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Ultrasonic Attenuation Due To Domain Walls Of Superconductors

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Abstract: Anisotropic superconductors with multicomponent order parameter can bear domains of different discretely degenerate superconducting states. These domains are separated by walls, regions where the order parameter is converted from one to another stable state.

Key wrods:

INTRODUCTION

Due to discrete degeneracy of the superconducting states, the extension of these domain walls cannot be large because the order parameter is strongly pinned to its stable bulk states due to its anisotropy energy (described by the fourth-order terms in the Ginzburg landau free energy). Therefore the order parameter has to overcome a certain energy barrier to pass from one state to the other. The competition between this energy barrier and the rigidity of the order parameter determines a finite width in space for this conversion, so that a well localized domain wall is created. The existence of such domain wall has led to the proposal that they could give a contribution to the unconventionally large, ultrasound absorption (Joynt R, Rice, T. M and Ueda, K 1986). We shall base our explanation of this mechanism on the work done by Truell and Elbaum, who considered allenuation of sound due to planar defects in solids Truell, R and Elbaum. C (1962).

A domain wall lowers locally the condensation energy of the superconducting state, i.e. a certain energy expense $\boldsymbol{E}_o$ per unit area has to be invested for their creation. Therefore, to keep the total energy small, they may be fixed at pinning centres where the superconducting condensation energy is lowered anyway, e.g. due to the presence of impurity atoms or crystal lattice defects. Fixed by these pinning centres, the domain wall behaves like an elastic membrane trying to be as flat as possible (equilibrium position). Therefore, for reference example, we consider a domain wall pinned at the corners of a square with side length L. For small deviation ξ from its equilibrium position in the plane we find an energy increase

$$\frac{\mathcal{E}_0}{2}[\nabla_{x,y}\xi(x,y)]^2 ds \tag{1}$$

with $\nabla_{x,y}$ as the gradient in the x, y plane. The integral goes over the square.

We now consider the coupling mechanism between the superconducting domain wall and the sound waves. We assume that the wavelength of the sound is much larger that the average size a of the domains. Therefore the strain $\boldsymbol{E}_{ij}$ induced by the sound is practically homogeneous over the range of several domains, With this assumption we can write the energy density due to lattice deformation as

$$f = \frac{1}{2}\sum_{\gamma,m} B(\gamma)\mathcal{E}^2(\gamma,m) + \frac{1}{aL^2}\sum_{\gamma,m}\Delta V(\gamma,m,\lambda)\,\mathcal{E}(\gamma,m)\int \xi ds + \frac{\mathcal{E}_0}{2aL^2}\int [\nabla_{x,y}\xi(x,y)]^2 ds \tag{2}$$

The symbols $\boldsymbol{E}$ (γ, m), B(γ), C(γ) and V(γ, m, λ) denotes a combination of $\boldsymbol{E}$, B, C and v-tenser elements and has the symmetry property of the basis function m in the representation γ of the original point group.

Theoretical Considerations and Calculations:

The second term describes the coupling. Since the strain $\boldsymbol{E}$ can couple differently to the superconducting phase in different domains, it induces an energy density difference between the domains i and j given by

$$\sum_{\gamma,m}\Delta V(\gamma,m,\lambda)_{ij}\,\mathcal{E}(\gamma,m) \text{ with}$$

$$\Delta V(\gamma,m:\lambda) = c(\gamma)[V(\gamma,m:\lambda)|_i - V(\gamma,m:\lambda)|_j] \tag{3}$$

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The bilinear form V(γ, m: λ)$_i$ is evaluated in the superconducting phase $_i$. Note that $\Delta V(\Gamma_1^{\pm})$ is always zero. The coupling term between domain wall and strain is determined by the change of energy if the domain wall is deformed.

The sound attenuation due to the domain walls can be calculated starting with the equation of motion in terms of strain $\mathcal{E}_{ij}$ and stress σ_{ij}.

$$\frac{\delta^2\sigma_{ij}}{\delta x^2_i} = \rho \frac{\delta^2}{\delta t^2} \mathcal{E}_{ij} \text{ and } \sigma_{ij} = \frac{\delta f}{\delta t_{ij}} \tag{4}$$

where ρ is the mass density. For simplicity we consider a longitudinal sound wave along z axis, $\sigma = \sigma_{zz}$ and $\mathcal{E} = \mathcal{E}_{zz}$. The relation between $\mathcal{E}_{zz}$ and σ_{zz} is given by

$$\mathcal{E} = \frac{\sigma}{B} - \frac{1}{BaL^2}\sum_{\gamma, m} \Delta V(\gamma, m, \lambda)_{ij}\, b(\gamma, m)\int \xi ds \tag{5}$$

where $b(\gamma, m) = \frac{\mathcal{E}(\gamma, m)}{\mathcal{E}_{zz}}$ is a constant. Note that the second term is small compared to $\frac{\sigma}{B}$. We now need an equation of motion for ξ. To consider the return of the deviated wall to its equilibrium position we neglect the kinetic energy of the wall, assuming that the movement is characterised by over damping (relaxational motion). Thus the equation of ξ has the form

$$\Upsilon \frac{\delta}{\delta t} \xi(x, y, t) = -\frac{\delta f}{\delta \xi} \tag{6}$$

