A New Economic Order Policy Model under Markovian Inflationary Conditions

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Abstract: This paper develops an inventory model under inflationary conditions. In many real-life situations, the practical experiences reveal that the inflation is non-deterministic and variable. Because of previous data don’t have great impact on inflation, so we consider markovian inflationary conditions, whereinflation rate changes as a discrete-time Markov chains. The developed model also implicates to finite replenishment rate, finite time horizon and deteriorating items with shortages. The objective is minimization of the expected present value of costs over the time horizon. The numerical example and sensitivity analysis have been provided for evaluation and validation of the theoretical results.

Key words: Inventory; Markovian Inflationary Conditions; Finite Production Rate; Deterioration.

INTRODUCTION

Inventory management is one of the crucial links of any supply chain. To manufacturers, it entails managing product stocks, in-process inventories of intermediate products as well as inventories of raw material, equipment and tools, spare parts, supplies used in production, and general maintenance supplies. As a consequence of high inflation, it is important to investigate how the time value of money influences various inventory policies. Buzacott (1975) made the first attempt in this field that dealt with an economic order quantity (EOQ) model with inflation subject to different types of pricing policies. Misra (1979) developed a discounted-cost model and included internal (company) and external (general economy) inflation rates for various costs associated with an inventory system. Inventoried goods can be broadly classified into four meta-categories:

1. Obsolescence: refers to items that lose their value through time due to rapid changes of technology or the introduction of a new product by a competitor.
2. Deterioration: refers to the damage, spoilage, dryness, vaporization, etc. of the products.
3. Amelioration: refers to items whose value or utility or quantity increase with time.
4. No obsolescence/deterioration/amelioration

If the rate of obsolescence, deterioration or amelioration is not sufficiently low, its impact on modeling of such an inventory system cannot be ignored. There are a few papers for obsolescing and ameliorating items. Moon, Giri, and Ko (2005) considered the ameliorating/deteriorating items on an inventory model with time-varying demand pattern. Against obsolescing and ameliorating items, the deteriorating inventory models under inflationary conditions are studied greatly. In a few of these works, deterioration rate is not constant. For example, Chen (1998) proposed an inflationary model with time proportionate demand rate and Weibull distribution for deteriorating items using dynamic programming. Balkhi (2004a) presented a production-lot-size inventory model where the production, demand and deterioration rates are assumed to be time-continuous differentiable functions. Shortages are allowed, but only a fraction of the stock out is backordered, and the rest is lost. Lo, Wee, and Huang (2007), developed an integrated production-inventory model with assumptions of varying rate of deterioration, partial backordering, inflation, imperfect production processes and multiple deliveries. If the rate of obsolescence, deterioration or amelioration is not sufficiently low, its impact on differentiable functions of time. Shortages are allowed, but only a fraction of the stock out is backordered, and the rest is lost. Lo, Wee, and Huang (2007), developed an integrated production-inventory model with assumptions of varying rate of deterioration, partial backordering, inflation, imperfect production processes and multiple deliveries. Most inventory systems for deteriorating items are considered in a constant deterioration rate, which will stay in continuance.

Some researches in inflationary inventory systems assumed time-varying demand rate. Datta and Pal (1991) investigated a finite-time-horizon inventory model with linear time-dependent demand rate when shortages are allowed. Yang, Teng, and Chern (2001) extended the inventory lot-size models to allow for inflation and fluctuating demand, which is more general than constant, increasing, decreasing, and log-concave demand patterns. Other works are performed by Chen (1998) and Balkhi (2004b). Several authors have considered finite replenishment rate for inflationary inventory systems. Wee and Law (1999) derived a deteriorating inventory model under inflationary conditions for determining economic production lot size when the demand rate is a
linear decreasing function of the selling price. Sarker and Pan (1994) surveyed the effects of inflation and the
time value of money on order quantity with finite replenishment rate.

Balkhi (2004b) proposed two flexible production lot-size inventory models for deteriorating items in which the production rate at any instant depends on the demand and the on-hand inventory level at that instant. Another research is performed by Lo et al. (2007). The stock-dependent demand rate models are prepared with some researchers. Vrat and Padmanabhan (1990) determined optimal ordering quantity for stock-dependent consumption-rate items, and showed that as the inflation rate increases, ordering quantity and the total system cost increase. Hou and Lin (2004) developed an inventory model under inflation and time discounting for deteriorating items with stock-dependent selling rate. The selling rate is assumed to be a function of the current inventory level and the rate of deterioration is assumed to be constant.

