Multi-objectives portfolio optimization: challenges and opportunities for Islamic Approach

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Abstract: In financial activity, a portfolio is a set of assets that can make one compound from several positions to investing. The main problem for this area is found out the optimal method to distribute a given founds on a set of existing assets. Maximization of expected return and minimize of risk are two main aims of this problem. This paper reviews Islamic approaches to multi-objective portfolio optimization problems. From this viewpoint, the present paper reviews several studies proposed for multi-objective optimization, and some research that they focus on Islamic approach. It is seen that most of recently studies have tried to improve the models of the risk of Islamic investing models. The main aims of them are restructured and making the new financial instruments that they are adaptive via Shariah as the supervision. Furthermore, it has been given two briefly review tables for recently investigates on multi-objective portfolio optimization in cases, conventional portfolio and Islamic approach. These tables led to find the best method to find a good strategy to research in this area. Next, they are presented some challenges and opportunities in this area based on literature and this investigate. Challenges and opportunities give a good viewpoint in this area. The scope of this research is the theoretical study on Islamic finance. Finally, results and some future works are presented as the conclusion.

Key words: Multi-objective, Portfolio optimization, Risk, Shariah-compliant

INTRODUCTION

In financial activity, a portfolio is a set of assets that can make one compound from several positions to investing. The main problem for this area is found out the optimal method to distribute a given founds on a set of existing assets. Maximization of expected return and minimize of risk are two main aims of this problem. The user’s risk aversion has a direct effect on optimal solution. Two criteria are necessary to optimization of the portfolio. First, the set of solution to the portfolio optimization problem that it called “efficient frontier”, or “Pareto-optimal front”. Second one is the measure against the risk of the portfolio. It is normal a financial institute wanted to present its customers different position to choose related to their risk aversion (Ayadi and Hyman, 2006; Markowitz, 2012).

The growing capital value of the Muslim countries and they require of these investors to invest their capital in financial products that do not divergence with the Shariah cause to the development of Shariah-compliant investment products such as Islamic equity funds (Elahi and Abd Aziz, 2011).

The rest of this paper is arranged as the following: first, we give an overview on multi-objective portfolio optimization. Then we describe multi-objective portfolio optimization via Islamic approach. We then present some challenges and opportunities in this area. Finally, we conclude and give future work for related researchers.

Multi-Objective Portfolio Optimization:

We face many decisions making in the real world. There are numerous methods to optimization of them. The aim of solving is finding the Pareto optimal solutions. However, in the theoretical problem cases, if Pareto set normally cannot be solved by algorithm, then we try to approximate of Pareto sets (Radziukynienė and Žilinskas 2008). Chiranjeevi and Sastry (2007) showed classical Markowitz model by figure 1.

For example, the Multi-objective model for five objective function \( f_1, \ldots, f_5 \) and some constraints indicate can show by following model (Chiranjeevi and Sastry, 2007):

\[
\begin{align*}
\text{Max} & \quad [f_1(x), \ldots, f_5(x)]^T \\
\text{s.t.} & \quad \sum_{i=1}^{M} x_i = 1 \\
& \quad x_i \geq 0, \quad \text{For} \quad i = 1, \ldots, M
\end{align*}
\]

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In the table 1 we review the Multi-objectives portfolio optimization in recently studies and denote Highlight (or main idea). In following table we show Highlight (or main idea) and research gap in each investigation by H, G respectively.

Table 1: Briefly review of Multi-objectives portfolio optimization.

| Portfolio revision under mean-variance and mean-CVaR with (Chen et al., 2012) | H: mean-conditional value-at-risk strategies | G: mean-variance-VaR | H: algorithm for the bi-objective model | ant colony optimization algorithm for the bi-objective (Obaori et al., 2010) |
| Ant colony optimization approach to portfolio optimization (Hiaoi, 2012) | H: Ant colony Portfolio | G: mean-variance-VaR | H: Max-Min Ant system | mean-variance |
| The mean-variance cardinality constrained portfolio optimiz... (Anagnostopoulos, 2011) | H: mean-variance portfolio | G: Ant colony | | |
| Ant colony of Mean-Variance Portfolio (Deng et al., 2010) | H: Ant Colony Optimization | G: mean-variance-VaR | H: envelope-based multi-objective algorithm | mean-variance-VaR |
| | | | H: two and three criteria portfolio optimization | G: ACO model |
| | | | | Methods for Multi-Objective Portfolio Optimization (Raczyniak, 2008) |

We see that most of recently studies try to improve the models of valuate the mean-variance and mean-CVaR risk of multi-objectives portfolio optimization that is useful on conventional financial activities. (Branke, Scheckenbach, Stein, Deb and Schmeck, 2009)

Recently some researchers tried to find the heuristic method to portfolio optimization. For this idea they used some new methods (e.g. ACO or Max-Min ACO system via Saeheaw et al. (2009)) to optimize the portfolio (Anagnostopoulos and Mamanis, 2011). Finally, the main aim of them is restructured and hybridization some methods to find the new strategy and algorithm to adaptive the currently optimization models.

