Cost Oriented Assembly Line Balancing Problem with Sequence Dependent Setup Times

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Abstract: Assembly lines are flow based production systems specially used to produce standardized products in high volume. Usually in the literature of assembly line research, it is assumed that, inside each station, Tasks can be performed in any feasible sequence without affecting the processing time of each other. However in practice, the sequence in which tasks are performed may have a significant effect on the processing time of each other due to existence of sequence dependent setup times (such as tool exchanges, walking times and etc). On the other hand auing to the high competition in the current production environment, reducing the production costs and increasing utilization of the current resources are of major importance for production managers. In this paper the General Assembly Line Balancing Problem with Setups (GALBPS) firstly proposed by Andres et al, 2008, is considered. Since production costs are one of the most important issues in production management, developing a model which directly attempts to minimize the production costs is of main interest. In this paper the GALBPS is considered with a cost oriented approach and a mathematical model is developed to solve the problem. In order to illustrate the proposed model, a numerical example is solved using Lingo 11 software.

Key words: Cost oriented; Assembly Line Balancing; Sequence-dependent setup time

INTRODUCTION

Assembly lines are a type of production systems used to produce one homogenous commodity in high volume. Each line consists of several workstations arranged along with a transportation system i.e. a conveyor belt. The transportation system moves the work-piece through the line from station to the next station with a constant pace which determines the production speed. In each workstation a set of tasks are performed on the work-piece, which is constrained by the cycle time (the duration that each work-piece stays in each station). Because of high capital requirements of installing an assembly line, configuration planning of lines is of great importance.

The decision problem of optimally partitioning the total work into the stations in order to optimize some objective is known as the assembly line balancing problem (ALBP). The total amount of work required to produce a product is divided into n elementary operations \( V = \{1, 2, \ldots, n\} \), these elementary operations are called tasks. Every task \( j \) needs \( t_j \) units of time to be performed; this duration is called task time. Additionally, due to technological constraints, a task can’t be started before all of its predecessors are completed. These precedence constraints can be depicted in a precedence graph, vertices of this graph represent the tasks and every arc represents a precedence relation between tasks \( i \) and \( j \), in other words each arc \((i, j)\) indicates that task \( j \) should be started after finishing task \( i \).

The most studied problem in the field of assembly line balancing is called the simple assembly line balancing problem (SALBP) and has the main assumptions such as paced line with fixed cycle time, mass production of one homogeneous product, serial line and etc. The assumptions of SALBP are very restricting with respect to real world assembly lines and cannot perfectly reflect the real industrial situations such as mixed model lines, parallelizing, two sided lines and etc. Therefore in recent years many researcher have focused on releasing some of the unrealistic assumptions of SALBP in order to produce more practical models; the resulting problems are called generalized assembly line balancing problems (GALBP). Several generalizations have been studied for the ALBP, some instances of these generalizations are considering U-shaped assembly lines balancing (Milenburg & Wijngaard, 1994), parallel workstations (Buxey, 1974), considering process alternatives (Pinto et al, 1983) and two sided assembly lines (Bartholdi, 1993), Recent surveys of generalized assembly line models are Erel & Sarin, 1998; Scholl, 1999; Rekiek et al, 2002 and Scholl & Becker, 2006.

In many real world situations there exist sequence dependent setup times between tasks. In other words the sequence in which tasks are performed has a significant effect on the processing time of each other. In spite of the fact that setups between jobs, in a mixed model environment, maybe ignored because of flexible machinery, there are numerous industrial occasions in which setup times affect the processing time of tasks assigned to
stations. Examples of these situations are tool exchanges, walking distances in a workstation and curing or cooling processes (Andres et al., 2008; Scholl et al., 2009). Therefore both balancing and scheduling of tasks inside each station must be determined due to the sequence dependent setup times.

Even though the sequence-dependent setup time between tasks is a regular situation in assembly lines, there have been few research papers that take this effect into account in assembly line balancing. Andres et al., 2008, proposed the GALBPS model in which it is assumed that there are sequence dependent setup times between tasks. They developed a mathematical model to solve the problem and proposed several priority rules and a GRASP meta-heuristic. For the same problem, Martino & Pastor, 2010, developed and examined several priority-rule-based heuristics. Scholl et al., 2009, further extended the problem by modeling the setups more pragmatically; they also proposed a new more compact mathematical model for the problem. They also developed efficient heuristics to solve the problem. Seyed-Alagheband et al., 2011, developed a mathematical model and a simulated annealing meta-heuristic to solve type-2 of the problem, which is minimizing the cycle time for a predetermined number of stations. They also employed the Taguchi method to tune different parameters of the algorithm. There is also another type of setup times called indirect setup times. This kind of setup time occurs when two tasks are related in such a way that executing one task before the other one result in an increase in processing time of the other one (Boysen et al., 2007; Scholl et al., 2008).

