Solving the Fuzzy Project Scheduling Problem Based on a Ranking Function

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Abstract: Quantitative methods such as the Critical Path Method (CPM) is being used extensively to analyze construction project networks. CPM, however, assume scheduling problem to be deterministic. In real-life operations, however, construction projects are normally executed under uncertain environments. With these uncertainties surrounding activities and resources data, it is unlikely that such deterministic methods can be used effectively. To circumvent such a limitation, two lines of research have been developed, these lines are probabilistic method and fuzzy set based methods. In this paper a new mechanism based on fuzzy theory is developed to use ranking function for solving the fuzzy project scheduling problem where fuzzy activities duration modeled by triangular fuzzy numbers. A new mechanism is also proposed to find fuzzy total slack (FTS), fuzzy free slack (FFS), fuzzy independent slack (FIS) for each activity in a fuzzy project network. To illustrate this mechanism a real life project was scheduled. The results that obtained from solving the fuzzy project scheduling problem give us some flexibility in planning, scheduling and controlling.

Key words: Fuzzy CPM approach; Fuzzy project scheduling; Fuzzy project network; Triangular fuzzy numbers; Ranking function.

INTRODUCTION

Critical Path Method (CPM) is one of the many network techniques which is widely used for planning, scheduling and controlling the projects. First of all it consists in the identification of the so-called critical paths, critical activities and critical events in the network, which is the project model, assuming the earliest possible completion time of the whole project. By chance certain values useful for the decision maker such as: events and activities slacks, the earliest and the latest moments of the start and finish of the particular activities, etc., are calculated. What is essential in the CPM method is that the activities duration times are deterministic and known. In practice, of course, this assumption not always can be fulfilled with the satisfying accuracy.

There are many literatures devoted to research about the fuzzy CPM theories and applications. Starting with the second part of the 1970s [see (Chanas and Radosinski,1976; Prade, 1979)] the other approach to the network project analysis, usually called the fuzzy PERT method or the fuzzy CPM, has been developed, in which it is suggested to use fuzzy numbers (sets) to model the activity times. The classic formulae used in the CPM as well as some dependencies true for a network with deterministic activity times are used where the common operations are replaced with the operations generalized on the fuzzy numbers. So modified formulae and dependencies are used to define many project characteristics, with the most important, such as degrees of criticality of the paths and activities, among them. Slyetsov and Tyshchuk (1999) noticed in the properties, which are equivalent in the deterministic case and lead to the unique identification of the critical path, cannot play such a role if they are automatically transferred to the fuzzy case. As result there are obtained different definitions of the fuzzy critical path which give different estimations of the degree of criticality for the same path in the network. Chanas and Zielinski (2001) analyzed critical patch in the network with fuzzy activity times. Dubois et al. (2003a) studied on latest starting times and floats in activity networks with ill-known durations. Dubois et al. (2003b) also planned fuzzy scheduling with incomplete knowledge. Slyetsov and Tyshchuk (2003) researched the fuzzy temporal characteristics of operations for project management based on the network models. Wang (1999) developed a fuzzy set approach to schedule product development projects with temporal information. Wang (2002) used a fuzzy project scheduling approach to minimize schedule risk for product development. Wang (2004) applied a genetic algorithm for solving the problem under the objective of maximizing the worst case scheduling. Nezhad et al. (2008) proposed a fuzzy number maximum operator approximation and its application in fuzzy shop scheduling. Ravi Shankar and Pardha Saradhi (2011) computed project characteristics such as earliest times, latest times and float times in terms of intervals. In this method, they introduced an approach to find latest times by removing negative intervals times which can be generated by other methods.
Fuzzy Set:

Fuzzy set theory was developed in the mid 70s by Zadeh in an effort to provide a basis to handle uncertainty that is non-statistical in nature. Basically, a fuzzy set is a class of objects (x) associated with their respective degrees of membership [μA(x)] within the set. The theory differs from the conventional crisp sets mainly in the degrees by which an object belongs to a set. In the crisp set theory, objects are either included or excluded from a set. In the fuzzy sets theory, on the other hand, objects are described in such a way to allow a gradual transition from being a member of a set to a nonmember. The following definitions of the fuzzy numbers and some basic arithmetic operations on it may be helpful (Zadeh, 1965).

