Accuracy Assessment for Processing GPS Short Baselines using Ionosphere-Free Linear Combination

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Abstract: The biases that affecting the GPS measurements fall into three categories, which are: satellite biases, station biases, and signal propagation biases. Accordingly, processing the GPS data using L1 & L2 data is the most proper way to minimize, but not to eliminate, all errors. The ionosphere refraction, which constitutes a major part in the biases, can be eliminated using linear combination of L1 and L2 data called Ionosphere-Free linear combination, which can affect the accuracy of the processing for GPS short baselines up to 20 km. This paper investigates the accuracy of GPS short baselines (up to 20km), in case of using Iono-Free linear combination in processing the GPS data, compared to processing data using L1 & L2 without combination. The results supported with statistical analysis showed that the difference between processing the GPS data using L1 & L2, and using the Iono-Free model gives discrepancies of mean values 3.6mm, 3.1mm, and 2.4mm in X, Y, and Z coordinates, respectively. In addition, the positional discrepancy between the two solutions has a mean value of 5.4mm. These findings are considered to be insignificant in the daily work of cadastral survey; but it should be taken into consideration in case of the monitoring the deformation of structures or ancient antiquities; where a sub-millimeter accuracy is considered in the measurement.

Key words: GPS; GPS Errors; GPS Observables; L1 & L2 GPS data; Ionosphere-Free Combination.

INTRODUCTION

Positioning with GPS can be classified into Single, Differential, and Relative positioning. Single positioning is meant by the process of finding out the 3-d coordinates of a certain point, while the Differential and Relative positioning are concerned with the determination of the differences in coordinates (vector) between two different points (baseline) (Hofmann et al, 2001). There are two types of GPS observables, namely the code and phase observables. In general, the code observations are used for coarse navigation, whereas the phase observations are used in high-precision surveying applications. That is due to the fact that the accuracy of the carrier phase observations is much higher than the accuracy of code observations (Langley, 1993). The code pseudorange is a measure of the distance between the satellite and the receiver. The P-code, C/A-code can be used to determine the code pseudorange. These ranges can be determined by multiplying the speed of light by the time shift required to match the code generated in the receiver with the code received from the satellite (Erickson, 1992). The phase observable is the difference between the phase of generated signal in the receiver and the carrier signal of the satellite, measured at the receiver. The phase measurement is made at instant time, so, the number of full cycles between the satellite and the receiver cannot be measured, and this is what so called initial phase ambiguity. This integer number of cycles is constant for the same receiver with same satellite, until a loss of lock happens, or the receiver is switched off (Leick, 1995).

Both GPS observables types are affected by many systematic biases, different in their source, nature, value and the suitable method of treatment. GPS biases can be classified into three groups, which are the satellite errors group; the receiver errors group; and the signal propagation group (Grant et al, 1990). In addition to these 3 groups, the accuracy of the computed GPS position is also affected by the geometric locations of the GPS satellites as can be detected by the receiver. The more spread out the satellites are in the sky, the better the obtained accuracy of the GPS derived 3-d coordinates. There are four methods to eliminate or at least reduce the GPS biases. The first method is by applying a mathematical model to correct some errors. The second method, is using the GPS difference modes. The third method is based on making linear combinations between the GPS observables (Abdel Mageed, 2006). The fourth method is the using the GPS precise products like IGS products (IGS, 2002).

This paper investigates the accuracy of the discrepancies in Cartesian coordinates X, Y, and Z and the spatial position P, in case of using Ionosphere–Free linear combination model compared to using the original GPS dual frequency data L1, and L2 for processing the GPS baselines up to 20 km. In this context, the GSP observations and errors will be presented. The different types of the Relative GPS technique will be discussed.
The methodology of investigation and the description of the field test will be presented. Finally, the analysis of the obtained data supported with the statistical analysis will be shown, from which the important conclusions and recommendations will be extracted.

**GPS Observations and Errors:**

The GPS observables are ranges which are deduced from measured time or phase differences based on a comparison between received signals and generated signals. Unlike the terrestrial distance measurements, GPS uses the so-called one-way concept, where, two clocks are used, namely one in the satellite, and the other in the receiver. Thus, the ranges are affected by satellite and receiver clocks errors and, consequently, they are denoted as pseudoranges.

