Determination of Lot Sizing in Supply Chain and Determination of Supplier Based on Simulated Annealing Algorithm Approach

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Abstract: The study introduces a scenario for determining multi-stage inventory shipment with several suppliers and products, while several discrete product demand for more than one planning perspective in under consideration. Several suppliers provide each item in products list. Value of transaction is a function of supplier and time period. Warehouse charges also a function of product at the time of delivery. Decision makers have to decide about type of products, supplier and period of procurement. Taking into account non-linear nature of the model and its complexity, the study uses meta-creative simulated annealing algorithm.

Key words: supply chain, choosing suppliers, inventory control, simulated annealing algorithm

INTRODUCTION

Choosing a proper group of suppliers is vital choice for survival on commercial foundations, and many scholars have emphasized importance of such choices. Recently, and following emergence of supply chain management (SCM), majority of scholars have agreed that finding right supplier and proper management system are of greatest importance to achieve competitive advantages.

Single-product multi stage inventory supplying model is a well known phenomenon in production and inventory management field. There are many researches conducted in this field. Including studies by Wangner and Whitin (1958) this led to introduction of a logarithm for dynamic programming. Development of this model results in deeper studies regarding multi-product & multi-supplier models. This study focuses on one of the developed models and takes a scenario into account, which considers discrete multiple products demand for more that one planning perspectives. Each item in the products list is supplied by a specific supplier and sum of transaction depends on the supplier and time period. Charges of warehouse for inventory at each period and for each product are also considered in the planning. Decision makers need to decide about type of product and supplier at the period of time.

One of the comprehensive studies in the field of lot sizing is conducted by Bahl et al., (1987). They defined four groups and teams for classifying the works in this field, including: mono-level unlimited resources, mono-level limited resources, multi-level limited resources, multi-level unlimited resources. The term “level” refers to different levels of raw materials structure, which depends on necessities, and the term “resources” refers to limitations of production capacity. The scenario under consideration of this study is located in first group. The study, therefore, does not take into account production capacity limitations and restrains. There are several academic works in this field beginning by works of Harris (1915). Wagner & Whitin, (1958) introduced a dynamic planning algorithm for the problem. Many others have developed their models including Aggarwal & Park (1993) and even introduced new algorithms. Researchers have also developed some approaches for achieving procurement costs reduction and attenuating demand uncertainty. New studies in this field have taken into account new issues such lot sizing, capacity limitations, time period for meeting demand, randomly suppliers’ capacity, over work decision making, reducing commissioning time, sale and emergency reserves, etc.

Although Wagner – Whitin’s (1958) algorithm is an optimum solution, its approach is very complex and needs great deal of calculations. Trying to find answer of the calculation, others such as Silver and Meal (1973) have devised an innovative algorithm. Moreover, other scholars have tried to improve the former solution. All of these are just quest to simply and lessen amount of required calculations through creating
functional data structures and more reliable rules.

Other studies on Wagner and Whitin (1958) have shown that the model for periods of T may be solved in time of O(T log T). These worked followed by Heady & Zhu (1994) have succeeded to lessen implementation time through almost linear algorithm on number of planning periods.

Along with emergence of chain management, choosing supplier has also received significant attention. Rosental et al., (1995) studied a procurement problem in case of discount on the side of suppliers and select the supplier for multiple products through formulating programming mixed integer. Chaudhry et al., (1993). focused on selecting suppliers under restricts of quality, shipment, capacity and price break regime. Ganeshan (1999) introduced a model for determining size of lot including several suppliers while there are several retailers and demand for stocking.

Kasilingam& Lee (1996) add to the problem fix cost of store keeping in a single period model including uncertainty of demand an role of quality for selecting specific supplier. Following the same way, Jayaraman et al., (1999) introduced a model for selecting supplier based on quality (considering number of unacceptable items provided by each supplier), production capacity (limitation of supplier), delay and warehousing limitation. In fact, this is a single-period model which assigns fix cost to supplier.

