Prediction of Crack Width in Panel Composite Deck Slab Under Repeated Loading

Nawir Rasidi, Agoes Soehardjono MD, Achfas Zacoeb

Department of Civil Engineering, Brawijaya University and State Polytechnic of Malang, East Java 65145, Indonesia

Abstract: This research presents a prediction method of crack width in panel precast composite deck slab under cyclic or repeated loading. Inspection of crack width prediction procedures proposed by various researchers indicates that each formula contains a different set of variables. A literature review also suggests that there is no general agreement among various researchers on the relative significance of different variables affecting the crack width, despite the large number of experimental work carried out during the past few decades. An analytical method is developed to determine the concrete stress distribution near flexural cracks in reinforced concrete one-way slabs and used to investigate the effects of various variables on the spacing and width of cracks. The formula is developed using a large number of tri-linear model values calculated from the concrete and steel strains at various sections between adjacent cracks for a number of composite precast deck slabs. The present method of incorporating the tension stiffening effect is verified by comparing calculated fracture mechanic and those measured by other researchers. The tri-linear model values at sections between adjacent cracks are calculated using an empirical formula. Development of this formula is based on the tri-linear model values calculated using the concrete and steel strains at various sections between successive cracks, for a number of composite panel deck slabs. Using the tri-linear model values evaluated by the proposed formula, short-term deflections were determined for a large number of flexural members and the results were compared with those measured by other researchers. This comparison indicated that the present method of incorporating the tension stiffening effect in fracture mechanic calculations is acceptable.

Key words: crack width, panel precast, composite, deck slab, tri-linear model

INTRODUCTION

The crack width is one of the important factors to the design of reinforced concrete structures. Currently, estimations of the crack width in the deck slab of bridges given by codes of practice are based on either theoretical or empirical approaches considering mainly the monotonic loading behavior. (Park & Paulay, 1975). Cracking in reinforced concrete structures is unavoidable due to the low tensile strength of concrete. Wider cracks may not only destroy the aesthetics of the structure, but also expose steel reinforcement to the environment leading to corrosion. To control the crack width at the member surface, designers may use the guidelines prescribed in various building codes. These guidelines are based on certain crack width prediction formulas developed by various researchers. (Australian Standard AS 3600- 2001)

Inspection of crack width prediction procedures proposed by various researchers indicates that each formula contains a different set of variables. A literature review also suggests that there is no general agreement among various researchers on the relative significance of different variables affecting the crack width, despite the large number of experimental work carried out during the past few decades. (Piyasena, 2002). This is at least partly due to the differences in the variables incorporated by different researchers in their experimental work. Taking all the parameters into account in a single experimental program is not normally feasible due to the large number of variables involved, and the interdependency of some of the variables. Analytical methods, on the other hand, can incorporate most of the variables without much difficulty. However, a literature search reveals that different researchers have concentrated on different sets of parameters in their calculation, to simplify the complex phenomenon of cracking in reinforced concrete. A major focus in this research is to incorporate as many parameters as possible in an analytical investigation. (Piyasena, 2002)

Cracking in a reinforced concrete member also causes a significant increase in deflection. This is a result of the reduction of bending stiffness at cracked sections when the effect of tensile concrete below the neutral axis diminishes. However, at sections between successive cracks, some tensile stress is retained in the concrete around steel bars due to the action of bond, contributing to the bending stiffness of the member. (Carpinteri, 1990). This is called the “tension stiffening” effect. If the tension stiffening effect is neglected, the calculated deflection may be overestimated by a large proportion. In simplified methods of deflection calculation, the tension stiffening effect is incorporated in a semi-empirical manner by using the effective moment of inertia method. (Carpinteri, 1990)
In analytical methods, the deflection is calculated using the curvature values, evaluated by adopting a non-linear stress-strain relationship for tensile concrete. This relationship allows the concrete to retain some tensile stress beyond the cracking strain. A new method is developed to evaluate the curvature values at sections between successive cracks by incorporating the bond force acting around steel bars in the calculation, instead of the concrete tensile force. (Piyasena, 2002)

**Allowable Crack Width In Reinforced Concrete:**

The maximum crack width that may be considered not to impair the appearance of a structure depends on various factors including the position, length, and surface texture of the crack as well as the illumination in the surrounding area. Crack widths in the range 0.25 mm to 0.38 mm may be acceptable for aesthetic reasons (Park & Paulay, 1975).