Mainly, there are two types of GPS observables, namely the code pseudoranges and carrier phase observables. In general, the pseudorange observations are used for coarse navigation, whereas the carrier phase observations are used in high-precision surveying applications. That is due to the fact that the accuracy of the carrier phase observations is much more higher than the accuracy of code observations, (Rizos, 1997).

Beside the two GPS observables, the GPS satellite transmits a navigation message. The navigation message is a data stream added to both L1 and L2 carriers as binary biphase modulation at a low rate of 50 Kbps. It consists of 25 frames of 1500 bits each, or 37500 bits in total. This means that, the transmission of the complete navigation message takes 750 seconds. The navigation message contains, along with other information, the coordinates of the GPS satellites as a function of time, the satellite health status, the satellite clock correction, the satellite almanac, and atmospheric data. Each satellite transmits its own navigation message with information on the other satellites, such as the approximate location and health status (Seeber, 1993).

**Code Pseudoranges Observations:**

The code pseudorange is a measure of the distance between the satellite and the receiver. The P-code, C/A-code can be used to determine the code pseudorange. These ranges can be determined by multiplying the speed of light by the time shift required to match the code generated in the receiver with the code received from the satellite (Figure 1).

![Image](https://via.placeholder.com/150)

**Fig. 1:** Pseudorange observables

Analogously, the delays of the clocks with respect to GPS system time frame will lead to timing error. The tropospheric and ionospheric delays affect the measured code pseudorange (Leick, 1995). The general form of code pseudorange observation equation is:

\[ P = \rho + c(\text{dt} - \text{dT}) + d_{\text{ion}} + d_{\text{iono}} + d_{\text{orb}} + e_{\text{p}} \]  \hspace{1cm} (1)

Where: \( P \) is the observed pseudorange, \( \rho \) is the unknown geometric satellite to receiver range, \( c \) is speed of light which is approximately equal to 300,000 km/s, \( \text{dt} \) and \( \text{dT} \) are satellite and receiver clock errors respectively, \( d_{\text{ion}}, d_{\text{iono}} \) are the error due to ionospheric, tropospheric refraction respectively, \( d_{\text{orb}} \) is the orbital error and \( e_{\text{p}} \) is the code measurement noise. The precision of a pseudorange derived from code measurement has been about 1% of the chip length. Consequently, a precision of about 3m, 0.3m is achieved with C/A-code and P-code pseudoranges respectively. However, recent development indicates that a precision of about 0.1% of the chip length may be obtained (El-Rabbany, 2002).

The bias term \( d_{\text{ion}} \) can be determined, with high percentage, in case of using dual frequency receivers, since the ionospheric effect is frequency dependant, and its estimation parameters are usually transmitted within the satellite message. Concerning the tropospheric effect term \( d_{\text{iono}} \), it can be evaluated to be more than 95%, using an adopted model, which is a function of measured meteorological quantities, such as humidity, pressure, and temperature, of the atmosphere surrounding the receiver position. Concerning the orbital bias term \( d_{\text{orb}} \), it can be estimated from satellite orbital dynamics continuous analysis, at the master control station, and included in the satellite-transmitted message also. The satellite and receiver clocks biases term (dt-dT), is usually treated as one unknown parameter. Hence, equation (1) of observed pseudorange will be left out with only four unknowns.
parameters, which are the 3-d geocentric cartesian coordinates (X, Y, Z) of the receiver antenna position, in addition to the clock bias term. Of course, in order to solve such an equation, for the four unknown parameters, one needs to have four of these observation equations.

**Phase Observations:**

The range between the receiver and satellite can be obtained through the carrier phase. The range would simply be the sum of the total number of full carrier cycles plus fractional cycle at the receiver and the satellite, multiplied by the carrier wave length (Figure 2). The ranges determined with the carriers are more accurate than those obtained by the codes (Leick, 1995). This is due to the fact that, the wavelength of the carrier signal (19cm in case of L1) is smaller than the codes. However, there is a problem that the carriers are just pure sinusoidal waves, which means that all cycles look the same. Therefore, the GPS receiver has no means to differentiate one cycle from the other. In other words, the receiver cannot determine the total number of the complete cycles between the satellite and receiver when switched on. The receiver can only measure a fraction of a cycle accurately, while the initial number of complete cycles remains unknown, or ambiguous (Kaplan, 1996). This initial cycle ambiguity remains unchanged over time as long as no signal loss or cycle slip occurs.

