On $t$-Best Coapproximation in Fuzzy 2-Normed Spaces

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Abstract: In this paper we study the concept of $t$-best coapproximation in fuzzy 2-normed spaces. We introduce the notions of $t$-best coapproximation, $t$-coproximinal sets, $t$-coChebyshev sets and $t$-orthogonality and prove some interesting theorems to characterization of $t$-best coapproximation elements. 2000 Mathematics subject classifications: Primary 46S40, Secondary 41A50;

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INTRODUCTION

The theory of fuzzy sets was introduced by L. Zadeh in 1965. Many mathematicians considered the fuzzy normed and 2-normed spaces in several angels (Mohiuddine(2011), Bag and Smanta(2003), Cheng and Mordeson(1994)). S. A. Mohiuddine has introduced the concept of $t$-best approximation in fuzzy 2-normed spaces. We investigate another kind of approximation, called best coapproximation, in fuzzy 2-normed spaces. The concept of best coapproximation was introduced by Franchetti and Furi(1972), in order to study some characteristic properties of real Hilbert spaces, and such problems were considered further by Papini and Singer(1979), and Rao et al.(1999). In this context, we shall consider the set of all $t$-best coapproximations in fuzzy 2-normed spaces and then prove several theorems pertaining to this set.

PELIMINARIES:

Definition 2.1: A $t$-norm is a continuous binary operation $*: [0,1] \times [0,1] \rightarrow [0,1]$ such that $([0,1],*)$ is a abelian monoid with unit 1 and $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0,1]$. It is clear that if we define $ab$ or $a * b = \min(a, b)$ then $*$ is a continuous t-norm.

The concept of linear 2-normed spaces was introduced by Gähler in 1965 as follows.

Definition 2.2: The 2-normed space is a pair $(X, \|.,.\|)$, where $X$ is a linear space of a dimension greater than one and $\|.,.\|$ is a real valued mapping on $X \times X$ such that the following conditions are satisfied:

(i) $\|x, y\| = 0$ if and only if $x$ and $y$ are linearly dependent,
(ii) $\|x, y\| = \|y, x\|$ for all $x, y \in X$,
(iii) $\|\alpha x, y\| = \|x, \alpha y\|$ whenever $x, y \in X$ and $\alpha \in \mathbb{R}$,
(iv) $\|x + y, z\| \leq \|x, z\| + \|y, z\|$ for all $x, y, z \in X$.

As an example of a 2-normed space we may take $X = \mathbb{R}^2$ being equipped with a 2-norm $\|x, y\| = \text{the area of the parallelogram spanned by vectors } x$ and $y$, which may be given explicitly by the formula

$\|x, y\| = |x_1 y_2 - x_2 y_1|$, where $x = (x_1, x_2), y = (y_1, y_2)$.

Now we define the concept of fuzzy 2-normed spaces.
Definition 2.3: The 3-tuple \((X, N, *)\) is said to be a fuzzy 2-normed space if \(X\) is a vector space, \(\ast\) is a continuous \(t\)-norm and \(N\) is a fuzzy set on \(X \times X \times \mathbb{R}\) satisfying the following conditions for every \(x, y, z \in X\) and \(t, s > 0\).

1. \(N(x, y; t) = 0\) for all \(t \in \mathbb{R}\) with \(t \leq 0\),
2. \(N(x, y; t) = 1\) if and only if \(x\) and \(y\) are linearly dependent for all \(t \in \mathbb{R}\) with \(t > 0\),
3. \(N(x, y; t) = N(y, x; t)\) for every \(x, y \in X\),
4. \(N(\alpha x, y; t) = N(x, y; t/|\alpha|)\), for all \(\alpha \neq 0\),
5. \(N(x, z; t) * N(y, z; s) \leq N(x + y, z; t + s)\), for all \(t, s \in \mathbb{R}^+\).

If in addition for \(t > 0\), \((x, y) \rightarrow N(x, y; t)\) is continuous map on \(X \times X\), then \((X, N, *)\) is said a strong fuzzy 2-normed space.