Liao and Chen (2003) surveyed a retailer’s inventory control system for the optimal delay in payment time for initial stock-dependent consumption rate when a wholesaler permits delay in payment. The effect of inflation rate, deterioration rate, initial stock-dependent consumption rate and a wholesaler’s permissible delay in payment is discussed. A deterministic EOQ inventory model is taking into account the inflation and time value of money developed for deteriorating items with price- and stock-dependent selling rates by Hou and Lin (2006). An efficient solution procedure is presented to determine the optimal number of replenishment, the time cycle and selling price. Hou (2006) prepared an inventory model for deteriorating items with stock-dependent consumption rate. Maiti, Maiti, and Maiti (2006) proposed an inventory model with stock-dependent demand rate constant deterioration rate, which will stay in continuance, and two storage facilities under inflation and time value of money where the planning horizon is stochastic in nature and follows exponential distribution with a known mean. Other efforts in inventory systems under inflationary conditions are performed under the assumption of the permissible delay in payments. Chang (2004) proposed an EOQ model for deteriorating items under inflation when the supplier offers a permissible delay to the purchaser, if the order quantity is greater than or equal to a predetermined quantity. Shah (2006) derived an inventory model by assuming constant rate of deterioration of units in an inventory, time value of money under the conditions of permissible delay in payments. Other models are prepared by Liao and Chen (2003).

The inflationary inventory models with two warehouses are proposed previously. Yang (2004) discussed the two-warehouse inventory problem for deteriorating items with a constant demand rate and shortages. Yang (2006) extended the models introduced in Yang (2004), to incorporate partial backlogging and then compare the two-warehouse inventory models based on the minimum cost approach. The above mentioned papers have considered a constant and well-known inflation rate over the time horizon. Horowitz (2000) discussed a simple EOQ model with a Normal distribution for the inflation rate and the firm’s cost of capital. He showed the importance of taking into account the inflation rate and time discounting, especially, when the former is relatively high or when there is a considerable uncertainty as to either the inflation rate or the marginal cost of capital.

Mirazazadeh and Sarfaraz (1997) presented a multiple items inventory system with budget constraint and the Uniform distribution for external inflation rate. These two recent models do not consider shortages and deteriorating items. Mirazazadeh et al. (2009) presented stochastic inflationary conditions with variable probability density functions (pdfs) over the time horizon and the inflation dependent demand rate. The developed model, also, implicates to a finite replenishment rate, finite time horizon, deteriorating items with shortages. The objective is the minimization of the expected present value of costs over the time horizon. Mirazazadeh (2010a) assumed the inflation is time-dependent and demand rate is assumed to be inflation-proportional. Mirazazadeh (2010b) proposed an inventory model with stochastic internal and external inflation rates for deteriorating items and allowable shortages.

Tolgari et al. (2012) investigated a profit-maximizing inventory model with incorporating both imperfect production quality and two-way imperfect inspection, i.e., Type-one inspection error of falsely screening out a proportion of no defects and disposing of them like defects and Type-two inspection error of falsely not screening out a proportion of defects, this model includes one more stages of inspection after rework process and there is no inspection error in this stage. Also, the purpose of this study is to determine the important factors of inventory system to optimize the present value of the total profit in the finite time horizon.

The present article differs from previous researches on the following aspect. The inflationary changes over the time horizon have been considered. In most previous models, the inflation rate has been considered as a constant value. In a few models, the inflation rates have been assumed stochastic with known pdfs over the time horizon. In the real world, especially, for long-term investment and forecasting, the fluctuations in the inflation rate cannot be disregarded. Therefore there is a need to consider the inflationary changes and inventory control problem in a fluctuating inflation rate environment, and a Markovian inflationary modeling approach provides an effective mechanism to address this problem. Because Previous data don’t have great impact on inflation rate.

Markov chain, a well-known subject introduced by Markov in 1906, has been studied by a host of researchers for many years Chung (1960), Doob (1953), Feller (1971) Kushner & Yin (1997). Markovian formulations (see Chiang (1980), Taylor & Karlin (1998), Yang, Yin, Yin, & Zhang (2002), Yin, Zhang, Yang, ...
& Yin (2001), Yin & Zhang (1997), Yin, Yin, & Zhang (1995) and the references therein) are useful in solving a number of real-world problems under uncertainties such as determining the inventory levels for retailers, maintenance scheduling for manufacturers, and scheduling and planning in production management. Markov chain approach has been applied in the design, optimization, and control of queueing systems, manufacturing processes, reliability studies and communication networks, where the underlying system is formulated as stochastic control problem driven by Markovian noise.