![Fig. 2: Phase Observables.](image)

The observation equation of the phase pseudorange is:

\[
\Phi = \rho + \alpha(c(t - dT) + \lambda N - d_{\text{ion}} + d_{\text{nrop}} + d_{\text{tot}} + \varepsilon_q) \tag{2}
\]

Where, the measured phase is indicated in meters by \(\Phi\), \(\lambda\) is the carrier wavelength, \(N\) is the phase ambiguity, and \(\varepsilon_q\) is the combined receiver and multipath noise, and the other remaining symbols are the same as defined in equation (1). The same analysis of the bias terms and unknown parameters, as given in the previous subsection, holds true here also for the case of carrier phase observation equation. The only difference here is the ambiguity term \(N\), which can be solved for, using a certain adopted technique. This means, again, that at least four satellites should be in view at the time, which can be simultaneously tracked from the same ground receiver. On the other hand, for both cases of code pseudorange and carrier phase pseudorange, most of the bias terms can be eliminated, or minimized by following a certain technique for collecting GPS measurements, such as single, double, and triple differences; and/or using mathematical model; and/or using linear combination.

**GPS Errors:**

GPS measurements are subjected to some errors, which will affect the accuracy of the final results. There are two basic types of errors, which are the systematic errors or biases, and the random errors. Generally, the biases affected the GPS measurements fall into three categories which are: satellite biases, receiver biases, and signal propagation biases (Grant *et al*. 1990). Satellite biases consist of biases in satellite ephemeris, satellite clock, and the effect of selective availability SA. The later was internationally terminated by the U.S. Government in May 1, 2000 (Divis, 2000). Satellite biases are affecting both code and phase pseudorange measurements. Receiver bias usually consist of receiver clock bias, receiver noise and antenna phase center variation. The signal propagation biases appear due to tropospheric refraction, ionospheric refraction, and multipath (Klobuchar, 1991). To give an idea about GPS errors and their values, table (1) shows the absolute navigation error budget contained in GPS observables. Beside the effect of these biases, the accuracy of the computed GPS position is also affected by the geometric locations of the GPS satellites as seen by the receiver. Generally, the more spread out the satellites are in the sky, the better the obtained accuracy, which is denoted as dilution of precision DOP.
Table 1: GPS absolute error values

<table>
<thead>
<tr>
<th>Source</th>
<th>Error</th>
<th>Absolute value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Space segment</td>
<td>Satellite clock error</td>
<td>3.0m</td>
</tr>
<tr>
<td></td>
<td>Ephemeris error</td>
<td>2.7m</td>
</tr>
<tr>
<td></td>
<td>Selective availability (not active)</td>
<td>27.0m</td>
</tr>
<tr>
<td>Atmospheric effect</td>
<td>Ionosphere effect</td>
<td>8.2m</td>
</tr>
<tr>
<td></td>
<td>Troposphere effect</td>
<td>1.8m</td>
</tr>
<tr>
<td></td>
<td>Multipath</td>
<td>0.6m</td>
</tr>
<tr>
<td>User segment</td>
<td>Receiver noise</td>
<td>0.3m</td>
</tr>
</tbody>
</table>

In order to achieve the expected high accuracy of GPS, the above mentioned errors must be taken into consideration in the data processing stage, then modeled and minimized as possible. The minimization of these errors can be done through four approaches (El-Maghaby et al., 2005). The first is by modeling these errors mathematically and counts for them in the adopted observation equation. The second approach is based on a differential solution to cancel out, or at least greatly reduce many of these errors. The third approach is concentrated on using linear combination between the GPS observables. The fourth approach is depending on using precise products such as precise satellite ephemeris and satellite clock offsets, through multinational GPS agencies such as the International GPS Service IGS (IGS, 2002).