Definition:

The characteristic function μA of a crisp set A ⊆ X assigns a value either 0 or 1 to each member in X. This function can be generalized to a function μA such that the value assigned to the element of the universal set X fall within a specified range i.e. μA:X → [0,1]. The assigned value indicate the membership grade of the element in the set A. The function μA is called the membership function and the set A = {x, μA(x)); x ∈ X} defined by μA (x) for each x ∈ X is called a fuzzy set (Dubois and Prade, 1980; Kaufmann and Gupta, 1985).

Definition:

A fuzzy number A is a Triangular-Fuzzy number denoted by (a, b, c) and it’s membership function μA(x) is given below (Dubois and Prade, 1980; Kaufmann and Gupta, 1985):  

\[
\mu_A(x) = \begin{cases} 
0 & \text{if } x < a \\
\frac{x-a}{b-a} & \text{if } a \leq x \leq b \\
\frac{c-x}{c-b} & \text{if } b \leq x \leq c \\
0 & \text{if } x > c 
\end{cases}
\]

Definition:

A triangular fuzzy number (a, b, c) is said to be non-negative fuzzy number iff a ≥ 0 (Dubois and Prade, 1980; Kaufmann and Gupta, 1985).

Arithmetic Operations:

Arithmetic operations between two triangular fuzzy numbers, defined on universal set of real numbers R, are reviewed (Dubois and Prade, 1980; Kaufmann and Gupta, 1985).

Let Ā1 = (a1, b1, c1) and Ā2 = (a2, b2, c2) be two triangular fuzzy numbers then 

(i) Ā1 ₀ Ā2 = (a1 + a2, b1 + b2, c1 + c2)  
(ii) Ā1 ₀ Ā2 = (a1 − a2, b1 − b2, c1 − c2)  
(iii) Ā1 = Ā2 iff a1 = a2, b1 = b2, c1 = c2

Ranking Functions:

An appropriate approach for comparing of fuzzy number is by use of ranking function (Zimmermann, 1969; Kaufmann and Gupta, 1985). A ranking function R: F(R) → R, where F(R) (a set of all fuzzy numbers defined on set of real numbers), maps each fuzzy number into a real number of F(R).

Let ā and ĉ be two fuzzy numbers in F(R), then

(i) ā ≥R č if and only if R(ā) ≥ R(č) 
(ii) ā >R č if and only if R(ā) > R(č) 
(iii) ā =R č if and only if R(ā) = R(č)

Let R be any linear ranking function. Then, ā ≥R č if and only if ā − č ≥R 0 and only if −č ≥R −ā.

if ā ≥R č and č ≥R ĉ, then ā + č ≥R č + ĉ.

Ranking Functions For Triangular Fuzzy Number:

For triangular fuzzy number Ā = (a, b, c) ranking function is given by R(Ā) = 1/4 ∫a b f(x) dx + sup a adx , where a is a –cut on Ā (Zimmermann, 1969; Kaufmann and Gupta, 1985). This reduces to R(Ā) = 1/4 (a + 2b + c).

Computing Fuzzy Time Values and Critical Path in a Fuzzy Project Network:

A fuzzy project network is an acyclic digraph, where the vertices represent events, and the direct edges represent the activities, to be performed in a project. Formally, A fuzzy project network is represented by N =
(V,A,T). Let \( V = \{ v_1, v_2, \ldots, v_n \} \) be a set of vertices, where \( v_i \) and \( v_n \) are the start and final events of the project, and each \( v_i \) belongs to some path from \( v_1 \) to \( v_n \). Let \( A \subseteq V \times V \) be the set of a directed edge \( a_{ij} = (v_i, v_j) \), that represents the activities to be performed in the project. Activity \( a_{ij} \) is then represented by one, and only one, arrow with a tail event \( v_i \), and a head event \( v_j \). For each activity \( a_{ij} \), a fuzzy number \( t_{ij} \in T \) is defined, where \( t_{ij} \) is the fuzzy time required for the completion of \( a_{ij} \). A critical path is a longest path from \( v_1 \) to \( v_n \), and an activity \( a_{ij} \) on a critical path is called a critical activity. Let \( ES_i \) and \( LS_i \) be the fuzzy earliest start of event \( i \), and the fuzzy latest start of event \( i \), respectively. Let \( EF_j \) and \( LF_j \) be the fuzzy earliest finish of event \( j \), and the fuzzy latest finish of event \( j \), respectively (Buckley and Feuring, 2000; Chen, 2007). Let \( D_i = \{ i / i \in V \text{ and } a_{ij} \in A \} \) be a set of events obtained from event \( j \in V \) and \( i < j \). If the forward pass calculations of CPM are entirely done in a fuzzy project network, the fuzzy earliest start of event \( i \) (\( ES_i \)) and earliest finish (\( EF_i \)) may be defined by the equations:

\[
ES_i = 0 \quad \text{(the fuzzy earliest time of the first event is traditionally set as zero)} \quad (1)
\]

\[
EF_j = \max \{ \Re (ES_i \oplus t_{ij}) \} \quad (2)
\]

Similarly, let \( H_i = \{ j / j \in V \text{ and } a_{ji} \in A \} \) be a set of events obtained from event \( i \in V \) and \( i < j \). If the backward pass calculations of CPM are entirely done in a fuzzy project network, the fuzzy latest finish of event \( j \) (\( LF_j \)) and latest start (\( LS_i \)) can be defined by the equations:

\[
LF_{\text{end}} = EF_{\text{end}} \quad \text{(Last activity in the project therein \( LF_j \) equal \( EF_j \))} \quad (3)
\]

\[
LS_i = \min \{ \Re (LF_j \ominus t_{ij}) \} \quad (4)
\]

**Fuzzy Total Slack (FTS), Fuzzy Free Slack (FFS), Fuzzy Independent Slack (FIS):**

Slack can be divided into three categories (Stevens, 1990; Nicholas, 2004; O’Brien and Plotnick, 2006):

**Fuzzy Total Slack (FTS) Calculations**

The term total slack (\( TS \)) refers to the difference between when an activity may start and must start, the number of time units that the activity may slip without impact to timely completion of the project. Similarly, the attribute measuring the difference between when an activity may finish and must finish is also known as total slack (or float) and is also expressed as \( TS \).

In the fuzzy environment, the attribute of fuzzy total slack of event \( i \) (\( FTS_i \)) may be defined by the equations:

\[
FTS_i = LF_i \ominus LS_i \quad (5)
\]

Or,

\[
FTS_i = EF_i \ominus ES_i \quad (6)
\]

**Fuzzy Free Slack (FFS) Calculations:**

The attribute measuring the number of time units that an activity may slip without impact to another activity that may follow (or successor activities) is known as free slack and is expressed as \( FS \).

The activity attributes of free slack is defined as the difference between the earliest of the early starts of all successors to an activity and the calculated early finish of that activity. In the fuzzy environment, the fuzzy free slack of event \( i \) (\( FFS_i \)) may be defined by the equation:

\[
FFS_i = \max \{ \Re (ES_{SUCC} \ominus LF_i) \} \quad (7)
\]

**Fuzzy Independent Slack (FIS) Calculations:**

The attribute measuring the number of time units that an activity may be deliberately deferred without reducing the ability to defer any other activity of the logic network is known as independent slack and is expressed as \( IF \).

The activity attribute of independent slack is defined as the difference between the earliest of the early starts of all successors to an activity and the latest of the early finishes of all predecessors of that activity. In the fuzzy environment, the fuzzy independent float of event \( i \) (\( FIS_i \)) may be defined by the equation:

\[
FIS_i = (\max \{ \Re (ES_{SUCC} \ominus LF_i) \} \max \{ \Re (EF_{PRED} \ominus t_{ij}) \}) \quad (8)
\]
Algorithm to Find Fuzzy Critical Path and Fuzzy Slacks in a Fuzzy Project Network:

In this section, a fuzzy critical path and fuzzy slacks algorithm is utilized to find a critical path and all kinds of the fuzzy slacks (fuzzy total slack (FTS), fuzzy free slack (FFS) and fuzzy independent slack (FIS)) of project network in a fuzzy environment. The description of the algorithm is presented in the following.