Definition 2.4: Let \((X, N, *)\) be a fuzzy 2-normed space. For \(t > 0\), the open ball \(B_z(x, r, t)\) and the closed ball \(\bar{B}_z(x, r, t)\) with the center \(x \in X\) and radius \(0 < r < 1\) are defined as follows:

\[
B_z(x, r, t) = \{y \in X : N(x - y, z; t) > 1 - r\},
\]
\[
\bar{B}_z(x, r, t) = \{y \in X : N(x - y, z; t) \geq 1 - r\}.
\]

Example 2.5: Let \((X, \|., .\|)\) be a 2-normed space. If we define \(a * b = ab\) or \(a * b = \min\{a, b\}\) and

\[
N(x, y; t) = \frac{kt^n}{kt^n + m \|x, y\|}, k, m, n \in \mathbb{R}^+.
\]

Then \((X, N, *)\) is a fuzzy 2-normed space. In particular if \(k = m = n = 1\) we have

\[
N_s(x, y; t) = \frac{t}{t + \|x, y\|},
\]

which is called the standard fuzzy 2-norm induced by the 2-norm \(\|., .\|\).

Definition 2.6: Let \((X, N, *)\) be a fuzzy 2-normed space. For \(t > 0\) and nonzero \(z \in X\), a subset \(A \subseteq X\) is called \(t\)-bounded if there exists \(0 < r < 1\) such that \(N(x, z; t) > 1 - r\) for all \(x \in A\).

Definition 2.7: For \(t > 0\) and nonzero \(z \in X\), a sequence \(\{x_n\}\) in a fuzzy 2-normed space \((X, N, *)\) is called a \(t\)-convergence sequence to \(x \in X\) if for each \(0 < \varepsilon < 1\) there exists \(N \in \mathbb{N}\) such that for all \(n \geq N\) we have \(N(x_n - x, z; t) > 1 - \varepsilon\), and we will denoted it by \(x_n \rightarrow_x t\).

Definition 2.8: Let \((X, N, *)\) be a fuzzy 2-normed space. For \(t > 0\) and nonzero \(z \in X\), a subset \(A \subseteq X\) is called \(t\)-closed, if for each sequence \(\{x_n\}\) in \(A\) such that \(x_n \rightarrow_x t\), then \(x \in A\). Also we recall that a subset \(A \subseteq X\) is called \(t\)-compact, if every sequence \(\{x_n\}\) in \(A\) has a subsequence \(\{x_{n_k}\}\) which \(t\)-converges to an element \(x_0 \in A\).

\(t\) - Best Coapproximation:
Definition 3.1: Let $A$ be a nonempty subset of fuzzy 2-normed space $(X, N, *), \mathbb{N}$ and $t > 0$. For $x \in X$ and $z \in X$, an element $y_0 \in A$ is said to be a $t$-best coapproximation of $x$ from $A$ if

$$N(y_0 - y, z; t) \geq N(x - y, z; t),$$

for all $y \in A$. The set of all elements of $t$-best coapproximation of $x$ from $A$ is denoted by $R^t_A(x; z)$; i.e.,

$$R^t_A(x; z) = \{y_0 \in A : N(y_0 - y, z; t) \geq N(x - y, z; t), \text{ for all } y \in A\}.$$

For $t > 0$ putting

$$\tilde{A}_t = \{x \in X : N(y, z; t) \geq N(y, z; t) \text{ for all } y \in A\} = (R^t_A)^{-1}(\{0\}).$$

It is clear $y_0 \in R^t_A(x; z)$ if and only if $x - y_0 \in \tilde{A}_t$.

Definition 3.2: Let $A$ be a nonempty subset of a fuzzy 2-normed space $(X, N, *)$. If for $t > 0$ and nonzero $z \in X$, each $x \in X$ has at least (respectively exactly) one $t$-best coapproximation in $A$, then $A$ is called a $t$-coproximinal (respectively $t$-coChebyshev) set. Also $A$ is called $t$-quasi-coChebyshev set if $R^t_A(x; z)$ is a compact set.