Multi-Objective Portfolio Optimization Via Islamic Approach:

The main result of last crisis in the world is the Islamic finance had been minimum damage and it seems that necessary to restructure the Islamic financial instruments for using in conventional finance (Zaher and Kabir Hassan, 2001). Furthermore, the growing capital value of the Muslim countries and they require of these investors to invest their capital in financial products that do not divergence with the Shariah.
development of Shariah-compliant investment products such as Islamic equity funds. Furthermore, the existing Islamic portfolio mathematical models (e.g. see (Derigs and Marzban, 2008)) are insufficient to take all the relevant variables in their design. Also, all variables are most being compliant by shariah. Since Shariah prohibits the participation in interest-based assets (Mosler and Scarsini, 1991), specific guidelines need to be introducing as more constraints into models for constructing Shariah-compliant portfolios (Whitmore and Findlay, 1978).

Common rules for Islamically allowable forms of financial activity can funded in The Quran, Islam’s Holy Book, and the hadith (Khan, 2010). The main characteristic of Islamic economic behavior is Increase risk sharing between the procurer of funds (investor) and both the financial agent and the user of fund (Bac, 2010).

In the figure 2 we Review the Islamic portfolio optimization in recently studies and denote Highlight (or main idea). In following table we show Highlight (or main idea) and research gap in each investigation by H, G respectively.

![Review of Islamic Portfolio](image)

**Fig. 2:** Briefly review of Islamic portfolio optimization.

We see that most of recently studies try to improve the models of valuate the risk of Islamic financial activities. The main aim of them is restructures and making the new financial instruments that they are adaptive via Shariah as the supervision (Ebrahim, 2009; Jobst, 2007). In other studies, some researchers tried to define of the islamic financial instrument models via improvement of conventional financial tools based on monitoring by Shariah (El-Hawary, Grais and Iqbal, 2007).

Now, we review more details about that how Shariah-compliance can be modeled within the framework of portfolio optimization. We let the standard condition, i.e. an asset space

\[ I = \{1, \ldots, n\} \]

from which a portfolio can be produced, and we imagine that the total wealth has to be invested and short-selling is not permissible, which is practical since short-selling is usually not allowed under Shariah. Then every portfolio can be exposed by a share vector

\[ x = (x_1, \ldots, x_n) \] where \( x_i \in [0,1] \) represents the weight or fractional capital invested in each asset, \( i \in I \) and \( \sum_{i=1}^{n} x_i = 1 \) it hold (Whitmore and Findlay, 1978).

For the following we as well suppose an objective function \( f(x) \) by which the efficiency of a portfolio is measured and a set \( C \) of constraints stemming from investment guidelines other than Shariah guidelines. The objective function is cogitative the investment strategy. Moreover, the return/risk transaction and could stem, for example, from the mean-variance model (Markowitz 2012) or index tracking senses (Corielli and Marcellino, 2006). Furthermore, we supposed to have the following conventional portfolio optimization model that before or lacking about Shariah-compliance:
Min \( f(x) \) Subject to \( x \) comply with constraints in \( C \)
\[
\sum_{i=1}^{n} x_i = 1 \\
x_i \geq 0 \quad \forall i \in I
\] (3)

Sector rules have to be satisfied on the single assets or the total portfolio, whereas compliance is measured as an element of them. We suppose an economic guideline \( g \in G \) we have to compute a financial ratio \( r_i(g) \) for each asset, \( i \in I \) which measures the level of participation in a non-compliant financial activity, and to evaluate the value with a maximum permissible value \( T(g) \), the so called threshold value. Thus, we have to organize a set of limitation of the following type:

\[
r_i(g) \leq T(g)
\] (4)

Clearly, these guideline’s consequences in an additional decrease of the asset universe and thus their achievement can be safe in a preprocessing phase analogously to the sector rules. Therefore, Shariah-fulfillment of a portfolio can be operationally generated by a preprocessing stage in which the asset universe for a conventional portfolio optimization model is specified. However, the useless way to model compliance with relate to a guideline is to formalize conditions (4) as set of mathematical inequalities, which are introduced as constraints into the portfolio model. Conceptually, a financial principle can be modeled by a set of logical limitations of the following type:

\[
x_i = 0 \quad \text{if} \quad r_i(g) > T(g) \quad \forall i \in I
\] (5)

