Simulation of Laminar Natural Convection in a Cavity with Cylindrical Obstacles

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Abstract: The laminar natural convection inside a rectangular cavity containing two cylindrical obstacles has been numerically investigated. A curvilinear coordinates system was used to transfer the physical space into a computational domain. The governing partial differential equations are solved using stream function and vorticity method. The vorticity and energy equations were solved by using an alternate difference scheme(ADI) while the stream function with an iteration method. The cavity was differentially heated. The effect of the distance between the obstacles has been tested for Rayleigh number range $10^3 \leq Ra \leq 10^5$. The documented results show that the fluid flow and temperature fields are significantly depend on the distance between the obstacles for the studied Rayleigh numbers.

Nomenclature

$c$ dimensionless distance between obstacles($e/H$)
$g$ gravitational acceleration, m/s$^2$
$H$ height of the cavity wall, m
$J$ Jacobian of the transformation
$Nu$ local Nusslet number
$Nu_{av}$ average Nusselt number
$R$ radius of the obstacle, m
$Ra$ Rayleigh number
$T_c$ cold wall temperature, $^\circ$C
$T_h$ hot wall temperature, $^\circ$C
$u,v$ velocity components, m/s
$x,y$ Cartesian coordinates, m
$X, Y$ dimensionless Cartesian coordinates
$\alpha, \beta, \gamma, \tau, \sigma$ Transformation parameters in grid generation
$\xi, \eta$ coordinates in the transformed domain
$\psi$ dimensionless stream function
$\omega$ dimensionless vorticity
$\rho$ density, Kg/m$^3$
$\alpha$ thermal diffusivity, m$^2$/s
$\theta$ dimensionless temperature

INTRODUCTION

The study of natural convection in cavities containing obstacles acquired a significant importance due to its diverse application. These include solar collectors, cooling of electronic packages and heat exchangers. Investigation of natural convection in these cavities needs extensive work compared with a free obstacles cavities. That work arises from a resulting complexity due to transfer of the physical model to the computational one and consequently application of the boundary conditions. In reviewing the related published studies in this field, there is few studies concerned with natural convection inside a cavity with two cylinders as shown in the Fig.1. So the present work tries to enhance the academic research in this field and finding the possible benefit from this configuration. However there is no similar study to the considered problem as shown in Fig.1. Many researchers performed numerical and experimental studies for the natural convection inside cavities including obstacles. Cesiniet et al. (1999) conducted a numerical and experimental analysis on natural convection heat transfer from a horizontal cylinder enclosed in a rectangular cavity. The temperature distribution in the air and the heat transfer coefficients were measured by a holographic interferometer and compared with numerical predictions obtained by finite element based on stream-vorticity formulation of the momentum equations. The study was emphasized on Rayleigh number and geometry of the cavity. Marcel
and Antonine, (1996) performed studies on natural convection heat transfer of air from two vertical walls of finite conductance and horizontal walls at heat sink temperature. The results were obtained for Rayleigh number between $10^3$ and $10^6$. Edimilson and Marcel, (2005) investigated the heat transfer characteristics inside a square cavity filled with a fixed amount of conducting solid material. The obtained results showed that the average Nusselt number for cylindrical rods were slightly lower than those of the square rods. The natural convection around a tilted heated square cylinder kept in an enclosure was studied by Arnab and Amaresh, (2006). Stream-function vorticity formulation of the Navier-stokes equations was solved numerically using finite difference method in non-orthogonal body-fitted coordinates system. It was found that the uniform wall temperature heating is quantitatively different from the uniform wall heat flux heating. Also it was found that the overall heat transfer coefficient was changed for different aspect ratios. The natural convection heat transfer for a heated cylinder placed in a square enclosure was investigated by Roychowdhury et al. (2001). The study was performed for different thermal boundary conditions. The laminar buoyancy driven flow around two heated cubes in an infinite medium was investigated numerically and experimentally by Don et al. (1995). They emphasized on changing the distance between the cubes centers and the Rayleigh number. Sambamurthly et al. (2008) studied numerically the two dimensional natural convection from a heated square cylinder placed inside a cooled circular enclosure. A correlation between Ra, aspect ratio and conductivity was reported. The natural convection in a cavity with a solid block obstacles was investigated by many researchers such as Merrikh and large, Das and Reddy, (2005) and Laguerre et al. (2008) House et al. (1990) performed a numerical study on natural convection heat transfer in a vertical square cavity in the presence of a hot conductivity body situated in the center. They conducted that the heat transfer across the enclosures was increased by the body as the conductivity less than one.