Theorem 3.3: Let $(X, N, *)$ be a fuzzy 2-normed space and $A$ be a subspace of $X$ and $t > 0$. Then for nonzero $z \in X$

(i) $A$ is a $t$-coproximinal subspace if and only if $X = A + \tilde{A}_t$.

(ii) $A$ is a $t$-coChebyshev subspace if and only if $X = A \oplus \tilde{A}_t$.

Proof. (i) $(\Rightarrow)$ Assume that $A$ is $t$-coproximinal, $x \in X$ and $y_0 \in R^t_A(x; z)$. Then $x - y_0 \in \tilde{A}_t$. Now, $x = y_0 + (x - y_0) \in A + \tilde{A}_t$. Hence $X = A + \tilde{A}_t$.

$(\Leftarrow)$ Let $x \in X = A + \tilde{A}_t$. Then $x = y_0 + \tilde{y}, y_0 \in A, \tilde{y} \in \tilde{A}_t$ and so $0 \in R^t_A(\tilde{y}, z) = R^t_A(x - y_0, z)$. Since $N(0 - (x - y_0), z; t) \geq N(y - (x - y_0), z; t)$, so $N(y_0 - x, z; t) \geq N((y + y_0) - x, z; t)$ where $y + y_0 \in A$; hence $y_0 \in R^t_A(x, z)$. Therefore $A$ is $t$-coproximinal.

(ii) $(\Rightarrow)$ Suppose that $A$ is $t$-coChebyshev subspace, $x \in X$, and $x = y_1 + \tilde{y}_1 = y_2 + \tilde{y}_2$, where $y_1, y_2 \in A$ and $\tilde{y}_1, \tilde{y}_2 \in \tilde{A}_t$.

We show that $y_1 = y_2$ and $\tilde{y}_1 = \tilde{y}_2$. Since $x = y_1 + \tilde{y}_1 = y_2 + \tilde{y}_2$, then $x - y_1 = \tilde{y}_1, x - y_2 = \tilde{y}_2$, this implies that $y_1, y_2 \in R^t_A(x, z)$. Therefore $y_1 = y_2$, it follows that $\tilde{y}_1 = \tilde{y}_2$. Thus $X = A \oplus \tilde{A}_t$.

$(\Leftarrow)$ Let $X = A \oplus \tilde{A}_t$, and suppose for $x \in X$, there exist $y_1, y_2 \in R^t_A(x, z)$. Then $x - y_1, x - y_2 \in \tilde{A}_t$ and therefore, $x = y_1 + \tilde{y}_1 = y_2 + \tilde{y}_2$, where $\tilde{y}_1 = x - y_1, \tilde{y}_2 = x - y_2$. It follows $y_1 = y_2$ and $\tilde{y}_1 = \tilde{y}_2$.

Theorem 3.4: Let $A$ be a nonempty subset of a fuzzy 2-normed space $(X, N, *)$. Then for $t > 0$ and
nonzero \( z \in X \).

(i) \( R_{x+y}^t(x+y,z) = R_x^t(x,z) + y \), for every \( x, y \in X \).

(ii) \( R_{\alpha A}^{\alpha t}(\alpha x,z) = \alpha R_A^t(x,z) \) for every \( x \in X \) and \( \alpha \in \mathbb{R} \setminus \{0\} \).

(iii) \( A \) is \( t \)-coproximinal(respectively \( t \)-coChebyshev) if and only if \( A + y \) is \( t \)-coproximinal(respectively \( t \)-coChebyshev), for any \( y \in X \).

(iv) \( A \) is \( t \)-coproximinal(respectively \( t \)-coChebyshev) if and only if \( \alpha A \) is \( |\alpha| t \)-coproximinal(respectively \( |\alpha| t \)-coChebyshev), for any given \( \alpha \in \mathbb{R} \setminus \{0\} \).

