Magnetization at the Surface of Unconventional Superconductors

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Abstract: The finite ferromagnetic moments do not occur macroscopically in the bulk region of a superconductor due to complete screening by diamagnetic currents. However, in homogeneities of the superconducting state can produce a local occurrence of magnetic field, especially if the screening length is large compared to the characteristic length scale of the inhomogeneity (mostly the coherence length). Then the screening of the partially modulated magnetic moment cannot be perfect.

Key words:

INTRODUCTION

The surface of a superconductor has properties similar in many respects to the domain wall (Sigrist, M., 1989) The order parameter also varies spatially towards the surface in a range $\xi$. We consider the example of a surface perpendicular to the x axis demonstrated there ($x > 0$: superconductor and $x < 0$: vacuum). The component $|u|$ is assumed to be stressed to zero as

$$|u(x)| = \tanh(\sqrt{2}x / \xi) \text{and } (\chi - \phi = \mp \Pi / \xi) = \text{constant}$$

with $\xi^2 = 16k_x / [(1 + 4k_x^2) - \beta^2]$, where as $|u|$ is assumed to remain (-1). The boundary condition prohibits a current perpendicular to the wall. $J_x = 0$. Thus the London equation for $a_x$ has the form:

$$\frac{\delta^2}{\delta x^2} a_x - \frac{1}{k_x^2} (k_x |u|^2 + k_y |v|^2) a_x - \frac{k_x + \Delta k}{k_x^2} |u| \frac{\delta}{\delta x} |u| \sin(x - \phi) = 0$$

Substituting equation (1) in equation (2), we see that the last term denotes a super current flowing parallel to the surface:

$$j_y = -(k_x + \Delta k) \sqrt{2} \sin(\chi - \phi) / k_x \cosh(\sqrt{2} x / \xi)$$

similar to the domain wall. The induced magnetic field can be obtained by replacing $j_y$ by

$$-(k_x + \Delta k) \sqrt{2} \sin(\chi - \phi) \exp(-2\sqrt{2} / x / \xi \kappa^2)$$

Then the magnetic field is

$$b_y(x) = (k_x + \Delta k) 8 / \kappa^2 - \xi^2$$

$$x(e^{-\phi} - e^{-\phi}\xi) \sin(\chi - \phi)$$

where we take the external magnetic field to be zero. The maximal magnitude in real units is about the same as in the case of the domain wall. Because the sign of the magnetic field depends on the relative phase ($\chi - \phi$), it is different in the two domains ($u, v = \pm 1$). Contrary to the case of the domain wall, there exists a finite net magnetization at surface:

$$|\gamma| = 4\sqrt{2}(k_x + \Delta k) H_x / (2\sqrt{2}k_x + \xi)$$

per unit surface areas. This has the consequence that a single domain superconductor has a finite, macroscopically observable magnetic moment due to the time – reversal breaking state.

Even if the currents found at the surface and the domain wall seem to be rather unconventional, their origins are in some ways analogous to those of super currents in conventional superconductors. They are the response to a spatial variation of the phase of the order parameter. In a multicomponent order parameter $|\Delta \Sigma n|/\exp(i\theta) \Delta n|$, the behaviour of this phase is determined by several variables, first of all by the phase of
each component. However, the total phase is also clearly changed if the magnitude of the moduli $|n_j|$ of each component varies in a different way and the value of the relative phase among them is somewhere between $0$ and $\pm \pi$ (note that the latter is the condition for a time reversal breaking state). The multicomponent structure of the order parameter yields a tensorial behaviour, so that currents can also be induced by gradients of the phase variation perpendicular to its direction.

**Theoretical Consideration and Calculations:**

In the weak coupling limit the gradient terms of a Ginzburg–Landau theory of a multicomponent order parameter can be written in the form

$$
F_c = \frac{\epsilon}{4} \sum_{j,a,m,n} \left[ d^3 R K_j (R) R_R R_R \right] \\
x \left[ \delta_n - \frac{2i\epsilon}{c} A_n (r) \right] n_j (r) \\
x \left[ \delta_n - \frac{2i\epsilon}{c} A_n (r) \right] n_j (r)
$$

which can be derived from the equation below

$$
n_j (r) = \sum \int d^3 r k_j (r, r) n_j (r)
$$

by assuming a slow variation of the order parameter $\xi (T) \gg \xi_0$ and gauge invariance (see for example (Abrikosov, A.A., 1963). The kernel has the structure

$$
K_n (R) = f l [\rho, [\lambda^*_n (R) \lambda_n (R)]
$$

as given in the equation below

$$
K_n (R, r^\prime) = \frac{V_k r}{2} \sum_e \int d\xi \left[ \frac{W_e}{i W_e - \epsilon} \sum \rho \right] \\
\left[ \rho \Delta^*_n \left( \frac{m}{2k_e} j (r) \right) \frac{m}{2k_e} j (r, r^\prime) \right] \delta (\epsilon - \xi)
$$

The constant $C$ is chosen to give the correct units. From this equation the supercurrent can be calculated by the derivative with respect to the vector potential $\langle j = -cF_c \rangle$.

$$
J (r) = Ce \left[ m \int d^3 R f (R) \frac{R r}{R} \Delta \left( \frac{2i\epsilon}{c} \right) \frac{\Delta (R, R) [\nabla, - \frac{2i\epsilon}{c} \Delta (R, R)] R} \right]
$$

We now introduce $\Delta (r, R) = \sum_j n_j (r) \Delta_j (R)$

For our discussion of the current induced by the phase variation, we neglect the diamagnetic currents, the part that depends on the vector potential.

