Stochastic Power Generation Unit Commitment in a Restructured Power Market

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Abstract: This paper proposes a new formulation of the unit commitment problem of electric power generators in a restructured electricity market. Under these conditions, an electric power generation company will have the option to buy or sell from a power pool in addition to producing electricity on its own. The unit commitment problem is expressed as a stochastic optimization problem in which the objective is to maximize expected profits and the decisions are required to meet the standard operating constraints. Under the assumption of competitive market and price taking, it is depicted that the unit commitment schedule for a collection of N generation units can be solved by considering each unit separately. The volatility of the spot market price of electricity is represented by a stochastic model. This paper uses probabilistic dynamic programming to solve the stochastic optimization problem pertaining to unit commitment. It is shown that for a market of 150 units the proposed unit commitment can be accurately solved in a reasonable time by using the normal, Edgeworth, or Monte Carlo approximation methods.

Key words: Wind Energy Conversion System, Dynamic voltage compensator, Reactive Compensation, Voltage and Energy Enhancement

INTRODUCTION

In the short-term, typically considered to run from twenty-four hours to one week, the solution of the unit commitment problem (UCP) is used to assist decisions regarding generating unit operations (Wood and Wollenberg, 1996). In a regulated market, a power generating utility solves the UCP to obtain an optimal schedule of its units in order to have enough capacity to supply the electricity demanded by its customers. The optimal schedule is found by minimizing the production cost over the time interval while satisfying the demand and a set of operating constraints. The minimization of the production costs assures maximum profits because the power generating utility has no option but to reliably supply the prevailing load. The price of electricity over this period is predetermined and unchanging; therefore, the decisions on the operation of the units have no effect on the firm’s revenues.

As deregulation is being implemented in various regions of the United States, the traditional unit commitment problem continues to remain applicable for the commitment decisions made by the Independent System Operator (ISO). The ISO resembles very much the operation of a power generating utility under regulation. The ISO manages the transmission grid, controls the dispatch of generation, oversees the reliability of the system, and administers congestion protocols (Galiana and Ilic, 1998). The ISO is a non-profit organization. Its economic objective is to maximize social welfare, which is obtained by minimizing the costs of reliably supplying the aggregate load. Under deregulation, the UCP for an electric power producer will require a new formulation that includes the electricity market in the model. The main difficulty here is that the spot price of electricity is no longer predetermined but set by open competition. Thus far, the hourly spot prices of electricity have shown evidence of being highly volatile. The unit commitment decisions are now harder and the modeling of spot prices becomes very important in this new operating environment. Different approaches can be found in the literature in this regard. Takriti et al., (Takriti et al., 1997) have introduced a stochastic model for the UCP in which the uncertainty in the load and prices of fuel and electricity are modeled using a set of possible scenarios. The challenge here is to generate representative scenarios and assign them appropriate probabilities. Allen and Ilic (Allen and Ilic, 1997) have proposed a stochastic model for the unit commitment of a single generator. They assume that the hourly prices at which electricity is sold are uncorrelated and normally distributed.

The purpose of this paper is to present a new formulation to the UCP suitable for an electric power producer in a deregulated market and consider computationally efficient procedures to solve it. We express the UCP as a stochastic optimization problem in which the objective is to maximize expected profits and the
decisions are required to meet operating constraints such as capacity limits and minimum up and down time requirements. We show that when the spot market of electricity is included, the optimal solution of a UCP with M units can be found by solving M uncoupled sub-problems. A sub-problem is obtained by replacing the values of the Lagrange multipliers by the spot market prices of electricity. The volatility of the spot market price of electricity is accounted for by using a variation of the stochastic model proposed by Ryan and Mazumdar (Ryan and Mazumdar, 1990). The model, which is referred to as the probabilistic production-costing model, incorporates the stochastic features of load and generator availabilities. It is often used to obtain approximate estimates of production costs (Mazumdar and Kapoor, 1995). This model ignores the unit commitment constraints and assumes that a strict predetermined merit order of loading prevails. This implies that a generator will be dispatched only when the available unit immediately preceding it in the loading order is working at its full capacity. We believe that this model provides a good approximation to the operation of an electricity market such as the California market in which no centralized unit commitment decisions are taken.