Proof. (i) For any \( x,y \in X \), \( t > 0 \) and nonzero \( z \in X \), \( y_0 \in R_{A+y}^t(x+y,z) \) if and only if,
\[
N(y_0 - (a + y); z,t) \geq N(x + y - (a + y), z; t)
\]
for all \( (a + y) \in A + y \) if and only if,
\[
N(y_0 - a, z; t) \geq N(x - a, z; t)
\]
for all \( a \in A \), if and only if, \( (y_0 - y) \in R_A^t(x,z) \), i.e. \( y_0 \in R_A^t(x,z) + y \).

(ii) For any \( x \in X \), \( \alpha \in \mathbb{R} \setminus \{0\} \), \( t > 0 \) and \( z \in X \), \( y_0 \in R_{\alpha A}^{\alpha t}(\alpha x,z) \) if and only if,
\[
N(y_0 - \alpha a, z; |\alpha| t) \geq N(\alpha x - \alpha a, z; |\alpha| t)
\]
for all \( a \in A \), if and only if, \( \frac{1}{\alpha} y_0 \in R_A^t(x,z) \) if and only if,
\[
y_0 \in \alpha R_A^t(x,z).
\]
Therefore, \( R_{\alpha A}^{\alpha t}(\alpha x,z) = \alpha R_A^t(x,z) \).

(iii) Is an immediate consequence of (i).

(iv) Is an immediate consequence of (ii).

Corollary 3.5: Let \( M \) be a nonempty subspace of \( X \). Then for \( t > 0 \) and nonzero \( z \in X \),

(i) \( R_M^t(x+y,z) = R_M^t(x,z) + y \), for every \( x, y \in X \).

(ii) \( R_M^t(\alpha x,z) = \alpha R_M^t(x,z) \) for every \( x \in X \) and \( \alpha \in \mathbb{R} \setminus \{0\} \).

Proof. The proof is an immediate consequence of theorem(3.4) and this fact that \( M + y = M \) and \( \alpha M = M \) for all \( y \in M \) and \( \alpha \in \mathbb{R} \setminus \{0\} \).

Definition 3.6: For \( x \in X \), \( a \in A \), \( 0 < r < 1 \), \( t > 0 \) and nonzero \( z \in X \), define
\[
e_a^t(x,z) = 1 - N(x-a,z; t).
\]

Theorem 3.7: Let \((X,N,*)\) be a fuzzy 2-normed space, \( A \) be a subset of \( X \), \( x \in X \setminus \overline{A} \), \( t > 0 \) and \( z \in X \). Then we have
\[
R_A^t(x,z) = \left[ \bigcap_{a \in A} B(a,e_a^t(x,z),t) \right] \cap A.
\]

Proof. By definition of \( R_A^t(x,z) \) for each \( a \in A \) we have
\[
R_A^t(x,z) \subseteq \left[ B(a,e_a^t(x,z),t) \right] \cap A.
\]
Therefore
\[ R_A^t(x,z) \subseteq \bigcap_{a \in A} B[a,e_a^t(x,z),t] \cap A. \]

Conversely, let \( y \in \bigcap_{a \in A} B[a,e_a^t(x,z),t] \cap A \), then we have \( y \in A \), and for each \( a \in A \),

\[ N(a-y,z;t) \geq 1 - e_a^t(x,z) = N(x-a,z;t), \]

which implies that \( y \in R_A^t(x,z) \). So \( \bigcap_{a \in A} B[a,e_a^t(x,z),t] \cap A \subseteq R_A^t(x,z) \), whence by (2), we have (1), which completes the proof.

**Corollary 3.8:** Let \( (X,N,*) \) be a fuzzy 2-normed space, \( A \) be a subset of \( X \), \( x \in X \setminus \overline{A} \), \( t > 0 \) and \( z \in X \). Then

(i) The set \( R_A^t(x,z) \) is \( t \)-bounded.

(ii) If \( A \) is \( t \)-closed, then \( R_A^t(x,z) \) is \( t \)-closed.

**Theorem 3.9:** Let \( (X,N,*) \) be a fuzzy 2-normed space and * has the condition \( a * a \geq a \) for all \( a \in [0,1] \). For \( t > 0 \), \( x \in X \) and nonzero \( z \in X \), if \( A \) is a convex subset of \( X \), then \( R_A^t(x,z) \) is a convex subset of \( A \) (for \( R_A^t(x,z) \neq \emptyset \)).