In the literature of sequence dependent assembly line balancing the objective is usually minimizing the number of stations for a given cycle time or minimizing the cycle time for a given number of stations. Therefore it is called the time oriented assembly line balancing problem. However due to the high competition in the current production environment, reducing the production costs and increasing utilization of available resources is of major importance for manufacturing managers. Therefore developing a model to directly minimize the production costs is of significant interest.

In this paper the GALBPS is considered with a cost oriented approach. Normally final assembly of a commodity is a labour-intensive process (Amen, 2006). In the cost oriented approach the aim is to minimize the total cost per production unit (Steffen, 1977; Rosenberg & Ziegler, 1992; Amen, 1997; Amen, 2000a). For this aim the significant factors that determine the cost must be considered. There are two main cost drivers in determining the costs of production: labor costs and the costs of capital.

First the costs of labor are considered. Wages of each worker is determined by the most difficult task assigned to him (or her). In order to be able to determine difficulty of each task, job values are determined using the well-known work measurement systems (Amen, 2000a). Using these job values for each task $i$ a wage rate $w_i$ $(MU/PU)$ is determined. As said before, the wage rate of each worker in the station is determined by the most difficult tasks assigned to the station. Therefore wage rate of the worker in station $j$, or the wage rate of station $j$, is: $w_j = \max\{w_i| i \in I_j\}$ where $I_j$ is the set of tasks assigned to the worker in station $j$. It is important to note that for each worker the wage rate is paid for the entire cycle time and not for the sum of duration of tasks assigned to him (or her).

The second cost driver considered in this paper is the costs of capital. These costs consist of the costs of machinery, material handling system and etc. in this paper it is assumed that the costs of capital are directly dependent on the length of the line i.e. number of stations. Generally the machinery needed to produce the final commodity can be classified into two categories: universal machinery and special machinery. Special machinery is needed to perform special tasks and it can be assumed that the number of them is fixed and is not dependent on the assignment of tasks to stations. It is also assumed that all of the stations need identical universal machines. Therefore the costs of capital for all stations are the same.

Other costs such as costs of material are assumed to be independent of the length of the line or assignment of tasks to stations (Steffen, 1977). Therefore the total capital cost per production unit can be calculated as $k = \sum_{j=1}^{m} C \times k_i + m \times k_{sc}$, where $k_{sc}$ is the total capital cost per station. To simplify the calculations the cost of capital can be rewritten as $m \times k_{sc} = m \times C \times k_{sc}/C$ and the total capital cost of a task can be calculated as $k_i = w_i + k_{sc}/C$; therefore the total capital cost of a station can be calculated as $k_j = \max\{k_i| i \in I_j\}$. Finally the total costs per production unit amount to: $k = \sum_{j=1}^{m} C \times k_j$ where $k_j = \max\{k_i| i \in I_j\}$.

Regarding the literature of cost-oriented assembly line balancing, Rosenberg and Ziegler, 1992, assigned a wage rate for each task and assumed that the wage rate of each station is equal to the maximum wage rate assigned to the station. The objective is to minimize the cumulative wage rate over the line. They assumed that the length of the line i.e. the number of stations is variable. They also proposed priority rule based heuristics to solve the problem; some of these rules are available for SALBP-1. Amen, 2000a additionally took the costs of capitals into account; examples of this kind of costs are costs of machinery and transportation system and etc. They also proposed a branch and bound algorithm to solve the problem. Amen (2000b, 2001) proposed station oriented heuristics which uses cost-based dynamic priority rules. He compared the proposed heuristics with the
existing ones using a large problem set which is generated randomly. The new rule named “best change of idle cost” had a better performance than all other rules. Amen, 2006, considered general formulations, which can be solved by standard optimization tools, for the same problem. These models include general branch and bound techniques with LP-relaxation and general enumeration techniques. He also showed that the “maximally-loaded-station-rule” is no longer valid for the cost oriented model. Maximally-loaded-station-rule implies that when filling stations with tasks, the current station must be maximally loaded in order to go to the next station. A station is said to be maximally loaded if no more tasks can be assigned to it. They showed that this rule must be replaced with a weaker rule named “two-stations-rule”. This results in an enormous increase in solution difficulty of the problem. Malakooti (1991, 1994) and Malakooti & Kumar (1996) considered a multi-objective ALBP with objectives that are based on cost and capacity.