Kumar et al., (2004) use fuzzy targeted programming to solve the problem of selecting vendor (supplier) considering that some parameters have fuzzy nature. They applied real world data to show effectiveness of their model and argued that their model is capable to manage real world situation in a fuzzy environment, and this provides decision makers with a tool to select supplier in a supply chain.

Verma & Pullman (1998) analyzed supplier selection process in their article and argued that customers select suppliers based on relative importance of different specifics such as quality, price, flexibility and delivery. In their research, they focused on difference of importance each manager assigned to each trait in experimental environment. Results showed that despite manager claim about quality as the most important factors, cost and performance are mainly two factors affecting on choosing supplier.

Basnet & Leung (2005) introduced their model for selecting multiple suppliers through multiple periods, and this research is following their works. Limitations taken into account in their research include meeting demands at each period and at the same period intact and selecting suppliers based on price. Meanwhile, by designing the model, they tried to solve it through decision tree and an innovative algorithm. In addition to these limitations, this study adds other limitations to the problem including maximum capacity of supplier at each period, and minimum demand at each supplier based on transportation costs. In addition, a simulated annealing algorithm is used for solving the problem.

Determining Size of Multi-period Lot Through Supplier Selection Problem:

This work tries to introduce a model for lot sizing for multiple products during multiple periods from multiple suppliers through applying following points;

Product index (inventory) \( i=1,\ldots, I \)

Suppliers index \( j=1,\ldots, J \)

Time period index \( t=1,\ldots, T \)

Parameters:

- \( D_{it} \) = demand for product \( i \) during period \( t \)
- \( P_{ij} \) = price of price \( i \) from supplier \( j \)
- \( H_{it} \) = warehouse cost for product \( i \) at each period
- \( O_{jt} \) = transaction costs from supplier \( j \)
- \( M_{jt} \) = maximum assigned capacity of supplier \( j \) for products at time \( t \)
- \( B_{jt} \) = minimum order to each supplier

Decision Making Variables:

- \( X_{ijt} \) = number of orders for product \( i \) from supplier \( j \) during period \( t \)
- \( Y_{jt} \) = 1 when order from supplier \( j \) during period \( t \) is met, 0 when order from supplier \( j \) during period \( t \) is not met)

Dependent Variables:

- \( R_{it} \) = inventory for product \( i \) remained from period \( t \) transferred to period \( t+1 \)
According to Basnet & Leung (2005) models and adding the restrictions on the maximum capacity of each provider in each period and minimum demand of the supplier of transportation costs which are acceptable, in this section in order to determine the lot sizing in multiple resourcing mode the model is presented as follows:

\[ \text{Min } Z = \sum_{i} \sum_{j} \sum_{t} p_{ij} x_{ijt} + \sum_{j} \sum_{t} o_{jt} y_{jt} + \sum_{i} \sum_{j} \sum_{t} H_{i} \left( \sum_{k=1}^{t} \sum_{j} x_{ijk} - \sum_{k=1}^{t} d_{ik} \right) \]

Subject to:

\[ R_{it} = \left( \sum_{j} \sum_{k} x_{ijk} \right) - \sum_{k=1}^{t} d_{ik} \geq 0 \text{ For all } i \text{ and } t \]

\[ \left( \sum_{k=1}^{t} d_{ik} \right) y_{jt} - x_{ijt} \geq 0 \text{ For all } i, j \text{ and } t \]

\[ \sum_{j} x_{ijt} \leq M_{jt} \text{ For all } j \text{ and } t \]

\[ (\sum_{j} x_{ijt} - B) y_{jt} + (B - \sum_{i} x_{ijt})(1 - y_{jt}) \geq 0 \text{ For all } j \text{ and } t \]

\[ y_{jt} = 0 \text{ or } 1 \text{ For all } j \text{ and } t \]

\[ x_{ijt} \geq 0 \text{ And integer } \text{ For all } i, j \text{ and } t \]

This method may be considered as a type of formulating through mixed integer programming method. Target function is comprised of 3 sections including: 1- cost of procured products, 2- cost of transaction for suppliers; 3- warehouse costs for remain of inventory at each period.