**Proof.** Let \( z_1, z_2 \in R_A^t(x,z) \), then for \( t > 0 \) and nonzero \( z \in X \),

\[ N(y - z_1, z_2; t) \geq N(x - y, z_1; t) \]

for all \( y \in A \). Now for each \( \lambda \in (0,1) \) we have

\[ N(y - (\lambda z_1 + (1 - \lambda)z_2), z; t) = N(\lambda y - \lambda z_1 + y - \lambda y - z_2 + \lambda z_2, z; t) \]

\[ = N(\lambda(y - z_1) + (1 - \lambda)(y - z_2), z; (1 - \lambda)t + \lambda t) \]

\[ \geq N(y - z_1, \frac{\lambda t}{(1 - \lambda)}; (1 - \lambda)t \)

\[ \geq N(x - y, z; t) \]

So \( \lambda z_1 + (1 - \lambda)z_2 \in R_A^t(x,z) \) and \( R_A^t(x,z) \) is convex.

**Theorem 3.10:** For \( t > 0 \) and nonzero \( z \in X \), let \( A \) be a \( t \)-coproximinal subspace of a fuzzy 2-normed space \( (X,N,*) \). Then

(i) If \( \overline{A}_z^t \) is a \( t \)-compact set then \( A \) is \( t \)-quasi-Chebyshev.

(ii) If \( \overline{A}_z^t \) is a \( t \)-closed set then \( R_A^t(x,z) \) is \( t \)-closed for every \( x \in X \).

**Proof.** (i) Suppose \( x \in X \) and \( \{y_n\} \) is a sequence in \( R_A^t(x,z) \). Since \( x - y_n \in \overline{A}_z^t \) and \( \overline{A}_z^t \) is a \( t \)-compact set, there exists a subsequence \( \{x - y_{n_k}\} \) that \( t \)-converges to \( x - y_0 \in \overline{A}_z^t \).

Consequently, \( \{y_{n_k}\} \) has a subsequence \( y_{n_k} \to y_0 \in R_A^t(x,z) \) and hence \( A \) is \( t \)-quasi-Chebyshev.

(ii) The proof is similar to (i).
Definition 3.11: A subset \( A \) of a fuzzy 2-normed space \((X, N, \ast)\) is called to be \( t \) - boundedly compact if every \( t \) - boundedly sequence in \( A \) has a subsequence \( t \) - converging to an element of \( X \).

Theorem 3.12: Suppose, for some \( t > 0 \) and nonzero \( z \in X \), \( A \) is a \( t \) - boundedly compact and \( t \) - closed subset of a fuzzy 2-normed space \((X, N, \ast)\), then, \( A \) is \( t \) - quasi-Chebyshev.

Proof. Let \( \{y_n\} \) be any sequence in \( R^t_A(x, z) \). Then \( N(y_n - y, z; t) \geq N(x - y, z; t) \) for every \( y \in Y \). Since \( R^t_A(x, z) \) is \( t \) - bounded, \( \{y_n\} \) is a \( t \) - bounded sequence in \( A \), and so \( \{y_n\} \) has a \( t \) - convergent subsequence \( \{y_{n_k}\} \), let \( y_{n_k} \to y_0 \in A \), as \( A \) is \( t \) - closed. Consider
\[
N(y_0 - y, z; t) = \lim_{k \to \infty} N(y_{n_k} - y, z; t) \geq N(x - y, z; t)
\]
for every \( y \in A \). So \( y_0 \in R^t_A(x, z) \), which implies that \( A \) is \( t \) - quasi-Chebyshev.