In addition, the GPS measurements include some observational random errors, moreover, the un-modeled small systematic errors inherent on the system due to multipath and imaging, antenna phase center movement, and residual biases, are usually treated in practice as contributing part of the resulting random errors. The main sources for the random errors in the GPS system can be stated as (Seeber, 1993):

1. Instrumental source, which causes multipath, satellite clock error, and receiver clock error.
2. Atmospheric source causing ionospheric and tropospheric refraction un-modeled effects.
3. Satellite orbital source, which includes shortage in dynamic models used to determine the relative motion of GPS satellites and the observing stations.

**Relative GPS Observation Technique:**

The GPS observation techniques include: Single Point Positioning SPP; Differential Positioning DGPS; and Relative GPS positioning. GPS Single Point Positioning employs one GPS receiver, while DGPS and Relative GPS positioning employ two or more GPS receivers, simultaneously tracking the same satellites. Surveying works with GPS have conventionally been carried out in the Relative and Differential positioning techniques. This is mainly due to the higher positioning accuracy obtained from the relative and differential techniques, compared to that of the GPS Single Point Positioning. A major disadvantage of GPS Relative and Differential techniques, however, is the dependency on the measurements or corrections from the reference receiver (Rizos, 1997).

There are different GPS Relative positioning techniques. These techniques are Static, Stop & Go, Post Processing Kinematic PPK, and Real Time Kinematic RTK. Static GPS technique, is an accurate and reliable technique, however, it is relatively slow in production. On the other hand, each one of other remaining techniques, is represented a fast solution to the problem of obtaining high productivity, such as measuring many baselines in a short period of time, or the ability to obtain results even while the receiver in motion, that is real time solution, however, with a relatively less accuracy than the static case (Abdel Mageed, 2006).

Static relative positioning by carrier phase is the most frequently used method by surveyors, as it is more accurate as compared to the code pseudorange measurements (Kaplan, 1996). The principle of static relative positioning is based on determining the vector between two stationary receivers, using code and phase data. The range of accuracy for static survey, is normally 3mm + 0.5ppm. The static surveying is usually applied in high accuracy surveying projects, such as establishing new geodetic networks, densification of existing first order control networks or lower order network, crustal movements, and structural deformation.

The intention of the Stop & Go and PPK techniques, is to determine the position of the antenna while it is in motion. The main difference between Stop & Go and PPK techniques, is that in PPK technique, the coordinates of roving receiver are calculated at all points separated by pre-specified time or distance interval, along the survey trajectory, whereas, in Stop & Go technique, the coordinates of the roving receiver are calculated at selected points. In many other respects, the PPK technique is similar to Stop & Go technique, that is the ambiguity must be resolved before starting the survey, and the ambiguity must be reinitialized if a cycle slip occurs during the survey. Providing that the ambiguities are resolved, in the initialization part of the chain, and lock to the satellites is maintained while moving, positional accuracy of about 10 mm + 1ppm can be achieved. Thus, the GPS Stop & Go, and PPK observing techniques, is well suited when many points close together, have to be surveyed. It may be also used in detailed engineering surveying. However, the main constraint is that, no obstacles are allowed between the satellite position and the roving receiver locations (Hofmann et al., 2001).
Real Time Kinematic RTK technique is used to determine the coordinates in real time. In this method, the base receiver remains stationary over the known point and is attached to a radio transmitter. The rover receiver is normally carried out and attached to a radio receiver. This method is similar to DGPS, except that in case of DGPS corrections are transmitted, however, in case of RTK the known coordinates of the base along with the receiver measurements are transmitted to the rover receiver through the communication link, using a data rate of 1Hz, which means one sample per second. The built-in software in the rover receiver combines and processes the GPS measurements collected at both the base and the rover receivers, to obtain the rover coordinates. The initial ambiguity parameters are determined instantaneously using a technique called on-the-fly OTF ambiguity resolution. Once the ambiguity parameters are fixed to integer values, the receiver will display the rover coordinates right in the field. That is, no post processing is required. The expected positioning accuracy is of the order of 15mm+1ppm (Shaw et al, 2000).