Crack width that will not endanger the corrosion of steel reinforcement depends on the environment surrounding the structure. Table 1 shows the maximum allowable crack widths recommended by ACI Committee 224 for the protection of reinforcement against corrosion. These values are taken as the basis for the development of rules prescribed in ACI 318 for the distribution of tension steel to limit the crack width.

<table>
<thead>
<tr>
<th>Exposure condition</th>
<th>Maximum allowable crack width (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dry air or protective membrane</td>
<td>0.4</td>
</tr>
<tr>
<td>Humid, moist air or soil</td>
<td>0.3</td>
</tr>
<tr>
<td>De-icing chemicals</td>
<td>0.2</td>
</tr>
<tr>
<td>Seawater and seawater spray; wetting and drying</td>
<td>0.15</td>
</tr>
<tr>
<td>Water retaining structures</td>
<td>0.1</td>
</tr>
</tbody>
</table>

**Causes Of Cracking:**

Cracks formed in reinforced concrete members can be classified into two main categories, namely cracks caused by externally applied loads, and those which occur independently of the loads. Flexural cracks and inclined shear cracks are the two main types of cracks caused by external loads. (Wallenfelsz., 2006). Flexural cracks are formed in the tensile zone of the member and have a wedge shape, with the maximum crack width at the tension face and zero width near the neutral axis. Inclined shear cracks usually develop in thin-web beams when subjected to high shear forces (Wallenfelsz., 2006).

Internal micro-cracks fall into the other type of cracks caused by external load. These cracks occur as a result of high concrete stresses near the ribs in deformed bars, and are confined in the immediate neighborhood of reinforcement without appearing on the concrete surface.

Cracks developed in restrained members due to concrete shrinkage or temperature change fall into the second category of cracks, which are independent of applied loads. In thin restrained members such as floor slabs these cracks may extend through the entire cross section, usually having an approximately uniform width. If the width of these cracks is not properly controlled, they may disrupt the integrity of the structure and reduce the bending stiffness considerably resulting in large deflections. (Wallenfelsz., 2006).

Flexural cracks begin to occur when concrete stress in the tension face of a member reaches the flexural strength of concrete. After formation of a crack some elastic recovery takes place in concrete on the member surface, contributing to the crack width. However, some stress and strain is maintained in concrete surrounding the reinforcement due to the action of bond. This contributes to a reduction in the crack width near the bar compared to that at the tension face. (Piyasena, 2002)

Flexural cracks in a varying moment region of a beam or one way slab develop at a regular interval; however, in a constant moment region, these cracks develop at discrete intervals. Their locations depend partly on the occurrence and distribution of zones of local weakness in concrete, and therefore cracking is somewhat a random process (Piyasena 2002). As a result, the exact locations of cracks in a constant moment region may not be predicted accurately. However, maximum and minimum spacing of adjacent cracks and the resulting maximum crack width may be predicted with sufficient accuracy by investigating concrete stresses developed in the tensile zone of a member.

**Crack Width Prediction:**

The development of crack width (w) prediction formulas is usually based on calculated concrete stress distributions within the tensile zone of a member. Some researchers have used various simplified analytical procedures to determine the concrete tensile stress. While some analytical investigations are coupled with experimental works to verify the new prediction formulas, there are some investigations totally based on test results.