Definition 3.13: Let \((X, N, \ast)\) be a fuzzy 2-normed space and \( A \) be a subset of \( X \). For \( t > 0 \) and \( z \in X \), an element \( x \in X \) is said to be \( t \) - orthogonal to an element \( y \in X \), and we denote it by \( x \perp z \), if \( N(x + \lambda y, z; t) \leq N(x, z; t) \) for all scalar \( \lambda \in \mathbb{R}, \lambda \neq 0 \). We say \( A \perp z \) if \( x \perp z \) for every \( x \in A \).

Theorem 3.14: For \( t > 0 \) and nonzero \( z \in X \), \( x \in X \) and \( y_0 \in A \), let \((X, N, \ast)\) be a fuzzy 2-normed space and \( A \) be a subspace of \( X \). If \( A \perp z \) then \( y_0 \in R^t_A(x, z) \).

Proof. Suppose \( t > 0 \), \( x \in X \) and \( A \perp z \), then \( a \in A \) and scalar \( \lambda \in \mathbb{R}, \lambda \neq 0 \). Then \( N(x - y_0 + \lambda^{-1} a, z; \frac{t}{|\lambda|}) \leq N(x - y_0, z; t) \) for all \( \lambda \in \mathbb{R}, \lambda \neq 0 \).

Hence \( N(x - a', z; \frac{t}{|\lambda|}) \leq N(y_0 - a', z; \frac{t}{|\lambda|}) \), where \( a' = y_0 - \lambda^{-1} a \). Now if \( \lambda = 1 \) then,
\[
N(y - y_0, z; t) \geq N(x - y, z; t)
\]
for all \( y \in A \), and so \( y_0 \in R^t_A(x, z) \).

4 \( F \) -Best Coapproximation:

Definition 4.1: Let \( A \) be a nonempty subset of a fuzzy 2-normed space \((X, N, \ast)\). An element \( y_0 \in A \) is said to be an \( F \) - best coapproximation of \( x \in X \) from \( A \) if it is a \( t \) - best coapproximation of \( x \) from \( A \), for every \( t > 0 \), i.e.
\[
y_0 \in \bigcap_{t \in (0, \infty)} R^t_A(x, z).
\]
The set of all elements of \( F \) - best coapproximations of \( X \) from \( A \) is denoted by \( FR^t_A(x, z) \), i.e.
\[
FR^t_A(x, z) = \bigcap_{t \in (0, \infty)} R^t_A(x, z).
\]
If each \( x \in X \) has at least (respectively exactly) one \( F \) - best coapproximation in \( A \), then \( A \) is called \( F \) - coproximinal (respectively \( F \) - coChebyshev) set.
Example 4.2: Let $X = \mathbb{R}^2$. For $a, b \in [0, 1]$, let $a \cdot b = ab$. Define $N : \mathbb{R}^2 \times (0, +\infty) \to [0, 1]$, by $N((x_1, x_2), (y_1, y_2); t) = \frac{t}{t + |x_1y_2 - x_2y_1|}$, for all $(x_1, x_2), (y_1, y_2) \in \mathbb{R}^2$. Then $(X, N, \cdot)$ is the standard fuzzy 2-normed space. Let $A = \{(x_1, x_2) \in \mathbb{R}^2; x_1 = x_2\}$. Then with a simple calculus can be shown that $FR^t_{\cdot}((x_1, x_2), (z_1, z_2)) = \frac{x_2z_1 - x_1z_2}{z_2 - z_1}$, for all $(z_1, z_2) \in \mathbb{R}^2 \setminus A$. So $A$ is a $F$-coChebyshev set.

Example 4.3: Let $X = \mathbb{R}^2$. For $a, b \in [0, 1]$, let $a \cdot b = ab$. Define $N : \mathbb{R}^2 \times (0, +\infty) \to [0, 1]$, by $N((x_1, x_2), (y_1, y_2); t) = \frac{\exp(|x_1y_2 - x_2y_1|)}{t}$, for all $(x_1, x_2), (y_1, y_2) \in \mathbb{R}^2$. Then $(X, N, \cdot)$ is a fuzzy 2-normed space.

Let $|}|1, 1; ,{,} = 1, 2, 2$, $x, x \in \mathbb{R},$ and $(0, 3) = x$, and $(0, 2) = z$. Then $(0, 2), (1, 1), 1, 1, x \in FR^t_{\cdot}(0, 2)$. So $A$ is not a $F$-coChebyshev set.