The model captures the fundamental stochastic characteristics of the system. At any moment, a power producer may not be fully aware of the exact characteristics of the units comprising the market at that particular time. But it is likely to possess information about the steady state statistical characteristics of the units participating in the market. Ryan and Mazumdar's probabilistic production costing model can be used to provide a steady-state picture of the market.

The hourly spot market price of electricity is determined by the market-clearing prices. The market-clearing price can be shown to be the variable cost or bid of the last unit used to meet the aggregate load prevailing at a particular hour. This unit is called the marginal unit. We determine the probability distribution of the hourly market-clearing price based on the stochastic process governing the marginal unit, which depends on the aggregate load and the generating unit availabilities. We model the aggregate load as a Gauss-Markov stochastic process and use continuous-time Markov chains to model the generating unit availabilities (Mazumdar and Kapoor, 1995). We assume that the information on mean time to repair, mean time to failure, capacity, and variable operating cost of each unit participating in the market required to characterize these processes is available. We use probabilistic dynamic programming to solve the stochastic optimization problem pertaining to unit commitment.

We also report results on the accuracy and computational efficiency of several analytical approximations as compared to Monte Carlo simulation in estimating probability distributions of the spot market price for electric power.

1. Formulation:

We consider the situation in which an electric power producer owns a set of M generating units and needs to determine an optimal commitment schedule of its units such that the profit over a short period of length T is maximized. Revenues are obtained from fulfilling bilateral contracts and selling electric power, at spot market prices, to the power pool. It is assumed that the electric-power company is a price taker.

If at a particular hour the power supplier decides to switch on one of its generating units, it will be willing to take the price that will prevail at this hour. We also assume that the power supplier has no control over the market prices and the M generating units will remain available during the short time interval of interest.

In determining an optimal commitment schedule, there are two decision variables which are denoted by \( P_k \) and \( v_k \). The first variable denotes the amount of power to be generated by unit \( k \) at time \( t \), and the latter is a control variable, whose value is “1” if the generating unit \( k \) is committed at hour \( t \) and “0” otherwise. The cost of the power produced by the generating unit \( k \) depends on the amount of fuel consumed and is given by a known cost function \( CF_k(p) \):

\[
CF_k(p) = a_k + b_k p + c_k p^2,
\]

where \( p \) is the amount of power generated. The start-up cost, which for thermal units depends on the prevailing temperature of the boilers, is given by a known function \( S_k(x_{kt}) \). The value of \( x_{kt} \) specifies the consecutive time that the unit has been on (+) or off (-) at the end of the hour \( t \). In addition, a generating unit must satisfy operating constraints. The power produced by a generating unit must be within certain limits. When the \( k \)th generating unit is running, it must produce an amount of power between \( P_{k\text{min}} \) and \( P_{k\text{max}} \) (MW). If the generating unit is off, it must stay off for at least \( t_{k\text{dn}} \) hours, and if it is on, it must stay on for at least \( t_{k\text{up}} \) hours.
1.1. Problem Formulation:

The objective function is given by the sum over the hours in the interval \([0,T]\) of the revenue minus the cost. The revenue is obtained from supplying the bilateral contracts and by selling to the power pool at a price of \(m_t\) per MWH the surplus energy \(E_t\) (if any) produced in each hour \(t\). The cost includes the cost of producing the energy, buying shortfalls (if needed) from the power pool, and the startup costs. Defining the supply amount stipulated under the bilateral contract by \(l_t\) (MWH) and by \(R\) ($/MWH) the price, the objective function (maximum total profit) is given by:

\[
\text{Max } \left\{ \sum_{t=1}^{T} \left[ l_t R - m_t E_t - \sum_{k=1}^{M} \left[ CF_k(P_{k,t}) + S_k(x_{k,t})(1-v_{k,t-1}) \right] v_{k,t} \right] \right\}
\]

subject to the following constraints (for \(t=1,\ldots,T\) and \(k=1,\ldots,M\))