The model considers 4 limitations, which need to be determined based on value of \( i, j, \) and \( t \). Generally, the model includes \( i \times j + 2 \times j \times t + i \times j \times t \). First limitation implies that all demands need to be met at its specific period (assuming that value of transaction is completely paid and all products are acceptable). Second limitation implies that it is no possible to place an order when the price is not reasonable. Thus it is a case when no order is placed to a supplier, variable \( Y_{it} \) of that supplier is 0. In case on only one supplier, it is possible to remove price limitation, and all Wagner & Whitin’s (1958) theorems for multiple product case are applicable, unless the case comprises of multiple inventories. Third limitation implies that within each programming perspective, each supplier is faced with supply (production) limitation, and the fourth one implies that should inventory volume of each supplier at each period be less that a specific level, no order will be placed on that supplier. Moreover, by considering \( X_{ijt} \) we imply that volume of order for product \( i \) supplier \( j \) during period \( t \) is an integer.

Processes and solutions given in classic methods such as search tree are considerably time consuming. Thus, this study introduces a meta-innovative method based on simulated annealing algorithm, which is very effective for big problems.

**Case study:**

Three products \( (A, B, C) \) are under consideration based on demands:

Demand matrix:

\[
D = \begin{bmatrix}
12 & 15 & 17 & 20 & 13 \\
20 & 21 & 22 & 23 & 24 \\
20 & 19 & 18 & 17 & 16
\end{bmatrix}
\]

First row of demand for product \( A \) is illustrated in horizontal planning for five periods:

Three suppliers \( (a, b, y) \) are under consideration with following price matrix:
First row denotes price for product \( A \) through three suppliers. Warehousing cost is indicated by vector \( H_j \), which comprises of \((1,2,3)\). Warehousing costs for a unit of product \( A \) during a period is equal to 1 unit of money. Transaction cost is indicated by vector \( O_j \) comprising of \((110, 80, \text{ and } 102)\). Cost of transaction for sale and order, for supplier \( \alpha \) is 110 of money unit.

Maximum assigned capacity to supplier \( j \) for different products in time \( t \) is as following matrix:
\[
M_{jt} = \begin{bmatrix}
50 & 64 & 45 \\
60 & 50 & 55 \\
57 & 68 & 70 \\
76 & 63 & 59 \\
68 & 57 & 73 \\
\end{bmatrix}
\]

Minimum supply to each supplier \( B \) in the research is at 30 units. The case study, therefore, includes 60 variables of decision (45 variable integer \((i\times j\times t)\) and 15 variables \(0\) and \(1\)\((j\times t)\), and 90 limitations \((i\times j\times 2\times j\times t+i\times j\times t)\)

To have better understanding, a reasonable answer is as follows: other \( X_{ijt} \)s are equal to 0.

Order for 44 units of product 1 from supplier 1 at period 1 \( \Rightarrow X_{111} = 44 \)
Order for 20 units of product 2 from supplier 1 at period 1 \( \Rightarrow X_{111} = 20 \)
Order for 20 units of product 3 from supplier 1 at period 1 \( \Rightarrow X_{111} = 20 \)
Order for 43 units of product 2 from supplier 2 at period 2 \( \Rightarrow X_{111} = 43 \)
Order for 37 units of product 3 from supplier 2 at period 2 \( \Rightarrow X_{111} = 37 \)
Order for 33 units of product 1 from supplier 4 at period 4 \( \Rightarrow X_{111} = 33 \)
Order for 47 units of product 2 from supplier at period 4 \( \Rightarrow X_{111} = 47 \)
Order for 33 units of product 3 from supplier 4 at period 33 \( \Rightarrow X_{111} = 33 \)

**Calculating Costs for Each Solution:**

Number of order to supplier 1 is equal to 1 (at period 1). Cost of transaction = \(1\times 110 = 110\)
Cost of purchasing product 1 is equal to: \(44\times 30 = 1320\)
No order gives to supplier 2.
Number of orders to supplier 3 is equal to 3 (at periods 1, 2 and 4). Cost of transaction = \(3\times 102 = 306\)