Let us consider the case of a planar surface. It is convenient to characterize spatial variation of the order parameter at the surface by separating it into components, which are classified by the parity $p(n)$ (reflection at the boundary which is represented by the normal vector $n$). This assumption allows us to write a simple expression for the final result without losing the general content. Therefore we decompose the order parameter, $\Delta (r, R) = \sum_j n_j (r) \Delta_j (R)$

$$\Delta (r, R) = \Delta_e (r) + \Delta_o (r, R) = \Delta_e (R) + \sum \Delta_e (\rho) (R) n_j (r)
$$

Where the indices $e$ and $o$ denote “even” and “odd” respectively, if we choose the basis gap function to be orthogonalized (also with $\int d^3 R (n R^\prime) [\Delta^*_n (R) \Delta (R)] \frac{\Delta \left( \frac{2i\epsilon}{c} \right) \frac{\Delta (R, R) [\nabla, - \frac{2i\epsilon}{c} \Delta (R, R)] R} \right]$, it is easy to see that there is no current perpendicular to the surface $n$. On the other hand, we find for the currents parallel to the surface
By making use of equation (9) and by partial integration we obtain

\[
J_o = -\frac{nx\text{e}}{2} \int d^3R \int [n(n,R) - \Delta_x(R)\delta_o\Delta_o(n,n,R) + (n,n,R)]
\]

This can be reduced to the form

\[
J_o = -\nabla_x Ce \int d^3R [\rho(n)/\Delta + (n,n,R)](RX\nabla \Delta o(n,n,R))
\]

(12)

Using the fact that the operation \( nx(Rn) \) changes the \( \rho(n) \) parity of a function \( \Delta e, O(R) \). The expression \( \rho(n)\Delta + (n,n,R) \) is defined as \( \Delta + (n,n,R - 2n)(n,n,R) \). From this final equation we can easily obtain the magnetic field using the Maxwell equation

\[
\nabla \times J = \frac{\partial B}{\partial t}
\]

Since the diamagnetic current was excluded from our discussion, in this expression the magnetic field is finite and constant in the bulk region. Therefore, inserting the bulk state gap function into the formula for the magnetic field, we obtain a criterion for the existence magnitude, and direction of a local magnetization at the surface with the normal vector \( n \), clearly, the result is that a time reversal invariant state \( 4* (R) = 4(R) \) or \( d*(R) = d(R) \) produces no such magnetization, since for these states the expression \( Im[\Delta (R)] \) vanishes.

Applying this formula to our example \( /5 \text{ of } D_{4h} \) for both even and odd parity states, we find a finite magnetization \( \mathbf{\nabla} \) axis, with the same sign as for any normal vector in the x – y plane for a fixed bulk state. However, for \( n \) parallel to the Z- axis, all currents are vanishing. Similarly, the time reversal breaking state \( O_h \) \( /7 \) generates a current for any direction of the surface except for a parallel to the main axis (Bares, P., 1988). In this case, however, the magnetization has directions for differently oriented surfaces. Consequently, this phase need not yield a finite magnetization for a single domain sample in contrast to the former example.

Since the behaviour of the surface and domains wall are analogous these results can also be transferred to the domain wall with the only difference that the sign of the current and the field can depend on the choice of some parameters (in our example on the ratio \( k_1 \) / \( k_2 \)). More precisely, the current direction depends on whether the even or the odd parity \( \rho(n) \) components of the order parameter are varying more strongly in the domain wall.

The weak coupling approach in our example requires \( k_3 \) and \( k_4 \) to be equal, leading to \( \Delta k = 0 \). Beyond this limit these two coefficients are in general different. It can be shown that their difference, due to particle hole asymmetry is similar by a factor of the order of \( (T_C/T_F) \) then the weak coupling coefficients (in conventional superconductors \( T_C/T_F \) is very small, or \( 10^4 \), but for heavy fermion superconductors it is considerably larger 0.1 (Serene, J.W. and D. Rainer, 1983). Therefore this term has to be discussed separately since it is not included in equation (4)

\[
\Delta k((d_u u^*)(d_j v^*) - (d_j u^*) (d_u v^*) + c. c), \Delta k = \frac{1}{2} (k_3 - k_4)
\]

(13)