Load: \(E_t + \sum_{k=1}^{M} v_{k,t} P^k_{k,t} = l_t\) \hspace{1cm} \text{(4)}

Capacity limits: \(v_{k,t} P^\text{min}_k \leq P_{k,t} \leq v_{k,t} P^\text{max}_k\) \hspace{1cm} \text{(5)}

Minimum down time: \(v_{k,t} \leq 1 - I(-t^\text{down}_k + 1 \leq x_{k,t-1} \leq -1)\) \hspace{1cm} \text{(6)}

Minimum up time: \(v_{k,t} \geq I(1 \leq x_{k,t-1} \leq t^\text{up}_k - 1)\) \hspace{1cm} \text{(7)}

where \(I(x) = \begin{cases} 0 & \text{if } x \text{ is false} \\ 1 & \text{if } x \text{ is true} \end{cases}\)

\(P_{k,t} \geq 0\) and \(E_t\) unrestricted in sign

\(v_{k,t} = \{0,1\}\)

After substituting in the objective function the value of \(E_t = l_t - \sum_{k=1}^{M} v_{k,t} P^k_{k,t}\), obtained from Equation 4, we re-write Equation 3 as follows:

\[
\text{Max } \left\{ \sum_{t=1}^{T} \left[ l_t R - m_t E_t - \sum_{k=1}^{M} \left[ CF_k(P_{k,t}) + S_k(x_{k,t})(1-v_{k,t-1}) \right] v_{k,t} \right] \right\}
\]

subject to the operating constraints. Because the constraints (5) to (7) refer to individual units only, the advantage of Equation 9 is that the objective function is now separable by individual units. The optimal solution can be found by solving \(M\) de-coupled sub-problems. Thus, the sub-problem \(D_k\) for the \(k\)th unit (\(k=1,\ldots,M\)) is:
subject to operating constraints of the \( k \)th unit. Equation 10 is similar to the sub-problem obtained in the standard version of the UCP (Bard, 1988) excepting that the value of the Lagrange multipliers are now replaced by the spot market price of electricity.

1.2. Stochastic Formulation of the Sub-problem:

We next consider the value of the spot market price of electricity, \( m_t \), which is determined by the market-clearing price, as a random variable. When the optimization sub-problem is being solved for a particular unit, we assume that the market, which includes the \( M-1 \) units owned by the power producer solving the problem, consists of \( N \) generating units (\( N>>M \)). The generating unit for which the sub-problem is solved is excluded from the market. We assume that the unit commitment decisions for any one unit have a negligible effect on the determination of the marginal unit of the market for a given hour.

To model the market-clearing price, we assume that the generators participating in the market are brought into operation in an economic merit order of loading. The \( i \)th unit in the loading order has a capacity \( c_i \) (MW), variable energy cost \( d_i \) ($/MWH), and a forced outage rate \( q_i \). Under the assumption of economic merit order of loading, the market-clearing price at a specific hour \( t \), is equal to the operating cost ($/MWH) of the last unit used to meet the load prevailing at this hour. The last unit in the loading order is called the marginal unit and is denoted by \( J(t) \). The market-clearing price, \( m_t \), is thus equal to \( d_{J(t)} \). The values of \( J(t) \) and \( d_{J(t)} \) depend on the prevailing aggregate load and the operating states of the generating units in the loading order.

We write the objective function of the sub-problem for one of the \( M \) generating units as follows:

\[
\text{Maximum profit} = \max \sum_{t=1}^{T} \{ m_t P_{k,t} - CF_k(P_{k,t}) + S_k(x_{k,t})(1 - v_{k,t-1})v_{k,t} \} 
\]  

subject to the operating constraints: capacity limits, minimum up time, and minimum down time.