In most investigations, a uniaxial tension member has been used to simulate the conditions around steel bars in the constant moment region of a member. In experimental investigations, a concrete prism with a steel bar embedded along its axis is subjected to a tensile force applied to the two protruding ends of the bar. The
resulting tensile cracks are considered to represent flexural cracks in a constant moment region of a beam. In analytical investigations the axial tensile stress distribution, developed in the concrete prism resulting from the bond force transferred from the steel bar, is calculated. This stress distribution is then used to predict the formation of new cracks in between existing cracks.

The literature review suggested that there is no general agreement among different researchers on the relative significance of various variables affecting the crack width, which sometimes leads to differing conclusions.

This is at least partly due to the absence of test data that describe the individual effects of each variable. Producing such a data set in the laboratory is expensive and time consuming because of the large number of variables involved, and due to the interdependency of some of the variables. A mathematical model capable of accurately predicting the spacing and width of cracks can be used to overcome this problem if it can include all variables involved in flexural cracking. This is not available at present, and is a main focus in this research.

Composite action between concrete and reinforcing steel cannot occur without bond. Therefore, the bond performance of reinforcing bars plays a major role in the behavior of reinforced concrete structures when subjected to static and dynamic loads. Insufficient bond can lead to a significant decrease in the load-carrying capacity and stiffness of the structure when subjected to monotonic, cyclic, or reversed cyclic loading. Aspects pertaining to bond behavior in reinforced concrete members include strength development, development length, anchorage of reinforcement, bar splicing, and ductility under monotonic and reversed cyclic loading.

Precast Composite Concrete Bridge Deck Panel System:

Precast bridge deck panels have been used for quite some time, however, an increase in the number of bridges undergoing reconstruction and rehabilitation has focused attention on the use of the fully precast system. A fully precast system can ensure quality and minimize hardship on the motoring public by minimizing construction related delays.

![Composite precast panel deck slab](image)

The construction of the bridge deck is the last component of bridge construction that requires refinement to achieve a fully functional totally pre-fabricated bridge system. Full-depth bridge deck panels have been developed and used extensively in Indonesia in the past decade.

Composite construction allows the designer to utilize the strength of the deck coupled with the girder to provide a more efficient and economical design. In order to account for this, the designer must accurately predict the horizontal shear developed at the interface between the slab and girder and provide adequate connectivity between the two to develop the full composite action.

Description Of Test:

To examine the crack width of a precast bridge deck panel system on precast girders 18 deck slab composite load tests were conducted. Concrete stress distribution patterns determined in formula are utilized in this experiment to predict the locations of cracks formed in a member when it is subjected to a gradually increasing load. Locations of primary cracks are determined based on the concrete stress distribution evaluated near the first flexural crack of the member. It is shown that the primary crackwidth in both constant and varying moment regions are governed by the slip length (bond length required to resist the steel stress increment at the first flexural crack). This prediction is verified by comparing the calculated and measured values of primary crack width in constant and varying moment regions.

The maximum crack width at a given load level is determined using the elastic extensions of steel and surrounding concrete. Using the present analytical procedure, the average maximum crack widths within constant moment regions are computed for 18 flexural members at various load levels.
The accuracy of the proposed calculation method is verified by comparing the calculated width of cracks with the measured values.

When a member is subjected to a gradually increasing load, the first flexural crack is developed at the location where the applied bending moment is equal to the cracking moment. Even in a prismatic member, the cracking moment may vary slightly along the length of the span due to the non-homogeneity of concrete and due to the presence of micro-cracks that may have occurred before the application of loading. Since the cracking moment $M_{cr}$ is proportional to the flexural strength of concrete $f_{r}$, the possible variation of $M_{cr}$ along the length of the slab is treated as a variation of $f_{r}$ for convenience, in the following discussion. Consequently, it is assumed that a primary crack is formed when the calculated tensile stress $f_{rc}$ at the tension face of the member reaches the flexural strength of concrete $f_{r}$.

Experimental Results:

Although many investigations have been carried out on cracking of reinforced concrete flexural members, results of individual crack width are rarely available; only the average crack widths are reported most of the times.