Characteristics of Ionosphere-Free Linear Combination:

GPS observables are obtained from the code information or the carrier phase in the broadcast satellite signal. Recall that the P-code is modulated on both carriers L1 and L2, whereas the C/A-code is modulated on L1 only; consequently, one could measure the code ranges \( P_{L1} \), \( P_{L2} \), the carrier phase \( \Phi_{L1} \), \( \Phi_{L2} \), and the Doppler shifts \( D_{L1} \), \( D_{L2} \) for a single epoch (Langley, 1993). GPS observables can be gathered for the L1 carrier wave only, when single frequency receivers are used, or it may be collected for both carrier waves L1 and L2, using dual frequency receivers. In the second case, when both L1 and L2 observables are available, it is possible to construct, mathematically, a new kind of observables, using different ratios of both L1 and L2 observables. These observables are naturally linear combinations. Hence, the main objective of GPS phase linear combinations is to eliminate, or at least greatly reduce, the different GPS biases. The general formulation of the resulting GPS phase linear combination can be given as, (Hofmann et al, 2001):

\[
\Phi_{a,b} = a \Phi_1 + b \Phi_2 \tag{3}
\]

\[
f_{a,b} = a f_1 + b f_2 \tag{4}
\]

\[
\lambda_{a,b} = \frac{c}{f_{a,b}} \tag{5}
\]

\[
d_{a,b}^{\text{ion}} = \frac{f_1}{f_2} \cdot \frac{a f_2 + b f_1}{a f_1 + b f_2} d_{1}^{\text{ion}} \tag{6}
\]

Where:

- \( a, b \) two arbitrary numbers, whose values are assigned by the considered type of the linear combination.
- \( \Phi_{a,b} \) the resulted GPS phase linear combination observable, \( \Phi_1 \) is the measured L1 phase, \( \Phi_2 \) is the measured L2 phase.
- \( f_{a,b}, l_{a,b} \) the corresponding frequency and wavelength of the considered phase linear combination.
- \( f_1, f_2 \) the frequencies of L1 and L2 signals which are 1575.42 MHz and 1227.60 MHz respectively
- \( d_{a,b}^{\text{ion}} \) the ionospheric delay on the linear combination as a factor of the ionospheric delay on L1 signal \( d_1^{\text{ion}} \)

Considering a certain noise level for the phases, the noise level is increased for the linear combination. Basically, the noise level is about 1% of the wave length. So, the noise of the p-code is about 30 cm, while the noise of the phase-carrier is about 2 mm., which indicates that the noise level of code observations is greater than that of phase observations by about 150 times (Hofmann et al, 2001). Applying the error propagation law and assuming the same noise for both phases, the noise of the linear combination is calculated from (Comery et al, 1989):

\[
\epsilon_{a,b} = \frac{\lambda_2}{a \lambda_2 + b \lambda_1} \cdot \sqrt{a^2 + b^2} \cdot \epsilon_1 \tag{7}
\]

Where: \( L_1 = 19 \) cm, \( L_2 = 24.4 \) cm, and \( \epsilon_1 \) is the noise on the L1 carrier

Regarding the ionosphere–free combination, the ionospheric refraction bias can be eliminated by defining the two coefficients a, and b as follows (Yang, 1995):

797
\[ a = \frac{f_1^2}{f_2^2 - f_1^2} = 2.54 \quad \text{and} \quad b = \frac{-f_2^2}{f_1^2 - f_2^2} = -1.54 \]  

(8)

Accordingly, the phase observation equation of the ionosphere-free is:

\[ \Phi_{IF} = \frac{f_1^2}{f_1^2 - f_2^2} \Phi_1 - \frac{f_2^2}{f_1^2 - f_2^2} \Phi_2 = 2.54 \Phi_1 - 1.54 \Phi_2 \]  

(9)

\[ \Phi_{IF} = \rho + c (d_T - dT) + \lambda_{IF} \cdot N_{IF} + d_{\text{trop}} + d_{\text{orb}} + \varepsilon_{\Phi_{IF}} \]

The resulting wavelength of the ionosphere-free linear combination is \( \lambda_{IF} = 14.2 \text{ cm} \). Also, the noise level is about 2.2 times that affecting L1 carrier. The phase ambiguity of the ionosphere-free can be calculated from the relation:

\[ N_{IF} = \frac{f_1^2}{f_1^2 - f_2^2} N_1 - \frac{f_2^2}{f_1^2 - f_2^2} N_{L2} = 2.54 N_1 - 1.54 N_2 \]  

(10)