Cost of product 1: \(33\times 32 = 1056\)
Cost of product 2: \((20+43+47)\times 30 = 330\)
Cost of product 3: \((20+37+33)\times 4 = 4050\)

Transitive matrix of inventory is as follows:
\[
R_p = \begin{bmatrix}
32 & 17 & 0 & 13 & 0 \\
0 & 22 & 0 & 24 & 0 \\
0 & 18 & 0 & 16 & 0 \\
\end{bmatrix}
\]
That \( R_{ij} \) implies for first input.

\[
R_{11} = X_{11} - D_{1} = 44 - 12 = 32
\]
\[
H_1 \sum R_{ij} = 1 \times (32 + 17 + 0 + 13 + 0) = 62
\]
\[
H_2 \sum R_{ij} = 2 \times (0 + 22 + 0 + 24 + 0) = 92
\]
\[ H_3 \sum R_{3i} = 3 \times (0 + 18 + 0 + 16 + 0) = 102 \]

So, total cost will be:
\[ 110 + 1320 + 306 + 1056 + 3300 + 4050 + 62 + 92 + 102 = 10398 \]

**Simulated Annealing Algorithm Solution Approach**

**General Simulated Annealing Algorithm:**
The general algorithm SA, which was introduced at the first time in 1953 by Metropolis, is as follows: (Majazi, 2011)

1. Select an initial Solution \( a = a_0 \in S_1 \).
2. Select an initial temperature \( T = T_0 \geq 0 \).
3. Set temperature change counter \( t = 0 \).
4. Repeat **Freezing Process**, set repetition counter \( i = 0 \),
   - Repeat - *Equilibrium Process*
     - Generate solution \( b \), a neighbor of \( a \),
     - \[ \Delta f = f(a) - f(b) \]
     - If (\( \Delta f \geq 0 \)) then \( a_i = b \),
     - Else if \( \text{random}(0,1) < \exp \left( \frac{-\Delta f}{T} \right) \) then \( a_i = b \).

\[ i = i + 1 \]

Until \( i = N(z) \)

\[ z + 1, \]

\[ T = T(z) \]

Until stopping criterion true

The main steps to implement a SA are as follows: (Majazi, 2011)
1. Determining the way of giving the first answer as starting the search point
2. Determining the initial temperature
3. Determining the rate of temperature decrease
4. Determining the way of neighborhood creation
5. Determining the number of reviewed neighborhoods in each temperature
6. Scale stop
    And the general procedure of SA is shown in Fig. 1.

**The Main Components of SA for Implementation Are as Follows:**

**Creating the Initial Answer:**
In this programme 1000 random answers by the length \( h \) have been created and the answer with the best objective function was selected as the start point.
Fig. 1: The general procedure of SA

**Initial temperature:**

Temperature is one of the parameters that plays an important role in acceptance or rejection the variation of the objective functions. Selection initial value of temperature should be such a way that in the early stages, many undesirable answers could be selected. This is because of the possibility of the development and changes the answer (Kirkpatrick, 1983).

Actually the initial temperature tells us how much we let the answer be deteriorated. It is more accurate to say how much does each deterioration have probability. The accept probability of each deterioration answer is:

\[-\frac{\text{Index of deterioration the answer}}{\text{Temp}}\]

In order to be our index, almost independent of the problem size we should place it equal to:

\[
\frac{\text{Scale of deterioration the objective function}}{\text{objective function}}
\]

**Determining the Rate of Temperature Decrease:**

In order to reduce the probability of accepting the unfavorable answers, we should reduce the temperature. The way of changing this parameter is on the base of the changes of the temperature functions \(T_k = \alpha \cdot T_{k-1}, \alpha < 1\) in this paper \(\alpha = 0.95\) is selected. (Kirkpatrick, 1983)

**Determining the Way of Creating Neighborhood:**

In this paper we change two time units of H. if the functions get better, we can accept this relocation And if it deteriorate, we should find the size of deterioration of objective function. We can find the index of deterioration of the objective function via the following equation (Kirkpatrick, 1983):
Index of deterioration the objective function = \[
\frac{\text{Scale of deterioration the objective function}}{\text{objective function}}
\]

And we produce a random number between 0 and 1 by uniform distribution.

\[
\text{if } \frac{\text{Index of deterioration the answer}}{\text{Temp}} > \text{rand}(0,1),
\]

Then we accept deterioration of the answer. If not, another neighborhood will be chosen.