In this paper a cost oriented approach is used to model assembly line balancing problem with sequence dependent setup times which to the best of our knowledge hasn’t been considered in the literature so far. A mathematical formulation is developed to solve the problem under consideration. In order to illustrate the proposed model, a numerical example is solved using Lingo 11 software and the result is compared to the classic time oriented model.

The remaining of this paper is organized as follows: in section 1 the problem under consideration and its main assumptions are described and a mathematical formulation is developed to solve the problem. In order to illustrate the proposed model, a numerical example is solved in section 2. In section 3 the paper is concluded by a summary of the main conclusions and suggestions for further research.

1. Proposed model and mathematical formulation

In this paper the assembly line balancing problem with sequence dependent setup times are considered with a cost oriented approach. The main assumptions of the problem under consideration are as follows:

a. Mass production of one homogenous commodity.
b. The production process, the tasks to be performed and the precedence relations, are already known.
c. Faced line with a predetermined cycle time $C$.
d. Each task $j$ has a Deterministic and integer operation time $d_j^i$.
e. Each task $j$ has a cost rate $k_j^i$ which determined the wage rates and the costs of capital for the task.
f. There are no assignment restrictions other than the precedence constraints
g. Serial line layout.
h. All stations are equally equipped with respect to machines and workers.
i. There are deterministic sequence dependent setup times between tasks.
j. Minimize the total costs of producing one product: $k = \sum_{j=1}^{N} C \times k_j^j$

The objective is to minimize the total cost per production unit and the decisions involved in this problem include the followings: (1) assignment of tasks to stations (2) the sequence in which tasks are performed in each station. The notations used in the mathematical model are presented in Table 1.

<table>
<thead>
<tr>
<th>Table 1: Notations used in the mathematical model</th>
</tr>
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<tbody>
<tr>
<td>$i, h$</td>
</tr>
<tr>
<td>$j$</td>
</tr>
<tr>
<td>$I$</td>
</tr>
<tr>
<td>$J$</td>
</tr>
<tr>
<td>$P_i(F'_i)$</td>
</tr>
<tr>
<td>$F_i(F'_i)$</td>
</tr>
<tr>
<td>$C$</td>
</tr>
<tr>
<td>$m_{\text{min}}$</td>
</tr>
<tr>
<td>$m_{\text{max}}$</td>
</tr>
<tr>
<td>$M$</td>
</tr>
<tr>
<td>$N$</td>
</tr>
<tr>
<td>$d_i^j$</td>
</tr>
<tr>
<td>$k_i^j$</td>
</tr>
<tr>
<td>$E_i(L_j)$</td>
</tr>
<tr>
<td>$T_j$</td>
</tr>
<tr>
<td>$t_{s\text{task}}$</td>
</tr>
<tr>
<td>$k_j^j$</td>
</tr>
<tr>
<td>$x_{ijs}$</td>
</tr>
<tr>
<td>$y_j$</td>
</tr>
<tr>
<td>$z_{i,j,k}$</td>
</tr>
<tr>
<td>$w_{ij}$</td>
</tr>
</tbody>
</table>
The problem under consideration is formulated as follows:

\[ \text{Min } k = \sum_{j=1}^{m_{\text{max}}} C_k j^z \]  

(1)

\[ \sum_{j=1}^{N_{mj}} \sum_{s=1}^{l_i} x_{ij}s = 1 \quad (\forall i) \]  

(2)

\[ x_{ij}s \leq 1 \quad (\forall j; s = 1, 2, \ldots N_{mj}) \]  

(3)

\[ \sum_{j=1}^{l_i} \sum_{s=1}^{N_{mj}} x_{ij}s \leq \sum_{j=1}^{l_i} \sum_{s=1}^{N_{mj}} (NT_{m, (j-1) + s}) x_{ij}s \quad (\forall i, h) | h \in P_j \]  

(4)

\[ k_j \geq k_i \sum_{s=1}^{N_{mj}} x_{ij}s \quad (\forall j; \forall i \in T_j) \]  

(5)