Similarly, the ionosphere-free linear combination for the code measurements is formed as [Rizos, 1997]:

\[ P_{IF} = \frac{f_1^2}{f_1^2 - f_2^2} P_1 - \frac{f_2^2}{f_1^2 - f_2^2} P_2 = 2.54 P_1 - 1.54 P_2 \]  

(11)

\[ P_{IF} = \rho + c (d_T - dT) + d_{\text{trop}} + d_{\text{orb}} + \varepsilon_{P_{IF}} \]

The advantage of the ionosphere-free linear combination is the totally removal of the ionosphere effect. The drawback of this linear combination is that due to the non-integer nature of the coefficients, the ambiguities have lost their integer characteristics, which make the process of fixing the phase ambiguity an impossible task.

**Methodology of Investigation:**

The objective of this paper is based on comparing the coordinates of GPS baselines processed using dual frequency L1, L2 data, and using the linear combined Ionosphere-Free model. The methodology of our investigation herein, will be based on the statistical analysis of the behavior of the discrepancies in the 3-D cartesian coordinates of 8 baselines with approximate distances from 2.5 km to 20 km.

The field test was done at New Cairo City on Aug 2, 2011 and Aug 4, 2011. The field procedure of the test was started by setting up a dual frequency GPS receiver of Topcon GR3 at a reference control point. A second dual frequency receiver of the same type of Topcon GR3 was set up on 8 control points at approximate distances from 2.5 km to 20 km, from the reference receiver. The observational operating parameters were the same for the two receivers, which are: static mode, elevation angle 15°, and 10 seconds rate of observations. The observational duration of each baseline was as follows: 25 minutes for the baselines of approximate distance 2.5 km, and 5 km; 45 minutes for the baselines of approximate distances 7.5 km, 10 km, and 12.5 km; 65 minutes for the baselines of approximate distances 15 km, 17.5 km, and 20 km.

After completing the GPS field campaign, the raw data were downloaded and transferred to RINEX format using TOPCON LINK software. The processing of the Rinex data was done using Leica Geo Office software, on two steps. The first step was the processing of the 8 baselines using L1, and L2 data. The second step was the processing of the same 8 baselines using the Ionosphere-Free. In each step the 3-D Cartesian coordinates were archived for the statistical analysis.

**Analysis of Results:**

The analysis of the results will be based on comparing the discrepancies in X, Y, and Z coordinates between processing 10 GPS baselines using L1, L2 data, and using Ionosphere-Free model. The discrepancies are:

\[ \Delta X = X_{IF} - X_{\text{Dual}}, \quad \Delta Y = Y_{IF} - Y_{\text{Dual}}, \quad \Delta Z = Z_{IF} - Z_{\text{Dual}} \]  

(12)

Where:

- \( X_{\text{Dual}} \) is the X coordinate from processing the data using L1, L2 data.
- \( X_{IF} \) is the X coordinates from processing the data using Ionosphere-Free model.

The same abbreviations are valid for Y and Z coordinates.

Also, the positional discrepancy and the standard deviation can be calculated from [Comery et. al, 1989]:

\[ \Delta P = \sqrt{(\Delta X)^2 + (\Delta Y)^2 + (\Delta Z)^2} \]  

(13)
\[ \sigma^2_{\Delta P} = \sigma^2_{\Delta X} + \sigma^2_{\Delta Y} + \sigma^2_{\Delta Z} \]  

(14)

The discrepancies in X, Y, Z and position P, as well as the approximate length of the 8 baselines are shown in Table 2.