The remainder of this paper is organized as follows. Section 2 includes the assumptions, notations and description of the inventory system. In Section 3, the objective of the problem is derived. Section 4 explains the solution procedure. Section 5 provides a numerical example to clarify how the proposed model is applied. The final section is devoted to the conclusion and Appendix.

The Assumptions, Notations and Description of the Model:

The following assumptions have been considered in this inventory system:

1. The inventory system costs are known at the beginning of the time horizon and during this time, they are increased with the internal and external inflation rates.
2. Inflationary rate changes as a discrete time Markov chain.
3. The demand rate is definite.
4. The replenishment rate is finite and lead time is zero.
5. A constant fraction of the on-hand inventory deteriorates per unit time.
6. The production rate is higher than the rates of consumption and deterioration. On the other hand, the inventory level will increase as the production continues.
7. Shortages are allowed and fully backlogged except for the final cycle.

R      The discount rate.
P      The constant annual production rate.
θ      The constant deterioration rate per unit time, where (0 ≤ θ ≤ 1).
C      Per unit cost of the item at time zero.
S      The ordering cost per order at time zero.
H      The fixed time horizon.

Additional notations will be introduced later. The graphical representation of the inventory system is shown in Figure 1. The time horizon, H, is divided into n equal cycle each of length T so that T=H/n. Initial and final inventory levels are both zero. Each inventory cycle except the last cycle can be divided into four parts. The production starts at time zero and the inventory level is increasing. This fact continues till the production stops at time α. Then, the level of inventory is decreasing by consumption and deterioration rates. At the moment of kT, the inventory level leads to zero and shortages occur. During the time interval [kT, β], we do not have any deterioration, and therefore the shortages level linearly increases by the demand rate. At time β, the production starts again and the shortages level linearly decreases until the moment of T. In this moment, the second cycle starts and this behavior continues till the end of the first inflationary period. The pdfs of inflation rates change during the time horizon and the second inflationary period starts at time (n-1)T. Similar to the first inflationary period, each inventory cycle can be divided into four parts. In the last cycle, shortages are not allowed and each inventory cycle can be divided into two parts. The production stops at time ((n-1)T+) and then the inventory level decreases until the end of time horizon.

Fig. 1: Graphical representation of the inventory system.
The Mathematical Modelling and Analysis:

The inventory cycles in the first inflationary period is divided into four different parts. Let \( I_i(t_i) \) denote the inventory level at any time \( t_i \) in the \( i \)th part of cycle \((i=1, 2, 3, 4)\). The amount of deteriorated units during a given time interval depends on the on-hand inventory and the elapsed time in the system during the period of positive inventory, then during part of \([0, \alpha]\), with finite replenishment rate, the inventory level is governed by the following differential equation:

\[
\frac{dI_1(t)}{dt} + \theta I_1(t) = p - D, \quad 0 \leq t \leq \alpha
\]  

(6)

During \([0, kT)\), the inventory level can be described as follows:

\[
\frac{dI_2(t)}{dt} + \theta I_2(t) = -D, \quad 0 \leq t \leq kT - \alpha
\]  

(7)

During \([kT, \beta]\), we have no deterioration. Therefore, the shortages level is governed by

\[
\frac{dI_3(t)}{dt} = -D, \quad 0 \leq t \leq \beta - kT - \alpha
\]  

(8)

Finally, during \([\beta, T)\) the shortages level be represented by

\[
\frac{dI_4(t)}{dt} = p - D, \quad 0 \leq t \leq T - \beta
\]  

(9)

In the last cycle, shortages are not allowed and the inventory level is governed by the following differential equations:

\((I_i(t_i))\) denotes the inventory level at any time \( t_i \) in \((i-4)\)th part of last cycle that is \(i=5, 6\).

\[
\frac{dI_5(t)}{dt} + \theta I_5(t) = p - D, \quad 0 \leq t \leq \gamma
\]  

(10)

\[
\frac{dI_6(t)}{dt} + \theta I_6(t) = -D, \quad 0 \leq t \leq T - \gamma
\]  

(11)

The solutions of the above differential equations after applying the following boundary conditions:

\(I_1(0)=0, I_2(kT-\alpha)=0, I_3(0)=0, I_4(T-\beta)=0, I_5(0)=0\) and \(I_6(T-\gamma)=0\), are:

\[
I_1(t_1) = \frac{p-D}{\theta}(1-e^{-\theta t_1}), 0 \leq t_1 \leq \alpha
\]  

(12)

\[
I_2(t_2) = \frac{D}{\theta}(1-e^{-\theta(kT-\alpha-t_2)}), 0 \leq t_2 \leq KT - \alpha
\]  

(13)

\[
I_3(t_3) = D t_3, 0 \leq t_3 \leq \beta - KT
\]  

(14)

\[
I_4(t_4) = (p - D)(t_4 - T + \beta), 0 \leq t_4 \leq T - \beta
\]  

(15)

\[
I_5(t_5) = \frac{p-D}{\theta}(1-e^{-\theta t_5}), 0 \leq t_5 \leq \gamma
\]  

(16)

\[
I_6(t_6) = \frac{D}{\theta}(1-e^{-\theta(T-\gamma-t_6)}), 0 \leq t_6 \leq T - \gamma
\]  

(17)

Using the above equations, we can calculate the values of \(\alpha, \beta\) and \(\gamma\) with respect to \(k\) and \(T\). Solving 

\(I_1(\alpha)=I_2(0)\) for \(\alpha\) we have,
\[ \alpha = \frac{1}{\beta} \ln \left( \frac{\beta - 1 - e^{-\beta \Delta T}}{\beta} \right) \]  

(18)

\[ \beta \text{ can be calculated by solving } I_3(\beta \cdot kT) = I_4(0) \]

\[ \beta = \frac{[1 - e^{-\beta \Delta T}]}{\beta} \]  

(19)

Finally, solving \( I_5(y) = I_6(0) \) for \( y \) we have

\[ y = \frac{1}{\beta} \ln \left( \frac{\beta - 1 - e^{-\beta \Delta T}}{\beta} \right) \]  

(20)

**Markov Chain and Markov Property:**

Markov chain is concerned with a particular kind of dependence of random variables involved: When random variables are observed in sequence, the distribution of a random variable depends only on the immediate preceding observed random variable and not on those before it. In other words, given the current state, the probability of the chain’s future behavior is not altered by any additional knowledge of its past behavior. This is the so-called Markovian property.

For a discrete-time process, \( T = (0, 1, 2, \ldots) \) and \( P\{\alpha_{t+1} = j \mid \alpha_0 = i_0, \ldots, \alpha_{t-1} = i_{t-1}, \alpha_t = i\} = P\{\alpha_{t+1} = j \mid \alpha_t = i\} \) If the index \( t \) of \( \alpha_t \) is continuous, i.e. \( T = [0, \infty) \), The Markov property means that the values of \( \alpha \) for \( t \) are not influenced by the value of \( \alpha \) for \( u \), \( u < t \).

We consider discrete-time processes in this work. A stochastic process is a Markov chain if it possesses the Markovian properties and its state space is finite or countable. In this paper, we consider changes of inflation rate as a Markov chain, and determined state space and Transfer matrix as follows:

\( X_n: \) inflation rate in the state \( n \).

Inflation rate of month \( n \) than month \( n-1 \):

\[ \left( \frac{C_n - C_{n-1}}{C_{n-1}} \right) \times 100 = X_n \]

\( C_n \) is price index in month \( n \), that is reported by the Central Bank.

Transfer Matrix and stationary distribution of inflation rate are calculated and steps required to calculations are contained in Appendix.

The objective of the problem is minimisation of the total expected present value of costs over the time horizon. Considering ECP as the expected present value (EPV) of costs of purchasing, ECH as the EPV of costs of holding, ECS as the EPV of costs of shortages and ECR as the EPV of costs of replenishment, respectively.

The total expected present value of costs over time horizon (ETVC) is:

\[ \text{ETVC}(n, k) = \text{ECR} + \text{ECP} + \text{ECH} + \text{ECS} \]  

(21)

The detailed analysis is given as follows.

