Australian Journal of Basic and Applied Sciences, 6(9): 530-536, 2012
ISSN 1991-8178

Measurement of Labour Productivity Through Stochastic Process: A Study in Organised Manufacturing Sector of Odisha, India

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Abstract: Labour productivity is not of constant magnitude, but it evolves with time. The process grows in the beginning of economical work unit but it reaches at its limit with respect to time. Any economical unit provides growth of labour productivity and in future, it may be replaced by more perfect economical units to enhance the growth of labour productivity. The main component of the labour productivity is the improvement of the efficiency parameters that is engineering and technology. The growth of economy of a country or efficiency of production system generally depends on its level of education, science, technology and strategic management.This paper presents a new analytical approach with the help of Stochastic process for calculating labour productivity using the data of Economy survey of Odisha, 2010-11, published by the State Govt. of Odisha, India. The role of educated workers to accept the technological changes which leads to improvement of total factor productivity (TFP) is also considered in this paper with the help of advanced calculus.

Key words: labour productivity, educated workers, total factor productivity (TFP), stochastic processes and Markov chain.

INTRODUCTION

It is well known that economic growth, as a means to enhancing the welfare of people, depends on the use of both the factors of production such as capital and labour with the efficiency in resource use. It is often referred to as productivity. Recent developments indicate the growing importance of productivity, particularly for the Indian economy at its present stage of development. The involvement of technological progress to modern economic growth and rising per capita income in the highly developed countries is widely acclaimed and acknowledged. Long-run economic growth is sustained through continuous rise in productivity and required fundamental non-reversible structural changes. The production structure has grown mature and technologically sophisticated across and within sectors.

The growth of economy of a country or efficiency of production system generally depends on its level of education, science, technology and strategic management. A well educated, skilled and developed work force must be required for the growth of a country's production level which also includes production facilities. Education at all levels contributes to economic growth through imparting general attitudes, discipline and specific skills necessary for a variety of workplaces. It also contributes to economic growth by improving health, reducing fertility and possibly by contributing to political stability. The major importance of the educational system to any labour market would depend majorly in its ability to produce a literate, disciplined, flexible labour force via high quality education. Consequently, with economic development new technology is applied to production, which results in an increase in the demand for workers with skill and better education.

The manufacturing sector of Odisha which constitutes 62 percent of its total industrial sector, plays a greater role in generating employment and supporting the large scale industries as well as small and medium scale industries (SMI_S).The majority of the manufacturing enterprises from small scale to medium scale grouped together are called "Micro, small and medium enterprises"(MSME). It is encouraging to note that the number of small scale industries (SSI) and MSME units set up during a year has been increasing over the years but their contribution to value- added and value of fixed assets are far less than that of large scale industries. The main cause behind this may be a low level of labour productivity and poor input quality. So this paper tries to study the impact of technological progress and changes in the industrial structure on the productivity growth of

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the manufacturing sector of the state of Odisha, India and also predicts the future trend of Total factor productivity.

Review of Literature:

Dr. Shri Prakash and Ritu Sharma(2011)[4] in a study of "Impact of Technology on Production in Indian Economy" evaluated the impact of change in technology on output with relation to economic factors which influence final demand. They evaluated the growth/output effect of technological change by the changes in input coefficients matrix, an in Input-Output Model. The model is used for examining growth effect with the change in technology as reflected by change in matrix, **A**. The model is:

$$\mathbf{X_t= (I-A_t)^{-1}* f_t}$$

Where X_t = Gross output vector;
A_t = Technology matrix;
f_t= Final demand vector;
t= Time,
$(I-A)^{-1}$ = Leontief inverse, and A is the technology matrix. Finally they had found that output grows with the change in technology.

Manoj Kumar Dash, Gaurav Kabra and Ajay Singh (2010)[3]in a study on "Productivity Growth of Manufacturing Sector in India an Inter-State Analysis "estimated the total factor productivity growth of the manufacturing industries of different states of India using the Translog production function. They examined the extent of employment concentration in Orissa's manufacturing industries relative to all India to know the structure and growth of registered manufacturing factory sector. They used total factor productivity as the measure of productivity which is defined as the difference between the rate of growth of output and rate of growth of combined inputs i.e. labour (L),Capital (K) and time (T) representing technical progress.