**Table 2**: The discrepancies in X, Y, Z, and position P

<table>
<thead>
<tr>
<th>Baseline No.</th>
<th>Approx. Length (km)</th>
<th>( \Delta X ) (mm)</th>
<th>( \Delta Y ) (mm)</th>
<th>( \Delta Z ) (mm)</th>
<th>( \Delta P ) (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.5</td>
<td>-7.7</td>
<td>10.7</td>
<td>-0.6</td>
<td>2.1</td>
</tr>
<tr>
<td>2</td>
<td>5.0</td>
<td>-12.5</td>
<td>-0.6</td>
<td>2.1</td>
<td>12.7</td>
</tr>
<tr>
<td>3</td>
<td>7.5</td>
<td>0.5</td>
<td>-4.0</td>
<td>-3.6</td>
<td>5.4</td>
</tr>
<tr>
<td>4</td>
<td>10.0</td>
<td>12.7</td>
<td>9.2</td>
<td>6.4</td>
<td>16.9</td>
</tr>
<tr>
<td>5</td>
<td>12.5</td>
<td>6.1</td>
<td>4.1</td>
<td>4.5</td>
<td>8.6</td>
</tr>
<tr>
<td>6</td>
<td>15.0</td>
<td>12.3</td>
<td>7.4</td>
<td>4.4</td>
<td>14.9</td>
</tr>
<tr>
<td>7</td>
<td>17.5</td>
<td>-0.9</td>
<td>-4.3</td>
<td>-0.9</td>
<td>4.5</td>
</tr>
<tr>
<td>8</td>
<td>20.0</td>
<td>3.1</td>
<td>2.6</td>
<td>1.8</td>
<td>4.4</td>
</tr>
</tbody>
</table>

Figures (3) shows the X, Y, and Z coordinate discrepancies for 8 baselines between the processing of data using L1, L2 and using the Iono-Free linear combination model. In addition, Figure (4) shows the positional discrepancies P for the same 8 baselines.

**Fig. 3**: Variation of the X, Y, and Z coordinate discrepancies

**Fig. 4**: Variation of the Positional discrepancies

The previous figures are supported by descriptive statistics to measure the quality of the obtained results. Table (3) shows these descriptive statistics. For instance, the X-coordinate discrepancies are ranging between 12.7mm and -12.5mm, with mean value 3.6mm and SD 8.2mm for single determination. The Y-coordinate discrepancies are fluctuating between 107mm and -4.3mm, with mean value of 3.1mm and SD for single observation of 5.8mm. The Z-coordinate discrepancies are varying between 6.4mm and -3.6mm, with mean value of 2.4mm and SD for single determination of 3.3mmmm. Finally, the positional discrepancies between the dual data and Iono-Free combination are differing from 4.4mm to 12.5mm, with most probable value of 5.4mm and SD 5.1mm respectively.

**Table 3**: Descriptive statistics of the discrepancies (mm)

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta X )</td>
<td>12.7</td>
<td>-12.5</td>
<td>25.2</td>
<td>3.6</td>
<td>8.2</td>
<td>2.9</td>
</tr>
<tr>
<td>( \Delta Y )</td>
<td>10.7</td>
<td>-4.3</td>
<td>15.0</td>
<td>3.1</td>
<td>5.8</td>
<td>2.0</td>
</tr>
<tr>
<td>( \Delta Z )</td>
<td>6.4</td>
<td>-3.6</td>
<td>10.0</td>
<td>2.4</td>
<td>3.3</td>
<td>1.2</td>
</tr>
<tr>
<td>( \Delta P )</td>
<td>16.9</td>
<td>4.4</td>
<td>12.5</td>
<td>5.4</td>
<td>5.1</td>
<td>1.8</td>
</tr>
</tbody>
</table>

**Conclusions**

The present study investigates an accuracy study for the discrepancies in Cartesian coordinates X, Y, Z in case of using Ionosphere–Free linear combination model compared to using the original GPS dual frequency
data L1, and L2. To achieve such an objective a field test was done to observe 8 GPS baselines varying from 2.5km to 20km. The GPS data were processed one time using the original dual frequency data L1, L2; and the second time the data were processed using the Iono-Free linear combination.

The results supported with statistical analysis showed that the difference between processing the GPS data using L1, L2, and using the Iono-Free model gives discrepancies of mean values 3.6mm, 3.1mm, and 2.4mm in X, Y, and Z coordinates, respectively. In addition, the positional discrepancy between the two solution has a mean value of 5.4mm.

The above findings are considered to be insignificant in the daily work of cadastral survey; but it should be taken into consideration in case of the monitoring the deformation of structures or ancient antiquities; where a sub-millimeter accuracy is considered in the measurement. Accordingly, in case of using GPS in the monitoring of deformation, it is highly recommended to process the all data using one technique either L1, L2; or Iono-Free combination; to sustain a sub-millimeter level between the initial observation and the repeated observations.

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