Finally, we obtain the following systems of coupled equations

$$\delta_z^2\sigma - \frac{1}{v_o^2}\frac{\delta^2}{\delta t^2}\sigma = -\frac{1}{av_o^2}\frac{\delta^2}{\delta t^2}\left[\int \xi ds\right] x \sum_{\gamma, m} \Delta V(\gamma, m, \lambda)_{ij}\, b(\gamma, m) \tag{7}$$

$$E_o \nabla^2_{x, y}\xi - \Upsilon \frac{\delta \xi}{\delta t} = \frac{\sigma}{B}\sum \Delta V(\gamma, m, \lambda)_{ij}\, b(\gamma, m) \tag{8}$$

where $v_o = \sqrt{B/\rho}$ is the sound velocity. The deviation ξ dues not directly depend on z. To solve this system we introduce the ansatz

$$\sigma(z, t) = \sigma_o e^{az} \exp iw[t - {}^{z}/_{v}] \tag{9}$$

Here a is the absorption coefficient and v the effective sound velocity. Using only the lowest frequency mode for the deviation ξ_1 we find

$$a(\omega) = \frac{4\omega_1}{\pi^2 a \Upsilon \rho \upsilon_o^3}\frac{\omega^2}{\omega^2 + \omega_1^2}\left[\sum_{\gamma, m} \Delta V(\gamma, m, \lambda)_{ij}\, b(\gamma, m)\right]^2 \tag{10}$$

$$\upsilon(\omega) = \upsilon_o\left[1 - \frac{4\omega_1}{\pi^2 a \Upsilon \rho \upsilon_o^3}\frac{\omega^2}{\omega^2 + \omega_1^2}\left[\sum_{\gamma, m} \Delta V(\gamma, m, \lambda)_{ij}\, b(\gamma, m)\right]^2\right] \tag{11}$$

with the lowest mode frequency $\omega_1 = \pi^2 \boldsymbol{E}_o / L^2$ for the square. Ultrasonic attenuation occurs if the energy density difference $[\Delta V(\gamma, m, \lambda)_{ij}\, b(\gamma, m)]$ induced by the sound wave is finite. At the same time the sound velocity is renormalized $\upsilon < \upsilon_o$.

As an example we consider the superconducting phase in the cubic representation Γ_5^+ with the three degenerate states $[D_{4h}(\Gamma_5^+)]$,

$$\psi_1(k) = \lambda k_y k_z,\ \psi_2(k) = \lambda k_z k_x, \text{ and } \psi_4(k) = \lambda k_x k_y, \tag{12}$$

for longitudinal sound waves along the z axis, only the strain component E_{zz} and thus $b(\Gamma_1^+) = 1/\sqrt{3}$ and $b(\Gamma_3^+, 1) = 2/\sqrt{6}$ are finite with the bilinear forms. we obtain

$$\Delta V(\Gamma_3^+, 1) = \begin{cases} 3c(\Gamma_3^+)|\lambda|^2 & i = 3 \text{ and } j = 1,2 \\ 0 & i = 1 \text{ and } j = 2 \end{cases} \quad (13)$$

Therefore the absorption coefficient a has the form

$$a(\omega) = \frac{4\omega_1}{\pi^2 a \Upsilon \rho \upsilon_o^3} \frac{\omega^2}{\omega^2 + \omega_1^2} [3c(\Gamma_3^+)|\lambda|^2]^2, \quad (14)$$

and a similar expression can be derived for the sound velocity.

RESULTS AND DISCUSSION

To show the temperature dependence of a, we assume that the only T – dependent quantities are the order parameter ($|\lambda| \propto |T - T_c|^{1/2}$) and the domain wall energy ($E_o \propto |T - T_c|$)

$$a(\omega, T) \propto \frac{\omega^2}{\omega^2 + \bar{\omega}_1^2 |T - T_c|^3} |T - T_c|^2,$$

where ϖ_1 is the constant in the lowest mode frequency ϖ_1 after extraction of the temperature dependence. From this equation we see that a has a peak immediately below the transition and decreases like $|T - T_c|^{1/2}$ for lower temperatures.

Conclusion:

The behaviour is modified if other quantities such as Υ are temperature dependent. The frequency dependence of a for a fixed temperature is quadratic in the low – frequency region ($\bar{\omega}|T - T_c|$ >> ω) and becomes constant in the high frequency limit.

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