**The EPV of Ordering Cost (ECR):**

Consider CR as the ordering cost, therefore,

\[ ECR = \sum_{t=-\infty}^{\infty} f(t)(1 + (\sum_{i=1}^{\infty} \phi \left( -\frac{-(s-t)}{100} \right) e^{-\frac{(t-\frac{s}{100}) - \frac{t-\frac{s}{100}}{\beta}}{\beta}})) \]  

(22)

**The EPV of Purchasing Cost (ECP):**

Let \( ECP_1 \) be the EPV of the purchase cost. The EPV of the purchase cost in the last cycle is shown with \( ECP_2 \). The first purchase is ordered at time zero and equals to: \( cP \). Then, next purchase will occur at time \( \beta \). Therefore,

\[ ECP_1 = \sum_{t=-\infty}^{\infty} f(t)(\sum_{i=1}^{\infty} CP \left( \frac{\ln \left( \frac{\beta - 1 - e^{-\beta \Delta T}}{\beta} \right) e^{-\frac{(t-\frac{s}{100}) - \frac{t-\frac{s}{100}}{\beta}}{\beta}} + \left( T - \frac{t-\frac{s}{100}}{\beta} \right) e^{-\frac{(t-\frac{s}{100}) - \frac{t-\frac{s}{100}}{\beta}}{\beta}} \right)) \]  

(23)

In the last cycle, one order will occur at time \( (n-1)T \) and the order quantity is \( yP \). The EPV of the purchase cost in the last cycle will be one of the following phrases:
The total expected purchase cost over the time horizon would be

\[ ECP = ECP_1 + ECP_2 \]  

(25)

**The EPV of Holding Cost (ECH):**

Let \( ECH_1 \) be the EPV of the holding cost. The EPV of the holding cost during the last cycle, can be defined with \( ECH_2 \).

\[
ECH_1 = \sum_{i=2}^{k} \sum_{j=1}^{t} f(j) \left( c_1 e^{\left( -\left( t-1 \right) \left( r-d \left( 1 + \frac{\ln\left( \frac{p-d(1-e^{\left( 1\cdot r\right) T})}{p} \right)}{\frac{d}{p}} \right) \right) \right)} \right)
\]

(26)

\[
ECH_2 = c_1 \left( \frac{1}{r} \ln\left( \frac{p-d(1-e^{\left( 1\cdot r\right) T})}{p} \right) \right) e^{\left( -\left( t-1 \right) \left( r-d \left( 1 + \frac{\ln\left( \frac{p-d(1-e^{\left( 1\cdot r\right) T})}{p} \right)}{\frac{d}{p}} \right) \right) \right)}
\]

(27)

So, the total EPV of the holding costs over the time horizon is
ECH = ECH1 + ECH2 \hspace{2cm} \text{(28)}

The EPV of Shortage Costs (ECS):
ECS show the EPV of the shortage costs. Shortages are not allowed in the last cycle.

$$ECS = \sum_{i=1}^{n} \int f(t) \left( c_1 (t) + \frac{(d(t+\Delta_t) - d(t))}{\Delta_t} \right) dt - \int_{0}^{T} \left( \Delta_t \frac{(d(t+\Delta_t) - d(t))}{\Delta_t} \right) dt$$ \hspace{2cm} \text{(29)}

Model Analysis:
The problem is to determine the optimal values of n, k, so as to minimise the total expected inventory system costs. For this, the algorithm begins by setting discrete variable n=1, and takes the partial derivatives of ETVC(n, k) with respect to k. Equating the partial derivatives to zero derives the following necessary conditions of optimality

$$\frac{d\text{ETVC}(n,k)}{dk} = 0$$

For a given value of n, derive k* from the above equation. ETVC(n, k*) is derived by substituting (n, k*) into Equation (21). Then, n increases by increment of one continually and ETVC(n, k*) drive again. The above stages repeat until the minimum ETVC(n, k*) can be found. The (n*, k*) and ETVC(n*, k*) values constitute the optimal solution and satisfy the following conditions:

$$\Delta \text{ETVC}(n*-1, k*) = 0$$

Where

$$\Delta \text{ETVC}(n*, k*) = \text{ETVC}(n* + 1, k*) - \text{ETVC}(n*, k*)$$

To ensure convexity of the objective function, the derived values of (n*, k*) must satisfy the following sufficient conditions:

$$\frac{d^2\text{ETVC}(n,k)}{dk^2} = 0$$

Numerical Example:
The following numerical example is provided to clarify how the proposed model is applied. The time horizon, H, is 10 years.
The company interest rate is 10% and the deterioration rate of the on-hand inventory per unit time is 0.01. The constant annual production rate is 5000 units.
r=$0.2$/year; $\theta$ = 0.01; $P$ = 5000 units/year; The ordering, production, holding and shortage costs at the beginning of the time horizon are $S = $100/order; $c =$5/unit; $c_1 =$0.1/unit/year; $c_2 =$0.2/unit/year; $d = 500$.
The demand rate, which is infinite.