Unequal of (5) can be turned into a set of mathematical inequality like follows: We describe a binary variable \( z_i \) for each asset, \( i \in I \) has been following form:

\[
z_i = \begin{cases} 
1 & \text{if} \quad i \text{ is complaint} \\
0 & \text{otherwise}
\end{cases}
\] (6)

and we use the constraints:

\[
x_i \leq z_i \quad \forall i \in I
\] (7)

\[
r_i(g) \cdot z_i \leq T(g) \quad \forall i \in I
\] (8)

Constraints (7) ensure that for every asset \( i \in I \)

\[
x_i > 0 \quad \text{only if} \quad z_i = 1
\] (9)

According to constraints (8) guarantee, that \( z_i \) is 0 if the guideline (6) is not fulfilled (Derigs and Marzban, 2009; Elahi and Abd Aziz, 2011).

**Challenges and Opportunities:**

The main problem of the classical problem (in theoretical and computational finance) is optimizing a portfolio of finitely numerous assets. As the determining work of Markowitz (1952, 1959, 1987) it is commonly agreed that portfolio performance must be measured in two distinct dimensions: the expected return rate with mean, and the risk for measured the uncertainty of the return rate. Another theoretic approach is stochastic dominance (Mosler and Scarsini, 1991). Current portfolio studies are in progress from original research work by Markowitz (Markowitz 2012). Markowitz was presented mean–variance model, which is the first portfolio selection model (Mansini and Speranza, 1999).

The portfolio selection problem refers to form a good portfolio. It is complicated to choose which assets should be selected because of the doubt on their returns. The central purpose in a portfolio selection Problem is to find optimal proportions of the stock for creating a portfolio which complements the investor’s preferences presumptuous that the investors’ wish to strike a balance between maximizing the return and minimizing the risk of their investment (Gupta, Mehlawat and Saxena, 2008).

Numerous types of risk measures for portfolio optimization have been introduced in the previous studies. They find many results from these proposed risk measures that have several applications in practice. Two types of risk measures are more popular to using for portfolio optimization. They are value at risk (VaR) and conditional value-at-risk (CVaR). VaR and CVaR are more useful than variance and allowable to use for asymmetric distributions (Chen, Fabozzi and Huang, 2008).
Aboulaich et al. (2010) introduced one multi-objective system for portfolio optimization that consists of three objective functions. The first objective function is to minimizing the conditional value at risk for portfolio optimization. The second objective function denotes minimizing the excepted value of return of portfolio. And the last objective function lets the minimizing the variance of returns. In that system Constraints show that we can manage our capital to invest on several assets.

On the other hand, one of the main practical methods of modeling the risk for portfolio optimization is mean-risk. The expected value and the value of a risk measure as two statistical quantities are computable via mean-risk models. Furthermore it is useful for decision making in finance (Aboulaich, Ellaia and El Moumen, 2010).

To sum up, several approaches have been used to address the Islamic Portfolio. This study reviews some of the studies, as well as conceptual information from the shariah, in an attempt to describe the structure of Islamic Portfolio. Most of this work will carried out in the conventional finance. Some of the results are however also of relevance to Islamic finance. the essential condition of such rules by the Shariah scholars the accessibility of modeling tools for setting up the Shariah-compliant portfolio optimization model is a main requirement, Although, not obligatory for trend the new paradigm in practice (Whitmore and Findlay, 1978).

**Conclusion and Future Work:**
This review showed that it is necessary to further explore and extend the research in Multi-Objective Portfolio Optimization model and its application in emerging market cases (especially Islamic portfolio optimization). The design of this investigate will present in this chapter. Specifically, improve the theoretical and practical models for Multi-Objective Portfolio Optimization, introduce the analytical method for measure the several risk of Multi-Objective Portfolio Optimization model via that suggested in this research are discussed.

Also, this model decreases complexity of computation and could be a fine replacement for Shariah-compliant portfolio optimization model, that present by Derigs and Marzban (2008). With sustaining this system, this model will develop in the future. Such as, Shariah-compliant Fuzzy risk and return optimization model and Shariah-compliant portfolio optimization model via mean-variance risk model.

On the other hand, this research reviews the traditional and new texts on Islamic finance referred to as Shariah-compliant Portfolio Optimization. Some researchers studied on the Model of Islamic Microfinance but there is a lack in developing a mathematical method. In addition, the new strategies to measure the risk of portfolio optimization model and application it algorithm on Islamic finance are not fully addressed in the literature. It can be the future work to illustrate application of the mathematical model to use in similar cases for measure the risk of Multi-objective Islamic portfolio problem with some related objective functions.

**ACKNOWLEDGMENT**

This publication is supported by Ministry of Higher Education and Research University Grant (RUG), Universiti Teknologi Malaysia.

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