Maiti and Mitra (2010)[8] have considered education, specifically, enrolment ratio in engineering and management studies, as a proxy for available skill formation. They argue that with higher levels of education, the quality of labour and thereby their employability in the formal sector of the economy would be enhanced which measures the skill content of the work force.

Idris Jajri and Rahmah Ismail (2009)[7] in a study of "Technical Progress and Labour Productivity in Small and Medium Scale Industry in Malaysia" observed that technical progress is a complement with more skilled labour but a substitute with less skilled labour. They examined the effect of technical progress on labour productivity using a Human capital method developed by Cörvers (1994)[1] which is based on the Cobb-Douglas production function. Finally they viewed that technical progress will have a positive effect on labour productivity.

Sabuj Kumar Mandal and S. Madheswaran(2009)[9] in a working paper (216) of Institute for Social and Economic Change (ISEC) titled "Technological progress, scale effect and total factor productivity growth in Indian cement industry: Panel estimation of stochastic production frontier" studied on Total Factor Productivity (TFP) growth in Indian cement industry during the period 1989-90 to 2006- 07 using company level data and applying Stochastic Frontier Approach. TFP growth is decomposed into technical progress (TP), technical efficiency change (TEC) and changes in scale component (SC) with a view to gaining some insights into the sources of productivity growth of this industry in the post reform era. The findings show that TFP growth is mainly driven by SC and TP and not by TEC, since TEC is time invariant in nature.

Nicholas F. R. Crafts(2003)[2] in a study of " Quantifying the Contribution of Technological Change to Economic Growth in Different Eras: A Review of the Evidence" found first that the estimate of TFP growth that results from standard growth accounting is not necessarily a good indicator of the contribution made by technological change to economic growth. The discrepancy can go either way and has varied considerably both over time and also across countries. Second, even the most powerful new technologies have modest impacts on productivity growth in their early stages. This is easily understood in the context of growth accounting which makes explicit the weights to be attached to new types of capital and production. He critically reviewed Solow method and Cobb-Douglas production function in this regard.

Humphrey (1997)[6] gives an outline of historical development of the concept and mathematical formulation of production functions before the enunciation of Cobb-Douglas function in 1928. Paul Douglas, on a sabbatical at Amherst, asked mathematics professor Charles W. Cobb to suggest an equation describing the relationship among the time series on manufacturing output, labor input, and capital input that Douglas had assembled for the period 1889–1922, and this led to their joint paper.

Productivity Growth and Technological Change:

Productivity is a relationship between production and the means of production or more formally a relation of proportionality between the output of a good or service and inputs which are used to generate that output. This relationship is articulated through the given technology of production.

Productivity growth is crucially affected by technological change. Their relationship is so close that the two terms are often used interchangeably. Productivity is a wider concept, even though a crucial one where technological change is only one of the many factors which affect productivity growth. Others are being social, cultural, educational, organisational and managerial factors. Better management of workers and machines and appropriate incentive structures can increase production and/or reduce costs. But these are different from technological change. It is not easy or straight forward to disentangle the effects of technological change from social and cultural factors.

Mathematical Formulation of Model Specification:

There are a good number of procedures exist to measure the total factor productivity (TFP) in industries (may be small scale or large scale). Generally, TFP as a measure of productivity is defined as the difference between the rate of growth of output and the rate of growth of combined inputs i.e. Mathematically:

$$P = \frac{dV}{dt} - \frac{d}{dt}\ f(K, L) \qquad (1)$$

where, P is Total factor productivity, V(t) is output and f(K,L) is the aggregate function of two inputs i.e. Capital(K),Labour(L) and time (t) which is representing the evolution of *technical progress.*

Thus,

$$P = \frac{dV}{dt} - \left[\frac{\partial f}{\partial K}\frac{dK}{dt} + \frac{\partial f}{\partial L}\frac{dL}{dt}\right]$$

Keeping Capital (K) as constant, for $\forall t$;

$$P = \frac{dV}{dt} - \frac{\partial f}{\partial L}.\frac{dL}{dt} \qquad (2)$$

which satisfies the idea of ***Law of variable proportion.*** The Law states that; keeping some inputs factor fixed and any change in other variable inputs lead to change in output for a certain limit.

A good number of studies are made on productivity growth in both developed and developing countries with the help of number of existing production functions like Cobb-Douglas production function, CES production function, VES production function, Translong production function etc. But in this paper an attempt has been made to measure labour productivity with the help of stochastic process and Markov chain. Then, the developed idea has been verified numerically with the help of the data source and a suitable example is also cited in it.