The problem is to determine the optimal ordering policy for minimising the EPV of the total inventory system costs (ETVC(n, k)). Considering the above information and using the numerical methods, the problem is solved and the results are illustrated in Table 1.

It can be seen that the number of replenishment=2 and time interval between replenishments is $T* = \frac{20}{2} = 5$ year.

In the first inflationary period, the shortages occur after elapsing 37% of the cycle time.(k* = 0.37).
The minimum value of the ETVC(n, k), under this condition:

<table>
<thead>
<tr>
<th>n</th>
<th>k</th>
<th>ETVC (n,k)</th>
<th>n</th>
<th>K</th>
<th>ETVC (n,k)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.37</td>
<td>0.528*10^7</td>
<td>11</td>
<td>0.37</td>
<td>0.103*10^8</td>
</tr>
<tr>
<td>2</td>
<td>0.37*</td>
<td>0.338<em>10^7</em></td>
<td>13</td>
<td>0.37</td>
<td>0.128*10^8</td>
</tr>
<tr>
<td>3</td>
<td>0.37</td>
<td>0.509*10^7</td>
<td>15</td>
<td>0.37</td>
<td>0.131*10^8</td>
</tr>
<tr>
<td>4</td>
<td>0.37</td>
<td>0.567*10^7</td>
<td>20</td>
<td>0.37</td>
<td>0.377*10^8</td>
</tr>
<tr>
<td>7</td>
<td>0.37</td>
<td>0.760*10^7</td>
<td>50</td>
<td>0.38</td>
<td>0.460*10^8</td>
</tr>
<tr>
<td>9</td>
<td>0.37</td>
<td>0.896*10^7</td>
<td>100</td>
<td>0.38</td>
<td>0.729*10^8</td>
</tr>
</tbody>
</table>
Sensitivity Analysis:

To study the effects of system parameters changes D, H, θ, r, s, p, c on the optimal cost, the replenishment time and k∗ which is derived by the proposed method, a sensitivity analysis was performed. This fact is done by increasing the parameters by 20, 50, 90% and decreasing the parameters to 20, 50, 90%, taking each one at a time, and keeping the remaining parameters at their original values. The following conclusion can be derived from the sensitivity analysis based on table 2.

1. The number of replenishments (n) is highly sensitive to the change of the parameters D, s, and H, and is insensitive to changes in r, θ, c, and p.
2. The optimal value of k is highly sensitive to the change of the parameters c, is moderately sensitive to r, and is insensitive to D, θ, H, p, and s.
3. The total expected inventory cost of the system is highly sensitive to the changes in the parameters D, r, H, and p and insensitive to θ, s, c.

### Table 2: Effects of changes in model parameters on n, k and optimal expected system cost.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>-90%</th>
<th>-50%</th>
<th>-20%</th>
<th>0%</th>
<th>20%</th>
<th>50%</th>
<th>90%</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>n</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>k</td>
<td>0.44157</td>
<td>0.41705</td>
<td>0.40439</td>
<td>0.40161</td>
<td>0.40876</td>
<td>0.52964</td>
<td>0.63476</td>
</tr>
<tr>
<td>ETVC</td>
<td>0.450*10^7</td>
<td>0.790*10^7</td>
<td>0.144*10^8</td>
<td>0.188*10^8</td>
<td>0.236*10^8</td>
<td>0.311*10^8</td>
<td>0.631*10^8</td>
</tr>
<tr>
<td>r</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>n</td>
<td>40</td>
<td>41</td>
<td>41</td>
<td>41</td>
<td>41</td>
<td>41</td>
<td>42</td>
</tr>
<tr>
<td>k</td>
<td>0.43241</td>
<td>0.42341</td>
<td>0.41556</td>
<td>0.40161</td>
<td>0.40351</td>
<td>0.40913</td>
<td>0.41568</td>
</tr>
<tr>
<td>ETVC</td>
<td>0.587*10^7</td>
<td>0.172*10^8</td>
<td>0.180*10^8</td>
<td>0.188*10^8</td>
<td>195*10^8</td>
<td>0.206*10^8</td>
<td>0.213*10^8</td>
</tr>
<tr>
<td>S</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>n</td>
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<td>46</td>
<td>41</td>
<td>41</td>
<td>41</td>
<td>41</td>
<td>41</td>
</tr>
<tr>
<td>k</td>
<td>0.40161</td>
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Discussion and Conclusion:

This paper presented a inventory model which considers changes of inflation rate and surveys effect of inflation on the total value of costs over time horizon. The inventory systems under inflationary conditions, it was assumed that the inflation rates are constant over the time horizon. But many economic factors may also affect the future changes of costs; such as changes in the world inflation rate, rate of investment, demand level, labour costs, cost of raw materials, rates of exchange, rate of unemployment, productivity level, tax, liquidity, etc. Therefore, the assumption of constant inflation rates is not valid in the real world situation.

The proposed model incorporates some realistic and practical features that are likely to be associated with the inventory of certain types of goods, such as (1) deterioration over time, (2) occurrence of inventory shortages and (3) finite production rate. From the inflation point of view, the developed model will be useful to the markovian and variable inflationary conditions as it gives a better and more general inventory control system. The numerical example and Sensitivity Analysis have been given to illustrate the theoretical results.

REFERENCES


**Appendix**

In this study, it is assumed that inflation rate has Markov property and then it will be predicted and estimated the stationary distribution of inflation rate in Country of Iran from 1991 to 2011.

\[ X_n; \text{inflation rate in the state } n \]

Now, by considering inflation rate of April 1991 to April 2011, will be determined transfer matrix.

**Assumptions:**
1. Each state has long been a month.
2. Using of the wholesale price of goods, rate inflation is calculated in each situation as compared to the previous situation.
3. Inflation rate in state \( n \) than state \( n-1 \) is computed as follows:

\[
(\frac{C_n - C_{n-1}}{C_{n-1}}) \times 100 = X_n
\]

\( C_n \) is price index in month \( n \), that is reported by the Central Bank. The base year (the year is the index is set equal to 100) is 2004.

**Annual growth index in year \( t \) than year \( t-1 \):**

\[
\text{Average in year } (t-1) = \text{Average in year } (t-1) / \text{Average in year } (t-1)
\]

**Table 1:** The total price index of consumer goods and services in urban areas (2004=100).

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Figures in Table 2 indicate the state space. Since elements of the state space must be integer, we convert the figures as follows:

Suppose $k$ is an integer.

If $X_n \in (k-0.5, k+0.5)$ then $X_n = k$

So, have:

$$-2, -1, 0, 1, 2, 3, 4, 5, 6 \} \{S =$$

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f_{ij} represents the number of observations from i to j.

determine transferMatrix using of frequencyMatrix:

This matrix calculated using of the frequency matrix and the following equation:

\[ P_{ij} = \frac{f_{ij}}{\sum_{j=-2}^{6} f_{ij}} \]

Transfer Matrix A:

\[
\begin{array}{cccccccc}
-2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\
\hline
-2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
-1 & 0 & 0.125 & 0.5 & 0.375 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.05 & 0.35 & 0.52 & 0.029 & 0 & 0.029 & 0 \\
1 & 0 & 0.03 & 0.15 & 0.64 & 0.13 & 0.019 & 0.009 & 0 \\
2 & 0 & 0 & 0.04 & 0.09 & 0.75 & 0 & 0 & 0 \\
3 & 0 & 0.125 & 0.5 & 0 & 0.5 & 0.125 & 0.125 & 0 \\
4 & 0.25 & 0 & 0.25 & 0 & 0.5 & 0 & 0 & 0 \\
5 & 0 & 0 & 0 & 0.33 & 0.33 & 0 & 0.33 & 0 \\
6 & 0 & 0 & 0 & 0.33 & 0.33 & 0 & 0 & 0 & 0.33 \\
\end{array}
\]

Calculation Stationary Distribution:

\[
[\{f(-2) , f(-1) , f(0) , f(1) , f(2) , f(3) , f(4) , f(5) , f(6)\}] \times A = [\{f(-2) , f(-1) , f(0) , f(1) , f(2) , f(3) , f(4) , f(5) , f(6)\}]
\]

So have:

f(-2) = 0.01, f(-1) = 0.06, f(0) = 0.3, f(1) = 0.15, f(2) = 0.3, f(3) = 0.07, f(4) = 0.04, f(5) = 0.03, f(6) = 0.04