1.3. Probabilistic Dynamic Programming Solution:

The maximum profit over the period \( T \) (Equation 11) is a random variable because the hourly market-clearing price is a random variable. We assume that at the time of the decision, hour zero, the marginal unit and the load for all the hours before hour zero are known. We denote the marginal unit at time zero by \( j_0 \), and solve the sub-problem by maximizing the conditional expected profit over the period \( T \). We express the objective function as:

\[
\text{Max} \ E[\text{profit} | j_0] = \max \sum_{t=1}^{T} \{ PE[d_{J(t)}] - CF(P_t) - S(x_t)(1 - v_{t-1})v_t \}
\]  

This equation is subject to the same operating constraints described earlier. We use probabilistic dynamic programming to solve this optimization problem. We define the function \( g_t(v_t, j) \) by the following equation:

\[
g_t(v_t, j) = \max \{ P_d j - CF(P_t) \} \quad 0 < t \leq T
\]

This function denotes the maximum profit at hour \( t \) given that at this hour the \( j \)th unit is determining the market-clearing price and the generator to be scheduled is in the operating state \( v_t \). We also define the recursive function \( F_t(x_t) \) to be the optimum expected profit from hour \( t \) to hour \( T \) of operating the generator that is in state \( x_t \) at time \( t \). Thus, the expression for hour zero is:

\[
F_0(x_0) = \max \{ F_t(x_t) - v_t[1 - v_0]S(x_t) \}
\]

and for hour \( t \) (\( 0 < t < T \)) the expression is given by the following recursive relation:

\[
F_t(x_t) = \max \{ F_{t+1}(x_{t+1}) - v_{t+1}[1 - v_t]S(x_{t+1}) + \sum_{j=1}^{N} \Pr[J(t) = j | J(0) = j_0] g_t(v_t, j) \}
\]  

Setting the expected incoming profit at time \( T+1 \) to be zero (\( F_{T+1}(x_{T+1})=0 \), we obtain the boundary condition for the last stage \( t = T \) to be:

\[
F_T(x_T) = \max \{ F_{T+1}(x_{T+1}) - v_{T+1}[1 - v_T]S(x_{T+1}) \}
\]
F_T(x_T) = \sum_{j=1}^{N} \Pr[J(T) = j | J(0) = j_0] \mathcal{E}_T(v_T, j) \quad \quad \quad (16)

The initial conditions are given by the initial state of the generator \( x_0 \) and \( v_0 \), and the marginal unit at hour zero \( j_0 \). Consequently, the optimal schedule is given by the solution of \( F_0(x_0) \). To solve the problem, the following conditional probabilities need to be computed.

\[
\Pr[J(t) = j | J(0) = j_0] = \frac{\Pr[J(t) = j \text{ and } J(0) = j_0]}{\Pr[J(0) = j_0]} \quad \quad \quad (17)
\]

Thus, the joint probability distribution of \( J(0) \) and \( J(t) \), and the marginal probability distribution of \( J(0) \) are needed.

2. Stochastic Model for the Market-clearing Price:

The stochastic model of the market-clearing price uses the production-costing model proposed by Ryan and Mazumdar (Ryan and Mazumdar, 1990). This model has been used in estimating the mean and variance of production cost and marginal cost (Shih and Mazumdar, 1998) of a power generating system.

2.1. Stochastic Model of the Market:

For a market with \( N \) generating units, the model uses the following assumptions:

The generators are dispatched at each hour in a fixed, pre-assigned loading order, which depends only on the load and the availability of the generating units. Operating constraints such as minimum up time, minimum down time, spinning reserve, and scheduled maintenance are not considered.

The \( i \)th unit in the loading order has a capacity \( c_i \) (MW), variable energy cost \( d_i \) ($/MWH), mean time to failure \( l_i \), mean time to repair \( m_i \), and a forced outage rate, \( q_i \), \( i = 1, 2, \ldots, N \).