In this paper, the natural convection heat transfer and fluid flow inside a rectangular cavity containing two cylinders has been numerically investigated. The emphasis was concentrated on changing the distance between the two cylinders. The cavity was partially heated as shown in Fig.1 and the two cylinders are situated on the bottom hot wall. The stream function and vorticity method was used to analyze the characteristics of the problem and the grid generation techniques was used to solve the complexity in the physical domain. The grid generation method proposed by Thompson, (1974) is used to map the non-rectangular grid in the physical space into a rectangular uniform grid in the computational space.

**Mathematical model:**

The governing equations of a viscous incompressible flow and heat transfer are described as flows. The viscous and inertia effects are assumed to be ignored and the Boussinesque approximation is valid.

\[
\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = -\omega 
\]  

(1)

\[
\frac{\partial \Omega}{\partial t} + u \frac{\partial \omega}{\partial x} + v \frac{\partial \omega}{\partial y} = \mu \left( \frac{\partial^2 \Omega}{\partial x^2} + \frac{\partial^2 \Omega}{\partial y^2} \right) + \rho g \beta \frac{\partial T}{\partial x} 
\]

(2)
The above governing equations can be written in dimensionless stream and vorticity method after using the following dimensionless parameters.

\[ \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \]  

(3)

The transformation of the new dependent variables \( (\zeta, \eta) \) leads to replacement of \( \psi(x, y) \) in to \( \psi(\zeta, \eta) \), \( \omega(x, y) \) to \( \omega(\zeta, \eta) \) and \( \theta(x, y) \) to \( \theta(\zeta, \eta) \) [12].

\[ \lambda \psi_{\zeta} + \sigma \psi_{\eta} + \alpha \psi_{\zeta \zeta} - 2\beta \psi_{\zeta \eta} + \gamma \psi_{\eta \eta} = -J^2 \omega \]  

(7)

\[ \omega_{\zeta} \left( \frac{-\psi_{\zeta \eta} + \psi_{\eta \zeta}}{J} \right) / J = \left( \lambda \psi_{\zeta} + \sigma \psi_{\eta} + \alpha \psi_{\zeta \zeta} - 2\beta \psi_{\zeta \eta} + \gamma \psi_{\eta \eta} \right) / J^2 \Pr + Ra \Pr \theta / J \left[ \left( \theta_{\eta \eta} - \theta_{\eta \zeta} \right) \right] \]  

(8)

\[ \theta_{\zeta} + \frac{1}{J} \left( \lambda \theta_{\zeta} + \sigma \theta_{\eta} + \alpha \theta_{\zeta \zeta} - 2\beta \theta_{\zeta \eta} + \gamma \theta_{\eta \eta} \right) / J^2 Ra \Pr \]  

(9)

where

\[ \lambda = \left( \chi Y_{\zeta} - \chi Y_{\eta} \right) / J \]  

(10)

\[ \sigma = \left( \chi Y_{\zeta} - \chi Y_{\eta} \right) / J \]  

(11)

\[ D_{\zeta} = \alpha Y_{\zeta \zeta} - 2\beta Y_{\zeta \eta} + \gamma Y_{\eta \eta} \]  

(12)

\[ D_{\eta} = \alpha Y_{\eta \eta} - 2\beta Y_{\zeta \eta} + \gamma Y_{\eta \eta} \]  

(13)

2-1 Boundary Conditions:

In order to solve the mathematical model, the following boundary conditions were used.

\[ U = V = 0, \theta = 0, \psi = 0, \omega = -\frac{\alpha}{J^2} \psi_{\zeta \zeta} \] on the upper cold wall

\[ U = V = 0, \theta = 1, \psi = 0, \omega = -\frac{\alpha}{J^2} \psi_{\zeta \zeta} \] on the lower hot wall

\[ U = V = 0, \frac{\partial \theta}{\partial \eta} = 0, \psi = 0, \omega = -\frac{\alpha}{J^2} \psi_{\eta \eta} \] on the two insulated walls

The mentioned boundary condition for the vorticity was imposed according to the Woods formula, (1982). The local and average Nusselt number along the hot bottom wall is calculated as follows.
\[
\text{Nu} = -\int_0^L \frac{d\theta}{dx}
\]

\[
\text{Nu}_w = \frac{1}{L} \int_0^L \text{Nu}dx
\]

**Numerical solution:**
Finite difference formulation is used to discretize the considered partial differential equations. The resulting algebraic equations for vorticity and temperature were solved by using alternate difference implicit (ADI) method. The iteration method with successive overrelaxation scheme (SOR) was used for solving the discretization equation of the stream function. The Relaxation factor used for stream function had the value of 1. A home computer program using Fortran 90 language was constructed to handle the considered problem. In order to ensure that the flow and heat transfer characteristics are not affected by the mesh, different grids were used, \((151 \times 51), (181 \times 51)\) and \((201 \times 51)\) respectively. The grid density \((201 \times 51)\) was used in this work.