1. Define the fuzzy project network and all of its significant activities or tasks. The fuzzy project network (made up of several tasks) should have only a fuzzy single start activity and a fuzzy single finish activity.
2. Develop the relationships among the activities. Decide which activities must precede and which must follow others.
3. Draw the "fuzzy Project Network" connecting all the activities. Each activity should have unique event numbers. Dummy arrows are used where required to avoid giving the same numbering to two activities.
4. Assign fuzzy time estimates to each activity.
5. Compute the fuzzy longest time path through the network by using appropriate ranking formula to:
   a. Find the fuzzy earliest finish of any activity j (\(\overline{EF}_j\)) (i.e. select the fuzzy earliest finish of any activity j (\(\overline{EF}_j\)) whose the (\(\overline{EF}_j\)) has the largest rank) as in the equations (1–2)
   b. Find the fuzzy latest star (\(\overline{LS}_i\)) of any activity i (i.e. select the fuzzy latest star (\(\overline{LS}_i\)) whose the (\(\overline{LS}_i\)) has the smallest rank) as in the equation (3–4):
6. Compute the fuzzy total slack of event i (\(\overline{FTS}_i\)), the fuzzy free slack of event i (\(\overline{FFS}_i\)) and the fuzzy independent slack of event i (\(\overline{FIS}_i\)) from the equations (5,6,7 and 8) respectively.
7. Use the fuzzy project network to help plan, schedule and monitor and control the project in the fuzzy environment.

Numerical Data of Al-SAMA Construction Project in a Fuzzy Environment:

The fuzzy data of Al-SAMA construction project is summarized in Table 1 in which there are 12 activities, where: \(A_i = (A_1, A_2, \ldots, A_{12})\).

<table>
<thead>
<tr>
<th>Activities Name</th>
<th>Activity Predecessor</th>
<th>Fuzzy Normal Time (T_n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A_1)</td>
<td></td>
<td>30,40,50</td>
</tr>
<tr>
<td>(A_2)</td>
<td>(A_1)</td>
<td>20,30,40</td>
</tr>
<tr>
<td>(A_3)</td>
<td>(A_2)</td>
<td>30,40,50</td>
</tr>
<tr>
<td>(A_4)</td>
<td>(A_2)</td>
<td>20,30,40</td>
</tr>
<tr>
<td>(A_5)</td>
<td>(A_2)</td>
<td>40,50,60</td>
</tr>
<tr>
<td>(A_6)</td>
<td>(A_5), (A_3)</td>
<td>30,40,50</td>
</tr>
<tr>
<td>(A_7)</td>
<td>(A_4)</td>
<td>20,30,40</td>
</tr>
<tr>
<td>(A_8)</td>
<td>(A_7)</td>
<td>50,60,70</td>
</tr>
<tr>
<td>(A_9)</td>
<td>(A_6)</td>
<td>60,70,80</td>
</tr>
<tr>
<td>(A_{10})</td>
<td>(A_8), (A_9)</td>
<td>30,40,50</td>
</tr>
<tr>
<td>(A_{11})</td>
<td>(A_{10}), (A_{11})</td>
<td>80,90,100</td>
</tr>
<tr>
<td>(A_{12})</td>
<td></td>
<td>90,100,110</td>
</tr>
</tbody>
</table>

Fig. 1: Fuzzy of Al-SAMA construction project network.
RESULTS AND DISCUSSION

Fuzzy Critical Path (FCP) Computations for the Project (Forward Pass):

From equations (1–2), we can calculate the fuzzy earliest starting (ES) and earliest finishing (EF), to make a Forward Pass through the fuzzy project network.