After adjusting for the variations in the ambient temperature and periodicity, the load at time \( t \), \( u(t) \), is assumed to follow a Gauss-Markov process with \( \mathbb{E}[u(t)]=\bar{u} \) and \( \text{Cov}[u(t), u(t)]=s_{ut} \), where \( \bar{u} \) and \( s_{ut} \) are assumed to be known.

The operating state of each generating unit \( i \) follows a two-state continuous-time Markov chain, \( Y_i(t) = \{0,1\} \), with failure rate \( l_i \) and repair rate \( m_i \). The forced outage rate \( q_i \) is related to these quantities by the equation \( q_i = l_i / (l_i + m_i) \).

For \( i \neq j \), \( Y_i(r) \) and \( Y_j(t) \) are statistically independent for all values of \( r \) and \( t \). Each \( Y_i(t) \) is independent of \( u(t) \) for all values of \( t \).

2.2. Probability Distribution of the Marginal Unit:

To derive an analytical expression for the probability mass function of the marginal unit at time \( t \), we first note that

\[
\Pr[J(t) = j] = \Pr[J(t) > j-1] - \Pr[J(t) > j] \quad \text{and that the events } J(t) > j \text{ and } u(t) - \sum_{i=1}^{j} c_i Y_i(t) > 0 \text{ are equivalent.}
\]

Thus, the following equality holds:

\[
\Pr[J(t) > j] = \Pr[u(t) - \sum_{i=1}^{j} c_i Y_i(t) > 0] \quad \quad \quad (18)
\]

Therefore, to obtain the probability mass function of \( J(t) \), the probability that \( u(t) - \sum_{i=1}^{j} c_i Y_i(t) \) is greater than zero for all values of \( j \) needs to be computed.

2.3. Bivariate Probability Distribution of the Marginal Unit:

An analytical approximation for the bivariate probability mass function of \( J(r) \) and \( J(t) \), needed for evaluating Equation 17, requires the following development. Writing and observing that the events \( m & J(t) = n \) = \( \Pr[J(r) > m-1 & J(t) > n-1] - \Pr[J(r) > m & J(t) > n] \)

\[
\text{and that the events } m & J(t) = n \text{ and } J(r) > m \text{ and } J(t) > n \text{ are equivalent, we obtain the following equality:}
\]

\[
\{ u(r) - \sum_{i=1}^{m} c_i Y_i(r) > 0 \} \text{ and } \{ J(r) > m \text{ and } J(t) > n \} \quad \text{and } \{ J(r) > m \text{ and } J(t) > n \} \text{ are equivalent, we obtain the following equality:}
\]

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Therefore, to compute the bivariate probability mass function of $J(r)$ and $J(t)$ the probability that
\[ \{ u(r) - \sum_{i=1}^{m} c_i Y_i(r) > 0 \text{ and } u(t) - \sum_{j=1}^{n} c_j Y_j(t) > 0 \} \]
needs to be evaluated for all values of $m$ and $n$.

The computational effort in evaluating equations 18 and 20 depends on the many values that the expression \( \sum_{j=1}^{i} c_i Y_i(t) \) can take, which in the worst case is \( 2^n \) (when \( j=N \)). Thus, the computational time increases exponentially as \( N \) increases and it would make an exact computational procedure prohibitive for large \( N \). In our numerical examples, we have used three approximation methods: the normal, Edgeworth and Monte Carlo approximations. The Edgeworth approximation is known in the power system literature as the method of cumulants. We also attempted the use of the large deviation or equivalently, the saddlepoint approximation method, but it turned out to be prohibitively time-consuming for very large systems.