**RESULTS AND DISCUSSION**

The present predicted results of the problem under consideration are accomplished for Rayleigh number ranges \(10^3 \leq \text{Ra} \leq 10^5\) and different values of distance between the obstacles.

Fig.2 demonstrates the distribution of stream function for the different values of the distance between the two cylinders (obstacles). As a general description, the flow field is represented by a large recirculation region and its strength strongly depends on the distance between the two cylinders. It is observed that the axisymmetric center of the whole recirculation region is situated at the zone between the two cylinders. It is noticed that the strength of the re-circulation increases as the distance between the two cylinders increases and the distance \(e=1.013\) exhibited the large values of the stream function. Also it is observed that the shape and location of the vorticity located at the region between the two cylinders is noticeably affected with the change of the distance between the two cylinders. The dimensionless temperature distribution for different distances between the obstacles is depicted at Fig.3. From the gradient of the colors, it can be seen that the temperature values are increased as the distance between the cylinders increases. This increase is clearly observed at the zone between the two cylinders. When the distance between the obstacles increase, the hot length of the bottom wall is increased and consequently increasing the temperature of the adjacent fluid. This situation is confirmed at Fig.7 where the Nusselt number increases at the mentioned zones as the distance \(e\) increases. However the Nusselt number is decreased at the region upstream the first cylinder and downstream the second cylinder as the distance \(e\) increases. This trend attributed to the decrease of the hot length of the bottom wall. The effect of Rayleigh number on stream function and temperature distribution is depicted at Fig.4 and Fig.5. At Fig.4, it is observed that the Ra has a noticeable effect on the evolution and shapes of the resulting vortices as shown in a, b, and c. At \(\text{Ra}=10^3\), the resulting vorticity is shifted towards the centers of the enclosure while with increasing the Ra, it seems to be moved towards the bottom wall. This attributed to the increase in convection currents. At Fig.5, the values of the temperature distribution is increased due to the increase of the hot wall of the bottom wall as shown in a, b, and c. The variation of the average Nusselt number versus Ra is seen in Fig.8 at different distances between the cylinders. It is observed that the average Nusselt number decreases for \(\text{Ra} \leq 10^4\) and increases for \(\text{Ra} > 10^4\). The validation of the present numerical method is tested with the available published results as shown in Fig. 9. The comparison indicated a good agreement.

**Conclusions:**
The laminar natural convection inside a rectangular cavity having two cylindrical obstacles has been numerically investigated for different Rayleigh numbers. The obtained concluding remarks can be summarized as follows.

The distance between the two cylindrical obstacles was a controlling parameter for describing the flow and thermal field.

The strength of the recirculation region increases as the distance between the two cylindrical obstacles increases.
Fig. 2: Stream function distribution for different distances between obstacles, $Ra=10^4$
Fig. 3: Temperature distribution for different distances between obstacles, Ra=10^4
Fig. 4: Stream function distribution for different Rayleigh numbers, $e=0.73$

da. $Ra=10^3$

b. $Ra=10^4$

c. $Ra=10^6$
Fig. 5: Temperature distribution for different Rayleigh numbers, e=0.73
Fig. 6: Effect of Rayleigh number on the local Nusselt number (on the hot wall) for different distances between obstacles.

Fig. 7: Nusselt number variation (on the hot wall) for different distances between the obstacles and $Ra=10^4$.

Fig. 8: Average Nusselt Number variation versus $Ra$ for different distances between baffles.
The distance (e) equal to 1.013 indicated the large values of the stream function.

The local Nusselt number increases as the distance (e) between the two cylindrical obstacles increases. The zone between the cylindrical obstacles exhibited the large values of Nusselt number for the studied Rayleigh number.

The increase in the value of the Rayleigh number, enhanced the rate of heat transfer.

REFERENCES