Even if it looks rather similar to the \( k \) term in the equation below

\[
F = \frac{1}{4g^2} H^2 \frac{1}{2} \int \frac{d^3r}{d^3r} \left[ 1/2(|u|^2 + |v|^2) + \frac{1}{2} \beta_2 (u^* v - u v^*)^2 - \beta_3/8(|u|^2 - |v|^2)^2 + k_1 (|d_u u|^2 + |d_v v|^2 + k_2 (|d_u u|^2 + |d_v v|^2) + [\Delta k((d_u u^*)(d_j v^*) - (d_j u^*) (d_u v^*) + c. c) + \Delta k((d_u u^*)(d_j v^*) - (d_j u^*) (d_u v^*) + c. c) + k_5 (|d_u u|^2 + |d_v v|^2) + k^2 b^2]
\]

(14)

It has essentially different properties in generating spontaneous currents and magnetic fields. It does not contribute at all to the diamagnetic currents for the meissner effect. In a time reversal breaking superconducting state, any inhomogeneity of the order parameter even without spatial variation of the phase of the order parameter – all \( |n| \) have the same spatial dependence and the phases \( \phi_j \) are constant leads to a current and a local magnetization.

**RESULTS AND DISCUSSION**

To understand the origin of this gradient term we consider a planar inhomogeneity. We now take the apposite approach to that used earlier. Starting with the induced magnetic field, we show that it leads to that kind of gradient term. Neglecting all screening effects, we write for the magnetic field.
where $g(|R|)$ is a function that includes strong coupling effects. Obviously this expression is related to an intrinsic magnetic moment of the superconducting phase deserved by the operator

$$m(R) = \frac{1}{i} (RX \nabla_x) + \sigma,$$

which is composed of the relative angular momentum and the spin polarization of the Cooper pairs (Leggett, A.J., 1975). In this formulation we neglect corrections due to spin orbit coupling. Considering this magnetic moment, (Volovik, G.E., 1989) classified the time reversal breaking states to be ferro or antiferromagnetic by the following natural definition

$$\langle \text{tr}[\hat{\Sigma}(R) \hat{m}(R) \hat{\Lambda}(R)] \rangle \begin{cases} \text{finite, ferromagnetic} \\
\text{finite, antiferromagnetic} \end{cases}$$

Where $\langle \rangle$ denotes the average over the direction of $R$. Obviously the magnetic field in equation (14) is only finite for ferromagnetic states. The total list of the classified ferromagnetic states is $D_{3d}(\frac{3}{4}), D_{4h}(\frac{5}{4} \pm), D_{6h}(\frac{5}{6} \pm), D_{4h}(\frac{5}{6} \pm),$ the only antiferromagnetic state is $O_h(\frac{3}{4} \pm)$. The finite ferromagnetic moments do not occur macroscopically in the bulk region of a superconductor due to complete screening by diamagnetic currents. However, inhomogeneities of the superconducting state can produce a local occurrence of magnetic field especially if the screening length is large compared to the characteristic length scale of the inhomogeneity (mostly the coherence length). Then the screening of the spatially modulated magnetic moment cannot be perfect.

Clearly, we can include the magnetic moment of the superconducting state in the Ginzburg – Landau theory by a Zeeman term,

$$F_z = -4\pi e^2 m(\nabla_x x A(r) [g(|R|) x \text{tr}[\hat{\Delta}^+ (r, R) \hat{m}(R) \hat{\Lambda}(r, R)] = -C[\nabla_x x A(r) \langle m(r) \rangle]$$

which is finite only for a ferromagnetic superconducting phase? This term does not produce additional boundary conditions, since the magnetic moment turns parallel to the normal vector $n$ at the surface.

$$\int [A(r) x \langle m(r) \rangle] \text{ind}^2 S_n = 0$$

Therefore equation (17) is equivalent to

$$F_z = -C[\nabla_x x A(r)]$$

which leads straight forward to equation (14) for the magnetic field. However, this term can also be expressed by

$$F = -4\pi e^2 m(\nabla_x x A(r) [g(|R|) \text{tr}((\hat{\Delta}^+ (r, R)] \hat{m}(R) \chi[\hat{\Delta}^+ (r, R)]],$$

which corresponds to a gradient term of the equation Ginzburg Landau free energy $D = \Delta - 2ieA/c$.

**Conclusion:**

The Zeeman term can be transformed to an equivalent gradient term. It is clear that this type of gradient term exists only for "ferromagnetic" states. For antiferromagnetic states, all magnetic properties are due only to the tensor character of the gradient terms and the corresponding currents.

A very simple criterion for the possible existence of a ferromagnetic state in an order parameter representation $|\rangle$ can be derived by considering equation (17). Since $\nabla x A$ transforms according to the vector representation $D_{G}$, and the magnetic moment is a bilinear form of the order parameter, the existence of such a term requires that the decomposition of $D_{G} \otimes \Phi$ contain the representation $|1^+\rangle$.
REFERENCES