<table>
<thead>
<tr>
<th>Activity</th>
<th>ES</th>
<th>EF</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>0</td>
<td>50</td>
</tr>
<tr>
<td>A2</td>
<td>50</td>
<td>90</td>
</tr>
<tr>
<td>A3</td>
<td>30</td>
<td>70</td>
</tr>
<tr>
<td>A4</td>
<td>20</td>
<td>60</td>
</tr>
<tr>
<td>A5</td>
<td>10</td>
<td>50</td>
</tr>
<tr>
<td>A6</td>
<td>0</td>
<td>50</td>
</tr>
<tr>
<td>A7</td>
<td>50</td>
<td>90</td>
</tr>
<tr>
<td>A8</td>
<td>60</td>
<td>100</td>
</tr>
<tr>
<td>A9</td>
<td>10</td>
<td>50</td>
</tr>
<tr>
<td>A10</td>
<td>30</td>
<td>70</td>
</tr>
</tbody>
</table>

ES \text{A1} = 0, ES \text{A2} = 0, ES \text{A3} = 30, ES \text{A4} = 20, ES \text{A5} = 10, ES \text{A6} = 0, ES \text{A7} = 50, ES \text{A8} = 60, ES \text{A9} = 10, ES \text{A10} = 30

EF \text{A1} = 50, EF \text{A2} = 90, EF \text{A3} = 70, EF \text{A4} = 60, EF \text{A5} = 50, EF \text{A6} = 90, EF \text{A7} = 100, EF \text{A8} = 100, EF \text{A9} = 50, EF \text{A10} = 70

Fuzzy Critical Path (FCPM) Computations for the Project (Backward Pass):

From equations (3–4) we can calculate the fuzzy latest finish (LF) and the latest start (LS), to make a Backward Pass through the fuzzy project network.

<table>
<thead>
<tr>
<th>Activity</th>
<th>LS</th>
<th>LF</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>10</td>
<td>30</td>
</tr>
<tr>
<td>A2</td>
<td>70</td>
<td>90</td>
</tr>
<tr>
<td>A3</td>
<td>50</td>
<td>90</td>
</tr>
<tr>
<td>A4</td>
<td>40</td>
<td>60</td>
</tr>
<tr>
<td>A5</td>
<td>30</td>
<td>50</td>
</tr>
<tr>
<td>A6</td>
<td>0</td>
<td>50</td>
</tr>
<tr>
<td>A7</td>
<td>90</td>
<td>100</td>
</tr>
<tr>
<td>A8</td>
<td>80</td>
<td>100</td>
</tr>
<tr>
<td>A9</td>
<td>40</td>
<td>50</td>
</tr>
<tr>
<td>A10</td>
<td>10</td>
<td>50</td>
</tr>
</tbody>
</table>

LS \text{A1} = 10, LS \text{A2} = 30, LS \text{A3} = 50, LS \text{A4} = 40, LS \text{A5} = 30, LS \text{A6} = 0, LS \text{A7} = 90, LS \text{A8} = 80, LS \text{A9} = 40, LS \text{A10} = 10

LF \text{A1} = 30, LF \text{A2} = 90, LF \text{A3} = 70, LF \text{A4} = 60, LF \text{A5} = 50, LF \text{A6} = 90, LF \text{A7} = 100, LF \text{A8} = 100, LF \text{A9} = 50, LF \text{A10} = 70

Fuzzy earliest start (ES) and latest start (LS) of all activities:

ES \text{A1} = 0, ES \text{A2} = 0, ES \text{A3} = 30, ES \text{A4} = 20, ES \text{A5} = 10, ES \text{A6} = 0, ES \text{A7} = 50, ES \text{A8} = 60, ES \text{A9} = 10, ES \text{A10} = 30

LS \text{A1} = 10, LS \text{A2} = 50, LS \text{A3} = 30, LS \text{A4} = 40, LS \text{A5} = 30, LS \text{A6} = 0, LS \text{A7} = 90, LS \text{A8} = 80, LS \text{A9} = 40, LS \text{A10} = 10

Fuzzy critical activities:

ES \text{A11} = 0, ES \text{A12} = 50, ES \text{A13} = 50, ES \text{A14} = 20, ES \text{A15} = 10, ES \text{A16} = 0, ES \text{A17} = 50, ES \text{A18} = 60, ES \text{A19} = 10, ES \text{A20} = 30