3. Solution of the Probabilistic Unit Commitment Problem: a Numerical Example:

For our purpose, we assume that a complete description of the electricity market is given by the data concerning the \( N \) power generators that comprise the market, historical data of the aggregate load, and the hourly temperature forecast for the day of trading. The description of the power generators includes the order in which they will be loaded by the ISO, their capacities, energy costs, mean times to failure, and mean times to repair. The data for the aggregate load gives the historically forecast ambient temperature and the corresponding load for each hour in the region served by the marketplace. In this example, a data set that gave the actual ambient temperature and the corresponding load for each hour in a region covering the Northeastern United States during the calendar years 1995 and 1996 was used. The last day of this data set, September 20, 1996, was chosen as the trading day. The actual temperatures on this day are given in Table 1. They were assumed to be the forecast temperatures.

### Table 1: Actual Average Hourly Temperatures on September 20, 1996.

<table>
<thead>
<tr>
<th>Hour</th>
<th>Temperature</th>
<th>Hour</th>
<th>Temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>59</td>
<td>12</td>
<td>72</td>
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<td>13</td>
<td>73</td>
</tr>
<tr>
<td>2</td>
<td>56</td>
<td>14</td>
<td>77</td>
</tr>
<tr>
<td>3</td>
<td>54</td>
<td>15</td>
<td>77</td>
</tr>
<tr>
<td>4</td>
<td>53</td>
<td>16</td>
<td>79</td>
</tr>
<tr>
<td>5</td>
<td>52</td>
<td>17</td>
<td>77</td>
</tr>
<tr>
<td>6</td>
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<td>19</td>
<td>70</td>
</tr>
<tr>
<td>8</td>
<td>55</td>
<td>20</td>
<td>63</td>
</tr>
<tr>
<td>9</td>
<td>59</td>
<td>21</td>
<td>61</td>
</tr>
<tr>
<td>10</td>
<td>64</td>
<td>22</td>
<td>59</td>
</tr>
<tr>
<td>11</td>
<td>68</td>
<td>23</td>
<td>58</td>
</tr>
</tbody>
</table>

**Example:**

The market is described by the aggregate load and a power generation system consisting of generators participating in the market. Using statistical time series analysis on the data at hand, we found that an ARIMA \((1,0,0)x(0,1,0)_{120}\) provided a very good description of the actual load observed. The model used is as follows:

\[
\hat{u}(t) = \hat{\beta}_{0,t} + \hat{\beta}_{1,t} \tau_t + \hat{\beta}_{2,t} (\tau_t - 65) \delta(\tau_t) + x(t)
\]

where \( t \) is the average ambient temperature at hour \( t \) and \( d(t) \) is defined as:

\[
\delta(\tau_t) = \begin{cases} 
0 & \text{if } \tau_t \leq 65 \\
1 & \text{if } \tau_t > 65 
\end{cases}
\]

and
\[ x(t) = x(t-120) + 0.879[ x(t-1) - x(t-121)] + z(t) \]  

(23)

where \( z(t) \) is a Gaussian white noise process with mean zero and estimated variance \( \sigma^2_z = 2032.55 \). The estimates of the least-square regression coefficients, \( \hat{\beta} \), relating load to temperature, are given in Table 2.

### Table 2: Least-square Estimates of Regression Coefficients Relating Load to Temperature.

<table>
<thead>
<tr>
<th>Hour</th>
<th>( b_0 )</th>
<th>( b_1 )</th>
<th>( b_2 )</th>
<th>Hour</th>
<th>( b_0 )</th>
<th>( b_1 )</th>
<th>( b_2 )</th>
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<td>24.92</td>
<td>12</td>
<td>1801</td>
<td>-6.19</td>
<td>27.8</td>
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<td>23.21</td>
<td>13</td>
<td>1828</td>
<td>-6.81</td>
<td>29.6</td>
</tr>
<tr>
<td>2</td>
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<td>-2.8</td>
<td>21.83</td>
<td>14</td>
<td>1837</td>
<td>-7.48</td>
<td>29.6</td>
</tr>
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</tr>
<tr>
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<td>25.52</td>
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<td>1891</td>
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</tr>
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<td>-5</td>
<td>24.35</td>
<td>18</td>
<td>2046</td>
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</tr>
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<td>-5</td>
<td>25.64</td>
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<td>2055</td>
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</tr>
<tr>
<td>8</td>
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<td>-4.2</td>
<td>25.59</td>
<td>20</td>
<td>2003</td>
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<td>32.1</td>
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<td>25.88</td>
<td>21</td>
<td>1699</td>
<td>-4.49</td>
<td>28.5</td>
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<tr>
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<td>1779</td>
<td>-5.2</td>
<td>26.6</td>
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<td>23</td>
<td>1393</td>
<td>-3.82</td>
<td>28.1</td>
</tr>
</tbody>
</table>