**Determining the Number of Neighborhoods Which Are Reviewed in Each Temperature:**

In order to reach the better answers, more iteration is necessary. These iterations should be determined so that the runtime will be minimized. At the same time the answer must be favorite. In this paper the number of the iteration are constant and equal to 1000.

**Scale stop:**

The runtime of the calculation is dependent on the scale stop. It is important how this scale is efficient in determining the favorite answer. The algorithm ends at that time when the answers in each temperature don’t be changed by increasing the temperature. This state is called freezing state. This status is assumed as the scale stop. In this paper the final temperature is assumed 0.002.

The answers obtained by simulated annealing (Table 1) for case study instance are more efficient than answers obtained by hand solution.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Simulated annealing solution</th>
<th>CPU time</th>
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<tbody>
<tr>
<td>$X_{122}$</td>
<td>30</td>
<td>58s</td>
</tr>
<tr>
<td>$X_{123}$</td>
<td>18</td>
<td></td>
</tr>
<tr>
<td>$X_{223}$</td>
<td>39</td>
<td></td>
</tr>
<tr>
<td>$X_{233}$</td>
<td>33</td>
<td></td>
</tr>
<tr>
<td>$X_{333}$</td>
<td>21</td>
<td></td>
</tr>
<tr>
<td>$X_{323}$</td>
<td>38</td>
<td></td>
</tr>
<tr>
<td>$X_{334}$</td>
<td>27</td>
<td></td>
</tr>
<tr>
<td>$X_{124}$</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>$X_{224}$</td>
<td>33</td>
<td></td>
</tr>
<tr>
<td>$X_{234}$</td>
<td>21</td>
<td></td>
</tr>
<tr>
<td>$X_{134}$</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>Objective function</td>
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<td></td>
</tr>
</tbody>
</table>

**Parameter setting:**

In this section, the results of the computational experiments are used to evaluate the performance of the proposed algorithm for determination of lot sizing in supply chain and determination of supplier problem. There are nine instances for each problem size. At this point, some information about parameter analysis would be useful. Initially, several experiments were conducted on test problems in order to determine the tendency for the values of parameters. Six test problems were used for this purpose.

In each step, only one of the parameters was tested. Each test was repeated four times. We considered the following values for the several parameters required by the proposed SA:

- Initial Temperature (IT): four levels (1, 0.90, 0.80 and 0.75).
- Temperature Decrease Rate (TDR): four levels (0.02, 0.04, 0.05 and 0.08).
- Final Temperature (FT): three levels (0.001, 0.002 and 0.005).
- Number of Iteration in each Temperature (NIT): one level (1000).

Test results showed that these values were suitable for the problem. Later, additional tests were conducted in order to determine the best values. After completing the tests, Taguchi analysis is applied for the different values of parameters. The best values of the computational experiments for determination of lot sizing in supply chain and determination of supplier problem were obtained for IT = 0.95, TDR = 0.04, FT = 0.002 and NIT = 1000. These values were set as the default value of the Parameters.
Conclusion:
Cost of raw materials and the items comprising the products is the main portion of costs in majority of industries, which in some cases exceeds 70% of end product. Therefore, selecting a proper supplier will be of great importance for attenuating costs and competitive advantages of the organization.

This study presented a model for selecting supplier through determining lot sizing of classic inventory; each item in the list may be provided by a set of suppliers. Cost of transaction depends on supplier and specific period of time. Cost of warehousing for each items and within its specific period is also taken into account in planning perspective. The model provides the answer for type of product, volume of order and supplier at each period. Considering nonlinear nature and complexity of the model, classic methods are too time consuming and complicated. Thus meta-innovative method of simulated annealing algorithm used for the case study. And we obtained very good solutions in very short time.

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