EF \text{A11} = 50, EF \text{A12} = 120, EF \text{A13} = 70, EF \text{A14} = 60, EF \text{A15} = 50, EF \text{A16} = 90, EF \text{A17} = 100, EF \text{A18} = 100, EF \text{A19} = 50, EF \text{A20} = 70

LS \text{A11} = 10, LS \text{A12} = 50, LS \text{A13} = 30, LS \text{A14} = 40, LS \text{A15} = 30, LS \text{A16} = 0, LS \text{A17} = 90, LS \text{A18} = 80, LS \text{A19} = 40, LS \text{A20} = 10

LF \text{A11} = 30, LF \text{A12} = 90, LF \text{A13} = 70, LF \text{A14} = 60, LF \text{A15} = 50, LF \text{A16} = 90, LF \text{A17} = 100, LF \text{A18} = 100, LF \text{A19} = 50, LF \text{A20} = 70

Fuzzy critical path:

A11→A12→A13→A14→A15→A16→A17→A18→A19→A20

10-11 = (300, 370, 440), 11-12 = (300, 370, 440), 12-13 = (300, 370, 440), 13-14 = (300, 370, 440), 14-15 = (300, 370, 440), 15-16 = (300, 370, 440), 16-17 = (300, 370, 440), 17-18 = (300, 370, 440), 18-19 = (300, 370, 440), 19-20 = (300, 370, 440)

Fuzzy critical path cost:

J = 300, 370, 440

\[ LF_{A_5} = LF_{A_8} = LF_{A_9} = LF_{A_{10}} = (180,230,280); \]
\[ LF_{A_2} = \min (LF_{A_1}, LF_{A_3}, LF_{A_4}) = (30,40,50). \]
\[ \min (\min (LF_{A_1}, LF_{A_2}), LF_{A_3}, LF_{A_4}, LF_{A_5}, LF_{A_6}, LF_{A_7}, LF_{A_8}) = (0,0,0). \]

The fuzzy earliest time schedule is completed at (300,370,440) weeks. This makes sense, since the project is finished as soon as activity \( A_{12} \) is completed and the earliest finish of activity \( A_{12} \) is (300,370,440) weeks later than for activity \( A_{11} \) (170,210,250) (see Fig.1).

As long as activity \( A_{12} \) stays on fuzzy schedule, the project will finish at (300,370,440) weeks if any delays in fuzzy starting activity \( A_{11} \) (which may be due to preceding activities taking longer than expected) and in performing activity \( A_{11} \) do not accumulate more than (40,60,80) weeks. Table 2 shows the fuzzy total slack for each of the activities. Note that some of the activities have zero fuzzy total slack, indicating that any delays in these activities will delay fuzzy project completion. This is how a fuzzy critical path method identifies the fuzzy critical path(s).

Fuzzy total slack for activity \( A_{11} \) is (40,60,80). This indicates that activity \( A_{11} \) can be delayed up to (40,60,80) weeks beyond the fuzzy earliest time schedule without delaying the fuzzy completion of the project at (300,370,440) weeks. However, the fuzzy project network (see Table 2). The activities on this path will be performed sequentially without interruption. Otherwise, this would not be the fuzzy longest path. Therefore, the fuzzy time required to reach the node \( A_{11} \) equals the length of this path. Furthermore, all shorter paths will reach the node \( A_{11} \) no later than this. This fuzzy longest path is called the fuzzy critical path. (If more than one fuzzy path tie for the longest, they all are fuzzy critical paths).

Fuzzy Free Slack (FFS):

We can compute fuzzy free slack (FFS) for each activity in the fuzzy project network from equation (7):

\[ FFS_{A_1} = \min (\min (LF_{A_1}, LF_{A_2}), LF_{A_3}, LF_{A_4}, LF_{A_5}, LF_{A_6}, LF_{A_7}, LF_{A_8}) = (0,0,0). \]
that are not dependent on another chain. In some cases, the fuzzy free slack will equal the fuzzy total slack value than the fuzzy early finish time of the series of activities under a study.

parallel path with the lowest fuzzy total slack and also for any series of initial activities with fuzzy early finishes zero fuzzy free slack value. Fuzzy free slack is, therefore, deceptive because it shows a zero value for the time at event 9, which is established by the fuzzy longer path A1

start times of any successor activities.

of fabricated materials that will not delay the fuzzy early start of a subsequent erection or installation activity. Instead, each of these preceding activities has a calculated fuzzy free slack of zero because their successors each have information relating to the submittal, approval, and fabrication activities preceding the delivery activity. Instead, it can be delayed (40,40,40) weeks without affecting the fuzzy early start of any of its successors.