Agoes Soehardjono MD has reported the results of individual crack width for any various reinforcement on two simply supported slabs. These measurements are compared with the predictions made in the previous section on primary crack width in constant and varying moment regions.

Width of cracks in reinforced concrete members were determined using the calculated concrete stress distributions near flexural cracks. To calculate the stresses, a free body concrete block bounded by top and bottom faces and two transverse sections of the member was isolated and analysed using the spreadsheet.

An investigation into the effects of various variables on the width of cracks revealed the following:

1. An increase in the width of the member or the concrete cover increases crack width if other variables are kept unchanged.
2. Concrete strength has no appreciable effect on the crack width, if other variables remain unchanged.
3. An increase in the number of bars, by reducing the bar diameter to have the same reinforcement ratio, will reduce width of cracks.
4. The steel stress at the cracked section will reduce the crack spacing while it increases the crack width.

![Fig. 4: Crack width at various analysis and experiment for precast deck slab](image)
Composite construction allows the designer to utilize the strength of the deck coupled with the precast panel to provide a more efficient and economical design. In order to account for this, the designer must accurately predict the horizontal shear developed at the interface between the topping slab and precast panel and provide adequate connectivity between the two to develop the full composite action.

![Fig. 5: Crack width versus number of load cycles for various reinforcement ratio](image)

**Fig. 5:** Crack width versus number of load cycles for various reinforcement ratio

![Fig. 6: Tri-linear model based on experimental analysis](image)

**Fig. 6:** Tri-linear model based on experimental analysis

Crack formation presents a complex mechanical and geometrical question to be modeled. The available crack width formulations are often based on simplifications. A rigorous formulation of crack widths should be based on the integration of strain differences of reinforcement and concrete between cracks, due to the accumulated slips. Considering Figure 4, Callister and Carpinteri analysis illustrates the significant increase in crack width, but the experimental results show a line between the two author analysis.

![Fig. 7: Tri-linear model based on proposed formula](image)

**Fig. 7:** Tri-linear model based on proposed formula

Experiment on reinforced concrete elements in flexure have demonstrated that concrete cover (c), spacing of reinforcing bars (s), reinforcement ratio and number of load cycles (N) have considerable influences on the crack width (Figure 5.).

The first crack is considered to appear when the tensile strain reaches the tensile capacity of concrete. In calculations it is taken into account with the crack width condition \( w = w_0 \) at the extreme tensile face. Figures 6 and 7 show the correlation between the experimental w-N curves and those predicted by the tri-linear model with reinforcement ratio.
Experiments have demonstrated that the nominal diameter of reinforcement ($d_b$), the reinforcement ratio ($\rho$), concrete cover ($c$) and number of load cycles ($N$) have considerable influences on the crack width ($w$). Basic parameters of the crack width ($w$) are the average strain in the reinforcement relative to that in the adjacent concrete. Available crack spacing and crack width formula are very numerous.

**Conclusions And Suggestions:**

Based on the results of a parametric study, simplified formulas were developed for the prediction of maximum crack width. A comparison of predicted crack widths for the flexural members and the measured values reveals that the proposed formulas perform adequately. In particular, the predictions of these formulas have almost the same accuracy as the results of the various researchers and various codes.

The tri-linear model for composite precast deck slab concrete structure under repeated loading is found from the proposed formula:

$$w_0 = 1.084 \cdot 10^{-6} \frac{A_s f_s}{(1 + n \bar{p})} p e^{0.013435 a}$$  (1)

First condition, initial crack width for $1 < N \leq N_0$

$$w = w_0 N_0 = U \frac{f_s}{f_y} + V$$  (2)

Second condition, the crack width increase for $N_0 < N < N_f$

$$w = S e^{T \frac{f_s}{f_y}} (N - U \frac{f_s}{f_y} - V) + w_0$$  (3)