To formulate the idea, we have taken here the help of **Johann Von Thünen** [10] exponential production function and derived the function as

$$\boldsymbol{P = A(1 - e^{a_1 K})(1 - e^{a_2 L^*})(1 - e^{a_3 T})}$$

where K=Capital,
L^*= effective labour,
A= Efficiency parameter and
T= Technology

$\mathbf{a_i}$ are the parameters associated with respective factors having their meanings at i =1, 2, 3.

It is pertinent to note that an effective labour can be defined as employment in three categories of works: management and professionals, technical and supervisors, and finally, direct production group. Thus the effective labour likelihood function can be defined as:

$$L^* = L.l_1 l_2 l_3$$

where

$L \equiv$ Quantity of labour,

$l_1 \equiv$ Number of workers in management and professional groups,

$l_2 \equiv$ Number of workers in technical and supervision groups and

$l_3 \equiv$ Number of workers in production groups (mainly the physical workers)

The total number of workers in production group i.e. l_3 type which consists of both educated and non-educated workers. Before defining educated workers, it is necessary to know the importance of educated workers in industries. To support any technical change for the growth of manufacturing sector Odisha, only educated workers will be effective enough to adopt the changes.

The employment of above defined educated workers will reduce the depreciation amount of fixed capital as well as the plant and machinery of the production unit. It is also observed from the study that the educated workers are cautious and able enough to use and handle the machineries, so that the depreciation amount occurring through wrong operation and misuse of machines will be certainly low.

Definition:

Educated workers can be defined as "the workers with no technical education (industrial or vocational training, diploma or degree engineering) but with formal school education (matriculation/10th standard) so that they are more able to adapt the technological change and can use new production techniques more proactively."

On the basis of the above, we have classified the above stated l_3 type of workers as:

$$l_3 = l_{3m} + l_{3n}$$

where

l_{3m} = physical worker without any school education

l_{3n} = worker with formal school education (educated labour category as stated above)

∴ The resulting production function will be:

$$\boldsymbol{P = A(1 - e^{-a_1 K})\left(1 - e^{-L\, l_1^{\theta 1} l_2^{\theta 2} l_3^{\theta 3}}\right)(1 - e^{-a_3 T})}$$

$$\boldsymbol{= A\left(1 - e^{-a_1 k}\right)\left(1 - e^{-L\, l_1^{\theta 1} l_2^{\theta 2} l_{3n}^{\theta 3}}\right)\left(1 - e^{-\beta T}\right).}$$

where

$\left(\boldsymbol{1 - e^{-\beta T}}\right) = \left(\boldsymbol{1 - e^{-\theta_3 T}}\right)\left(\boldsymbol{1 - e^{-L\, l_1^{\theta 1} l_2^{\theta 2} l_{3n}^{\theta 3}}}\right)$ and $\theta_1 + \theta_2 + \theta_3 = a_2$.

Thus; $\frac{dP}{dL} = A(1 - e^{-a_1 k})\left(-e^{-L\, l_1^{\theta 1} l_2^{\theta 2} l_{3n}^{\theta 3}}\right)\left(1 - e^{-\beta T}\right)\left(-l_1^{\theta 1} l_2^{\theta 2} l_{3n}^{\theta 3}\right)$

$$= \mathrm{A}\, l_1^{\theta 1} l_2^{\theta 2} l_{3n}^{\theta 3}\left(\boldsymbol{1 - e^{-a_1 k}}\right)\left(\boldsymbol{1 - e^{-\beta T}}\right)\left(\boldsymbol{e^{-L\, l_1^{\theta 1} l_2^{\theta 2} l_{3n}^{\theta 3}}}\right) > 0$$

$$\boldsymbol{\frac{d^2P}{dL^2}} = -\mathrm{A}\, l_1^{2\theta_1} l_2^{2\theta_2} l_{3n}^{2\theta_3}\left(\boldsymbol{1 - e^{-a_1 k}}\right)\left(\boldsymbol{1 - e^{-\beta T}}\right)\left(\boldsymbol{e^{-L\, l_1^{\theta_1}\, l_2^{\theta_2}\, l_{3n}^{\theta_3}}}\right) < 0$$

Which suffices the above classification theoretically signifies that the total production increases with respect to the classification and reduction of the parameter.