Non-zero fuzzy free slack can only exist where an activity has more than one predecessor. It is the consequence of the merger of multiple fuzzy paths of logic.

For a string of more than one activities, such as A1, A2, A3, A4, A6, A7 in which the fuzzy early finish for the j event is determined only by the fuzzy early start figure coming out of the junction point, the formula necessarily produces a fuzzy free slack of 0. It is only when the string of activities joins another junction event, at which a new fuzzy early start figure is determined by the fuzzy longest path leading into the new juncture, that the fuzzy free slack formula produces a non-zero number. This number is produced because one or more other paths coming into the junction point establish a fuzzy early start for that key juncture, which is greater than the fuzzy early finish time of the series of activities under a study.

Fuzzy free slack is really a comparative value of slacks in parallel paths. the fuzzy free slack is 0 on the activity A1, which initiates the path A1→A2→A3→A4→A1 but it is (10,10,10) on the fourth activity because that is the last activity before a junction point.

The fuzzy free slack for activity A8, in path A1→A2→A3→A4→A5→A6→A8 is dependent on the fuzzy early event time at event 9, which is established by the fuzzy longer path A1→A2→A3→A4→A5→A6 and, therefore, has a non-zero fuzzy free slack value. Fuzzy free slack is, therefore, deceptive because it shows a zero value for the parallel path with the lowest fuzzy total slack and also for any series of initial activities with fuzzy early finishes that are not dependent on another chain. In some cases, the fuzzy free slack will equal the fuzzy total slack value where a path of fuzzy non-critical activities re-enter a fuzzy critical path string of activities. It may be less than total float, but it will never be more.

Fuzzy free slack, is included in a report, it is usually to note the amount of slippage permitted for delivery of fabricated materials that will not delay the fuzzy early start of a subsequent erection or installation activity. But as noted previously, the calculated attribute is misleading as project personnel would desire similar information relating to the submittal, approval, and fabrication activities preceding the delivery activity. Instead, each of these preceding activities has a calculated fuzzy free slack of zero because their successors each have but one predecessor.

Non-zero fuzzy free slack can only exist where an activity has more than one predecessor. It is the consequence of the merger of multiple fuzzy paths of logic.

For fuzzy project network, in Fig.1 Activity A7 has fuzzy total slack time of (40,40,40) weeks but fuzzy free slack of (0,0,0) weeks. It has zero fuzzy free slack because any delay in it will also delay the fuzzy start of activities A8 (and if A8, then A10 also). Activity A8, on the other hand, has fuzzy free slack of (40,40,40) weeks. It can be delayed (40,40,40) weeks without affecting the fuzzy early start of any of its successors.

The importance of knowing fuzzy free slack is that managers can quickly identify activities where slippage has consequence for other activities. When an activity has no fuzzy free slack, any slippage also will cause at least one other activity to slip. If, for a numerical real life project, Activity A8 slips, so will A11 and A30 and work teams in the latter two should be notified. As with fuzzy total slack, the computed fuzzy free slack for an activity assumes the activity will begin at its ES time. Thus, the fuzzy free slack for Activity A8 is (40,40,40) weeks only as long as Activity A8, its predecessor, is completed at its EF time. If any fuzzy slack is used up by ActivityA5, then the fuzzy free slack of Activity A8 will be reduced by that amount.
Fuzzy Independent Float (FIS):

We can compute Fuzzy Independent Slack (FIS) for each activity in the project from equation (8).

