</td>
<td>efficient</td>
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<td>inefficient</td>
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<td>0.37</td>
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<td>0.56</td>
<td>inefficient</td>
<td>13</td>
<td>0.92</td>
<td>inefficient</td>
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<tr>
<td>11</td>
<td>0.38</td>
<td>0.68</td>
<td>0.26</td>
<td>inefficient</td>
<td>13</td>
<td>0.46</td>
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<tr>
<td>12</td>
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<td>0.93</td>
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<tr>
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<td>0.05</td>
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<td>13</td>
<td>0.09</td>
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<tr>
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<td>0.46</td>
<td>0.15</td>
<td>inefficient</td>
<td>13</td>
<td>0.21</td>
<td>inefficient</td>
</tr>
<tr>
<td>18</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>efficient</td>
<td>13</td>
<td>1</td>
<td>efficient</td>
</tr>
<tr>
<td>19</td>
<td>0.21</td>
<td>0.57</td>
<td>0.12</td>
<td>inefficient</td>
<td>13</td>
<td>0.33</td>
<td>inefficient</td>
</tr>
<tr>
<td>20</td>
<td>0.43</td>
<td>0.88</td>
<td>0.38</td>
<td>inefficient</td>
<td>13</td>
<td>0.77</td>
<td>inefficient</td>
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</tbody>
</table>

Conclusions:

Technical efficiency depends on effective use of inputs in the bank's technology. The efficient banks can produce more outputs with lower interest expenses. In other words, specific efficiency is related to the way inputs affect the production process. If a bank has a lower specific efficiency than another, it can be inferred that by changing the way its inputs are used it can increase its outputs. In table 4 we suggest how much each branch of Saderat bank in Guilan province can get close to efficiency level by decreasing its inputs. As an example, if unit 1 reduces its number of personnel for one unit and make it seven, and also reduces the assets, expenses, and incomes to 40232.52, 1328.25, and 2153.49 respectively, it can become efficient.

Table 4: Inputs and Intermediate Output Of Reference Units for Cooperation Model.

<table>
<thead>
<tr>
<th>Bank</th>
<th>Personnel $X_1$</th>
<th>Assets $X_2$</th>
<th>Expenses $X_3$</th>
<th>Income $Z$</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>6.9</td>
<td>40232.52</td>
<td>1328.25</td>
<td>2153.49</td>
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<tr>
<td>2</td>
<td>7.1</td>
<td>41398.68</td>
<td>1366.75</td>
<td>2215.91</td>
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<tr>
<td>3</td>
<td>6.1</td>
<td>35567.88</td>
<td>1174.25</td>
<td>1903.81</td>
</tr>
<tr>
<td>4</td>
<td>5.6</td>
<td>32652.48</td>
<td>1078</td>
<td>1747.76</td>
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<tr>
<td>5</td>
<td>5.7</td>
<td>33235.56</td>
<td>1097.25</td>
<td>1778.97</td>
</tr>
<tr>
<td>6</td>
<td>9.6</td>
<td>55975.68</td>
<td>1848</td>
<td>2996.16</td>
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<td>4.05</td>
<td>25221.24</td>
<td>421.2</td>
<td>1229.04</td>
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<td>27404.76</td>
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<tr>
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<td>9.3</td>
<td>71897.16</td>
<td>1819.08</td>
<td>5746.47</td>
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<tr>
<td>14</td>
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<td>762.12</td>
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</tr>
<tr>
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<td>19</td>
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<td>673.75</td>
<td>1092.35</td>
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<tr>
<td>20</td>
<td>3.04</td>
<td>18246.46</td>
<td>619.02</td>
<td>1374.08</td>
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</table>

In this research we pursued models that, unlike the conventional DEA model, could measure efficiency of DMU and its stages, doing so we showed the weaknesses of each inefficient branch in each of the production and profitability stages. We identified reference units for each of the inefficient units and suggested those units what to do to become efficient.

REFERENCES