The market is assumed to consist of 150 power generating units. This system was obtained by repeating ten times each unit of a 15-unit system, which in turn is a smaller version of the IEEE Reliability Test System (RTS). The load data was also multiplied by a factor of ten. Defining \( C_i \) as the cumulative capacity of the first \( i \) units:

\[ C_i = \sum_{j=1}^{i} c_j \]  

(24)

we assume that the variable cost of each unit \( i \) is given by the following function:

\[ d_i = 6 + 0.00073C_i + 0.000000045C_i^2 \]  

(25)

This assumption allows for the unit operating costs to increase in order of the position of the units in the loading order. The relevant characteristics of the units, in their loading order, are given in Table 3.

The problem is to schedule one of the generators of the power producer for the next 23 hours given the information about the electricity market and the known initial operating conditions for the generating unit. The characteristics of this generator were taken from (Wood and Wollenberg, 1996), and they are reproduced in Table 4. We modified the fuel-cost function of the unit to be consistent with the range of the individual units’ energy costs. The objective is to maximize the expected profit over this period. We assumed the generator to have been on for eight consecutive hours. As mentioned in section 2.3, this generator is not included in the set of generators that comprise the market. We also assumed that the variable cost of the 61st unit is currently determining the market-clearing price, which is $19.73 /MWH.