Estimation Results:

The observations of the study reveal that from 2001 to 2010 in the manufacturing sector of Odisha, the growth of total factor productivity (TFG) is minimum and fluctuating with the average of 1.378 and it is only due to less exposure to technical progress (Table- 1).

Table 1: No. of Registered industries, value of output, value of input and Net value added. Depreciation.

PERIOD	VALUE OF INPUT(RS)	VALUE OF OUTPUT(RS)	DEPRECIATION (RS, CRORE)	NET VALUE ADDED (RS/CRORE)	TFP(O/I)
2000-01	10163.24	13242.67	727.75	2351.68	1.302
2001-02	10654.16	14113.82	910.41	2549.25	1.324
2002-03	12465.72	15618.84	798.11	2355.01	1.252
2003-04	13606.49	18643.66	1205.63	3831.54	1.370
2004-05	15899.22	23294.00	1349.36	6045.42	1.465
2005-06	20230.76	27977.11	1458.86	6287.49	1.382
2006-07	25943.87	36641.60	1675.02	9022.70	1.412
2007-08	32504.72	48013.83	1997.61	13511.50	1.477
2008-09	50337.61	69532.92	2521.25	16674.06	1.381
2009-10	51439.62	72971.18	2586.44	18945.12	1.418

Source: Economics Survey of Odisha, India, 2011-12

After surveying more than 100 small and medium scale industries as a sample, it is recorded that near about 40% to 50% workers are working in these industries without having formal school education. Though it is discussed that educated workers can allocate the inputs optimally and efficiently, so as a result it will increase the output .As the educated workers are skilled enough to handle the machineries carefully so the depreciation occurring due to accident, wrong use and mishandling will be reduced. So a prescribed model Table -2 is prepared with the help of correlation and regression analysis to verify our discussion with reference to the Table -1.

Table 2: (Numerical verification).

PERIOD	VALUE OF INPUT(RS)	VALUE OF OUTPUT(RS)	TFP(O/I)
2000-01	10163.24	16455.64	1.619
2001-02	10654.16	17149.02	1.609
2002-03	12465.72	19707.66	1.580
2003-04	13606.49	21318.89	1.566
2004-05	15899.22	24557.143	1.544
2005-06	20230.76	30675.00	1.516
2006-07	25943.87	38744.20	1.493
2007-08	32504.72	45909.66	1.412
2008-09	50337.61	73197.91	1.454
2009-10	51439.62	74754.39	1.453

Source: Authors' calculation

*The above results for the new output is obtained by the regression analysis, taking output as dependent variable[Y] and input as independent variable [X]. The regression line equation is
Y=1.412399832X + 2101.08738

It is also observed that the output level is increasing as well as the average value of TFP is increased from 1.378 to 1.542.

Measuring Total Factor Productivity Through Stochastic Process:

Since in an industry (small or large scale); the total production for future year depends upon the present financial year but not on the past, thus the production processes follows the Markovian property and the process is a Markov chain.

The analytical studies to increase the labour productivity of small scale industries as discussed in the preceding section, is verified by the stochastic process in this section. The model is formulated and mathematically justified with suitable examples with numerical data taken from the data source.

Formulation:

As an $S-$valued stochastic process is a probability space $(\Omega, \mathbb{F}, P)$ together with random variables:

$X_t : \Omega \to S$, for $t \in \mathbb{N}_0 = \{0,1,2,......\}$;

where

$$\Omega \equiv Sample\ space,$$
$$\mathbb{F} \equiv \mathrm{P}(\Omega) \equiv power\ set\ of\ \Omega,$$
$$P \equiv Transition\ \ probability,$$

and

$$S \equiv State\ space;$$

the problem of labour productivity is formulated as follows.

" By replacing the physical labour with less number of educated labour $(l_{3n} < l_3)$ *if the production is increasing in present year, then the probability that it will increase in next year be* p *and if the production is decreasing in present year, then the probability that it will be increased in next year be* q *".*

Referring to the above stated model, the state-space is consists of only two states s_0 and s_1 as defined below:

s_0 : if the production is increasing in present year, then will increases in next year,

s_1 : if the production is decreasing in present year, then will increases in next year.