Third condition, the crack width for fatigue loading $N_f < N$

$$w = w_f = \left(W \frac{f_s}{f_y} + X\right)w_0$$  (4)

$$N_f = \left(Y \frac{f_s}{f_y} + Z\right)N_0$$  (5)

where,

- $w$ = crack width (mm)
- $w_0$ = initial crack width (mm)
- $N$ = number of load cycles (cycles)
- $A_s$ = area of steel reinforcement (mm$^2$)
- $f_s$ = reinforcement stress (N/mm$^2$)
- $f_y$ = reinforcement yield stress (N/mm$^2$)
- $n$ = $E_s / E_c$ = modular ratio of steel and concrete
- $p$ = precast modification factor
- $\bar{p} = A_s / s.h$ = reinforcement concrete ratio
- $a = 1.69 f_s - 347.1 = $ crack depth (mm)
- $S,T,U,V,W,X,Y,Z$ = non dimensional parameters depend on $\bar{p}$

<table>
<thead>
<tr>
<th>$\bar{p}$</th>
<th>S</th>
<th>T</th>
<th>U</th>
<th>V</th>
<th>W</th>
<th>X</th>
<th>Y</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0030</td>
<td>2E-7</td>
<td>1.82</td>
<td>-5173</td>
<td>20550</td>
<td>1.08</td>
<td>1.37</td>
<td>0.72</td>
<td>1.36</td>
</tr>
<tr>
<td>0.0040</td>
<td>2E-7</td>
<td>2.43</td>
<td>-8310</td>
<td>25617</td>
<td>1.15</td>
<td>1.41</td>
<td>0.80</td>
<td>1.28</td>
</tr>
<tr>
<td>0.0050</td>
<td>3E-7</td>
<td>3.03</td>
<td>-11447</td>
<td>30683</td>
<td>1.23</td>
<td>1.46</td>
<td>0.88</td>
<td>1.20</td>
</tr>
<tr>
<td>0.0059</td>
<td>3E-7</td>
<td>3.58</td>
<td>-14584</td>
<td>35750</td>
<td>1.30</td>
<td>1.50</td>
<td>0.96</td>
<td>1.12</td>
</tr>
<tr>
<td>0.0082</td>
<td>4E-7</td>
<td>4.98</td>
<td>-20420</td>
<td>42604</td>
<td>1.52</td>
<td>2.20</td>
<td>1.05</td>
<td>1.01</td>
</tr>
<tr>
<td>0.0098</td>
<td>5E-7</td>
<td>5.95</td>
<td>-26256</td>
<td>49457</td>
<td>1.74</td>
<td>2.90</td>
<td>1.14</td>
<td>0.89</td>
</tr>
</tbody>
</table>

Analytical results of crack widths are greatly influenced by the assumed bond stress distribution and bond stress-bond slip relationship. The constitutive relationships proposed by different researchers vary considerably. Further research in to the measurement of bond stress and bond slip is proposed.

It was shown that the crack width increases with concrete cover. In spite of this, provision of a large cover is considered to be the most practical means of protecting the reinforcement against corrosion. Further research in to the effect of varying concrete cover on the crack width is proposed.
The proposed analytical procedure can also be extended to determine the increase in the crack width with time by incorporating the creep and shrinkage effects in the calculation of concrete and steel strains.

ACKNOWLEDGEMENTS

The first author would like to acknowledge the Indonesian Directorate General of Higher Education (DIKTI) for providing the PhD scholarship. The first author would also like to acknowledge the employer, State Polytechnic of Malang for providing with support.

REFERENCES

Agoes Soehardjono, M.D., 2006. The Influence of Reinforcement Ratio ($\rho$) and Cyclic Loading (N) to Crack Width (w) of One Way Slab, Dissertation, Civil Engineering ITS, Surabaya, Indonesia.
Park, R and T. Paulay, 1975. Reinforced Concrete Structures, John Wiley & Sons, Inc, Canada, USA.
Ratnamudigedara Piyasena, 2002. Crack Spacing, Crack Width and tension Stiffening Effect in Reinforced Concrete Beams and One Way Slab, Dissertation Doctor of Philosophy, Gold Coast Campus Griffith University.