### Table 3: Characteristics of Generating Units Participating in a Hypothetical Market.

<table>
<thead>
<tr>
<th>Unit</th>
<th>Capacity ( c_i ) (MW)</th>
<th>MTTF1/1, (hour)</th>
<th>MTTR1/m, (hour)</th>
<th>Energy cost ( d_i ) ($/MWH)</th>
<th>Energy cost ( d_{e,o} ) ($/MWH)</th>
</tr>
</thead>
<tbody>
<tr>
<td>29229</td>
<td>350 1150</td>
<td>100 6.26</td>
<td></td>
<td>9.11</td>
<td></td>
</tr>
<tr>
<td>29544</td>
<td>150 960</td>
<td>40 9.26</td>
<td></td>
<td>10.78</td>
<td></td>
</tr>
<tr>
<td>21-30</td>
<td>150 960</td>
<td>40 10.95</td>
<td></td>
<td>12.65</td>
<td></td>
</tr>
<tr>
<td>31-40</td>
<td>150 960</td>
<td>40 12.84</td>
<td></td>
<td>14.72</td>
<td></td>
</tr>
<tr>
<td>41-50</td>
<td>150 960</td>
<td>40 14.94</td>
<td></td>
<td>17</td>
<td></td>
</tr>
<tr>
<td>51-60</td>
<td>150 960</td>
<td>40 17.24</td>
<td></td>
<td>19.48</td>
<td></td>
</tr>
<tr>
<td>61-70</td>
<td>150 1960</td>
<td>40 19.73</td>
<td></td>
<td>22.16</td>
<td></td>
</tr>
<tr>
<td>71-80</td>
<td>200 950</td>
<td>50 22.53</td>
<td></td>
<td>26.05</td>
<td></td>
</tr>
<tr>
<td>81-90</td>
<td>200 950</td>
<td>50 26.46</td>
<td></td>
<td>30.3</td>
<td></td>
</tr>
<tr>
<td>91-100</td>
<td>200 950</td>
<td>50 30.74</td>
<td></td>
<td>34.91</td>
<td></td>
</tr>
<tr>
<td>101-110</td>
<td>100 1200</td>
<td>50 35.15</td>
<td></td>
<td>37.35</td>
<td></td>
</tr>
<tr>
<td>111-120</td>
<td>100 1200</td>
<td>50 37.6</td>
<td></td>
<td>39.88</td>
<td></td>
</tr>
<tr>
<td>121-130</td>
<td>100 1200</td>
<td>50 40.13</td>
<td></td>
<td>42.5</td>
<td></td>
</tr>
<tr>
<td>131-140</td>
<td>50 2940</td>
<td>60 42.63</td>
<td></td>
<td>43.84</td>
<td></td>
</tr>
<tr>
<td>141-150</td>
<td>100 450</td>
<td>50 44.11</td>
<td></td>
<td>46.6</td>
<td></td>
</tr>
</tbody>
</table>
Table 4: Characteristics of a Generator for which the Commitment Decision is Being Made.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{max}$</td>
<td>250 MW</td>
</tr>
<tr>
<td>$P_{min}$</td>
<td>60 MW</td>
</tr>
<tr>
<td>$t_{up}$</td>
<td>5 hours</td>
</tr>
<tr>
<td>$t_{dn}$</td>
<td>3 hours</td>
</tr>
<tr>
<td>Initial State</td>
<td>8 hours</td>
</tr>
<tr>
<td>Fuel Cost</td>
<td>$585.62 + 16.95p + 0.0042p^2 \ ($/MWH)</td>
</tr>
<tr>
<td>Start-up Cost</td>
<td>$\begin{cases} $170 &amp; \text{if } -x \leq 5 \text{ hours} \ $400 &amp; \text{otherwise} \end{cases}$</td>
</tr>
</tbody>
</table>

Table 5 summarizes the unit commitment solutions obtained using the different algorithms. The optimal schedule produced by the Monte Carlo simulation (200,000 replicates used in estimating the probability distributions) is to turn the generating unit off during the first four hours. Then, the unit is turned back on for the next nineteen hours. The Monte Carlo procedure estimates that the execution of this schedule will generate an expected profit of $37,509. The normal and Edgeworth approximation methods provide this schedule as well. However, they estimate expected profits of $37,483 and $37,351, respectively.

Table 5: Unit Commitment Results.

<table>
<thead>
<tr>
<th>Solution Method</th>
<th>Schedule</th>
<th>Expected Profit ($)</th>
<th>CPU Time' (sec.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monte Carlo Simulation</td>
<td>111111111111111200</td>
<td>37509</td>
<td>571</td>
</tr>
<tr>
<td>Normal Approximation</td>
<td>111111111111111200</td>
<td>37483</td>
<td>137</td>
</tr>
<tr>
<td>Edgeworth Approximation</td>
<td>111111111111111200</td>
<td>37351</td>
<td>329</td>
</tr>
</tbody>
</table>

'CPU time is based on a 166 MHz Pentium Processor.

5. Conclusion:

In this paper, we have proposed a new formulation of the unit commitment problem that is valid under deregulation. We have shown that when a competitive market and price taking are assumed the unit commitment problem can be solved separately by each individual generating unit.

The solution method for the new formulation requires the computation of the probability distribution of the spot market price of electricity. The power generation system of the marketplace has been modeled using a variation of the Ryan-Mazumdar model. This model takes into account the uncertainty on the load and the generating unit availabilities.

The probability distribution of the spot market price, which is determined by the market-clearing price, is based on the probability distribution of the marginal unit.

The exact computation of the probability distribution is prohibitive for large systems. Three approximation methods were evaluated. From the computational experience, it appears that the proposed unit commitment can be accurately solved in a reasonable time by using the normal, Edgeworth, or Monte Carlo approximations.

REFERENCES