As a result, since the state transition probabilities:

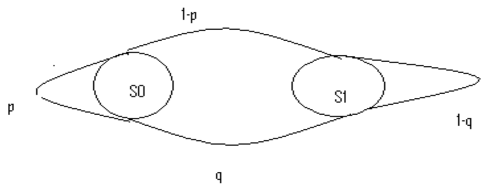
$$p_{ij} = p\left\{X_{t+1} = s_j \left| X_0 = s_0, X_1 = s_1, ..., X_{t-1} = s_v, X_t = s_i \right.\right\}$$
$$= p\left\{X_{t+1} = s_j \left| X_t = s_i \right.\right\}; \forall i, j = 0,1.$$

only depends on the present state but not on the past state. Thus, the collection:

$\{X_t : t = 0,1,2,3,..........\}$;

is a Markov chain and the corresponding state transition matrix is:

$$\begin{array}{ccc} States & s_0 & s_1 \end{array}$$
$$P = \begin{array}{c} s_0 \\ s_1 \end{array} \begin{pmatrix} p & 1-p \\ q & 1-q \end{pmatrix}.$$



(State-Transition Diagram)

To observe the labour productivity for a long period of years, we have extended our studies to the ***long-run properties*** of Markov chain in this model by taking the help of ***Chapman-Kolmogorov[5] equations:***

$p_{ij}^{(n)} = \sum_{k=0}^{1} p_{ik}^{(m)} p_{kj}^{(n-m)}; \ \forall i, j = 0,1 \quad and \quad m < n$, where $p_{ij}^{(n)}$ is the $n-$step transition probabilities which signifies the labour productivity for $n-$subsequent years.

Applying the algebraic method, the $n-$step transition matrix $P^{(n)}$ can be computed by the following recurrence relation:

$$P^{(n)} = P.P^{(n-1)} = P^{(n-1)}.P$$
$$= P^{n-1}.P = P^n$$

Moreover, since the formulated Markov chain is irreducible; thus

$\lim_{n\to\infty} p_{ij}^{(n)}$ exists and converges to a specific value π_j (say).

That is, for large $n-$(years after years) the total production will attain a constant value (***Steady-state***) one stage of laws of variable proportion.

However, when the production attains the steady-state; to observe the production process for future trends, we have determined the ***expected first passage time*** f_{jj} and which can be computed from the steady-state production probabilities π_j as:

$$f_{jj} = \frac{1}{\pi_j}; \forall j = 0,1.$$

Numerical Verifications:

To verify numerically, we have considered the source date given in Table- 1and the author's calculation given in Table-2. Since in Table-2 with the ***base year as 2004-05***, the calculated probabilities are $p = 0.5 \quad and \quad q = 0.4$; thus the one step state-transition matrix obtains as:

$$\begin{array}{ccc} States & s_0 & s_1 \end{array}$$
$$P = \begin{array}{c} s_0 \\ s_1 \end{array} \begin{pmatrix} 0.5 & 0.5 \\ 0.4 & 0.6 \end{pmatrix}$$

and therefore subsequently by applying the Chapman-Kolmogorov equations, the process attains the steady state after **six** years i.e. the total production will be constant which is a justification of law of variable of proportion. The 6-step transition matrix is given as:

$$P^{(6)} = \begin{array}{c|cc} States & s_0 & s_1 \\ s_0 & 0.4444 & 0.5556 \\ s_1 & 0.4444 & 0.5556 \end{array}$$

Since here the states are communicative and recurrent; representing the production for future years, thus the production will be increased in a periodical form if the process attains steady state after some year of productions which has been viewed by computing the first passage time f_{jj} .

Here

$$f_{00} = 2.25 \quad years$$

and

$$f_{11} = 1.799 \quad years$$

which signifies that after attaining the steady-state, in an interval of $2.25 \approx 03 \; years$; the total production will be increased.

Conclusion:

This study mainly focuses on improvement of labour productivity in manufacturing sector of Odisha which comprises of maximum number of medium and small scale industries. To improve the labour productivity, it is indeed necessary to accept and make use of advanced technology. But due to lack of educated workers, the total factor productivity grows at a slower pace which ultimately hampers the profit level of industries. For overall improvement of the industries of Odisha, it is concluded that at least the formal school education, vocational education, engineering education must be given top most priority by the government so that it leads to the acceptance and absorption of the technological changes to compete in the global market.

The paper develops analytical equations for calculating the labour productivity rate, which includes the equation of 'productivity rate of technology' which is only possible through educated workers. Consequently, manufacturing sector of Odisha needs skilled and educated workers to become an important source of growth and development of the economy.

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