Theorem 4.4: Let $(X, \| \cdot \|, \| \cdot \|, \| \cdot \|, \| \cdot \|, \| \cdot \|, \| \cdot \|, \| \cdot \|)$ be a 2-normed space and $N_s$ be the its standard induced fuzzy 2-norm. Then $y_0 \in A$ is a best coapproximation to $x \in X$ in the 2-normed space $(X, \| \cdot \|, \| \cdot \|, \| \cdot \|, \| \cdot \|, \| \cdot \|, \| \cdot \|, \| \cdot \|)$ if and only if $y_0$ is a $F$-best coapproximation to $x$ in the standard induced fuzzy 2-normed space $(X, N_s, \cdot)$.

Proof. For nonzero $z \in X$, $y_0$ is a best coapproximation to $x \in X$ in the 2-normed space $(X, \| \cdot \|, \| \cdot \|, \| \cdot \|, \| \cdot \|, \| \cdot \|, \| \cdot \|, \| \cdot \|)$ if and only if $t \| y - y_0, z \| \leq \| x - y, z \|$, for every $y \in A$, if and only if $\frac{t}{t + \| y - y_0, z \|} \geq \frac{t}{t + \| x - y, z \|}$, for every $y \in A$ and $t \in (0, \infty)$, if and only if $N_s(y - y_0, z; t) \geq N_s(x - y, z; t)$, for every $y \in A$ and $t \in (0, \infty)$, if and only if $y_0 \in FR^t_{\cdot}(x, z)$.

Definition 4.5: Let $(X, N_s, \cdot)$ be a fuzzy 2-normed space and $A$ be a subset of $X$. For nonzero $z \in X$, an element $x \in X$ is said to be $F$-orthogonal to an element $y \in X$, and we denote it by $x \perp_z F y$, if for every $t > 0$, $x \perp_z^t y$. We say $A \perp_z F y$ if $x \perp_z^t y$ for every $x \in A$.

Theorem 4.6: Let $(X, \| \cdot \|, \| \cdot \|, \| \cdot \|, \| \cdot \|, \| \cdot \|, \| \cdot \|, \| \cdot \|)$ be a 2-normed space and $N_s$ be the its standard induced fuzzy 2-norm. Then $x \in X$ is Brikhoff orthogonal to $y \in X$ in the 2-normed space $(X, \| \cdot \|, \| \cdot \|, \| \cdot \|, \| \cdot \|, \| \cdot \|, \| \cdot \|, \| \cdot \|)$ if and only if $x$ is $F$-orthogonal to $y$ in the standard induced fuzzy 2-normed space $(X, N_s, \cdot)$.

Proof. For nonzero $z \in X$, $x$ is Brikhoff orthogonal to $y$ in the 2-normed space $(X, \| \cdot \|, \| \cdot \|, \| \cdot \|, \| \cdot \|, \| \cdot \|, \| \cdot \|, \| \cdot \|)$ if and only if $\| x, z \| \leq \| x + \lambda y, z \|$, for every scalar $\lambda \in \mathbb{R} \setminus \{0\}$, if and only if $\frac{t}{t + \| x, z \|} \geq \frac{t}{t + \| x + \lambda y, z \|}$, for every $\lambda \in \mathbb{R} \setminus \{0\}$ and $t > 0$, if and only if $N_s(x, z; t) \geq N_s(x + \lambda y, z; t)$ for every
\[ \lambda \in \mathbb{R} \setminus \{0\} \text{ and } t > 0, \text{ if and only if } x \perp^F_y. \]

**Remark 4.7:** The converse of theorem (3.14) is true, if we replace \( t \) - orthogonality with \( F \) - orthogonality.

**Conclusion:**
In this paper we introduced the concept of \( t \) - best coapproximation and \( F \) -best coapproximation in fuzzy 2-normed spaces and then prove several theorems pertaining to this sets.

**REFERENCES**