In this formulation equation (1) implies the objective function which is the total cost of producing one production unit. Constraints (2) ensure that every task is assigned to exactly one position in the schedule of one station. Constraints (3) imply that in each position in every station at most one task can be assigned. Constraints (4) indicate that the positions in each station are filled in an increasing order. With (5) the precedence constraints are observed, this involves both between stations and the positions inside a workstation. Constraints (6) determine the cost rate of each station which is equal to the maximum cost rate of the tasks assigned to the station. With (7) and (8) it is ensured that the processing time of each station, including task times and setups, is at most equal to the cycle time. Constraints (9) ensure that if tasks and are assigned to the same station and are respectively scheduled as the first and the last task in the station, then a setup time must be considered between tasks and . Constraints (10) determines the last task in the schedule of each station. Constraints (11) ensure that if tasks and is assigned to the same station and are respectively scheduled as the first and the last task in the station, then a setup time must be considered between tasks and .

Numerical example:

To illustrate the proposed model an example is presented, this example is an extension of the well-known instance of Mertens, which can be found in the homepage for assembly line balancing (http://www.assembly-line-balancing.de/). Figure 1 shows the precedence graph for this example along with duration and cost rate for each task. The setup times between tasks are presented in table 2. The cycle time is assumed to be equal to 15.
This instance is solved using both the proposed cost oriented model and time oriented model proposed by Anders et al., 2008. In the classic time oriented model the aim is to minimize the number of stations for a given cycle time. The optimum solutions found by each model is presented in figures 2 and 3 respectively. In these figures each station is represented by a shaded rectangle and the sequence of performing tasks in each station is presented by directed lines. Also the station loads are presented below the corresponding rectangle for each station. As seen in these figures the obtained number of stations for both models is 3. This means that the optimum solution found by the cost oriented model is also an optimum solution in terms of the number of station.

Fig. 1: example of a problem instance

Fig. 2: The optimum solution found by the proposed model

Fig. 3: The optimum solution found by the time oriented model

Due to the high competition in the production environment, minimizing the costs of production in order to minimize the final price of the product is of major importance. Therefore it is interesting to consider costs of production into consideration when comparing the models.

The costs of production for the solutions found for each model is presented in Table 3. As seen from this table, the production cost of the solution obtained by the time oriented model is 240 monetary units, while this number could be reduced to 210 using the proposed model. Thus, 30 monetary units are saved. Besides, the required number of stations is 3 which is the same number of stations needed by the solution obtained using the time-oriented model. This means that the solution is also optimal in terms of number of stations. Consequently,
the solution is the best in terms of the total cost, and while reaching this best, the best number of stations is also achieved.

### Table 3: Optimal solutions for the time oriented and cost oriented versions of the problem

<table>
<thead>
<tr>
<th>Station</th>
<th>Time-oriented optimal solution</th>
<th>Cost-oriented optimal solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>{1,2,5} 5</td>
<td>{1,2,3} 6</td>
</tr>
<tr>
<td>2</td>
<td>{6,4} 6</td>
<td>{5,6} 5</td>
</tr>
<tr>
<td>3</td>
<td>{3,7} 5</td>
<td>{4,7} 3</td>
</tr>
<tr>
<td>$\sum_{j \in S} k_j$</td>
<td>16</td>
<td>14</td>
</tr>
<tr>
<td>$k = C \times \sum_{j \in S} k_j$</td>
<td>240</td>
<td>210</td>
</tr>
</tbody>
</table>

**Conclusions and Future Research:**

Usually in the literature of assembly line research, it is assumed that, inside each station, Tasks can be performed in any feasible sequence without affecting the processing time of each other. However in practice, the sequence in which tasks are performed may have a significant effect on the processing time of each other due to existence of sequence dependent setup times (such as tool exchanges, walking times and etc). On the other hand owing to the high competition in the current production environment, reducing the production costs and increasing utilization of the current resources are of major importance for production managers.

In this paper the GALBPS is considered. Since production costs are one of the most important issues in production management, developing a model which directly attempts to minimize the production costs is of main interest. In this paper the GALBPS is considered with a cost oriented approach and a mathematical model is developed to solve the problem. In order to illustrate the proposed model a numerical example is solved using lingo 11 software and the result is compared with the results obtained by the classical time oriented approach.

Developing heuristic or meta-heuristics such as genetic algorithm or simulated annealing to solve the proposed model and considering multiple-objective optimization problem by simultaneously taking into account many other objectives, such as smoothing and load balancing are recommended for future research in this area.

**REFERENCES**