FIS_{A12}=Earliest or Min\{\text{ES} (ES_{Succ})\} \oplus \text{Latest or max} \{\text{EF} (EF_{Pred})\} \oplus \text{F}_{12}

FIS_{A12}=Earliest or Min\{\text{ES} (ES_{Succ})\} \oplus \text{Latest or max} \{\text{EF} (EF_{A12})\} \oplus \text{F}_{10-11}

FIS_{A12}=(300,370,440) \oplus \text{Latest or max} \{90,100,110\} \oplus (90,100,110)

FIS_{A12}=(300,370,440) \oplus (210,270,330) \oplus (90,100,110)

FIS_{A12}=(90,100,110) \oplus (90,100,110)=(0,0)

FIS_{A11}=Earliest or Min\{\text{ES} (ES_{Succ})\} \oplus \text{Latest or max} \{\text{EF} (EF_{Pred})\} \oplus \text{F}_{A11}

FIS_{A11}=Earliest or Min\{\text{ES} (ES_{A12})\} \oplus \text{Latest or max} \{\text{EF} (EF_{A6}, EF_{Adum})\} \oplus \text{F}_{6-10}

FIS_{A11}=(210,270,330) \oplus \text{Latest or max} \{90,120,150, 80,110,140\} \oplus \text{F}_{6-10}

FIS_{A11}=(210,270,330) \oplus \text{Latest or max} \{90,120,150\} \oplus (80,90,100)

FIS_{A11}=(210,270,330) \oplus (90,120,150) \oplus (80,90,100)

FIS_{A11}=(120,150,180) \oplus (80,90,100) = (40,60,80)

FIS_{A10}=Earliest or Min\{\text{ES} (ES_{Succ})\} \oplus \text{Latest or max} \{\text{EF} (EF_{Pred})\} \oplus \text{F}_{A10}

FIS_{A10}=Earliest or Min\{\text{ES} (ES_{A12})\} \oplus \text{Latest or max} \{\text{EF} (EF_{A6}, EF_{A5})\} \oplus \text{F}_{6-10}

FIS_{A10}=(210,270,330) \oplus (180,230,280) \oplus (30,40,50)

FIS_{A10}=(30,40,50) \oplus (30,40,50)=(0,0)

FIS_{A9}=Earliest or Min\{\text{ES} (ES_{Succ})\} \oplus \text{Latest or max} \{\text{EF} (EF_{Pred})\} \oplus \text{F}_{A9}

FIS_{A9}=(180,230,280) \oplus (120,160,200) \oplus (60,70,80)=(0,0)

FIS_{A8}=Earliest or Min\{\text{ES} (ES_{Succ})\} \oplus \text{Latest or max} \{\text{EF} (EF_{Pred})\} \oplus \text{F}_{A8}

FIS_{A8}=(180,230,280) \oplus (90,130,170) \oplus (50,60,70)

FIS_{A8}=(90,110,140) \oplus (50,60,70) = (40,40,40)

FIS_{A7}=Earliest or Min\{\text{ES} (ES_{Succ})\} \oplus \text{Latest or max} \{\text{EF} (EF_{Pred})\} \oplus \text{F}_{A7}

FIS_{A7}=(90,130,170) \oplus (70,100,130) \oplus (20,30,40)=(0,0)

FIS_{A6}=Earliest or Min\{\text{ES} (ES_{Succ})\} \oplus \text{Latest or max} \{\text{EF} (EF_{A5}, EF_{dum})\} \oplus \text{F}_{A6}

FIS_{A6}=(120,160,200) \oplus \text{Latest or max} \{\text{EF} (EF_{A5}, EF_{dum})\} \oplus \text{F}_{A6}

FIS_{A6}=(120,160,200) \oplus (90,120,150) \oplus (30,40,50)=(0,0)

FIS_{A5}=Earliest or Min\{\text{ES} (ES_{Succ})\} \oplus \text{Latest or max} \{\text{EF} (EF_{Pred})\} \oplus \text{F}_{A5}

FIS_{A5}=Earliest or Min\{\text{ES} (ES_{Succ})\} \oplus \text{Latest or max} \{\text{EF} (EF_{A2}, A_{dum})\} \oplus \text{F}_{A5}

FIS_{A5}=(90,120,150) \oplus (50,70,90) \oplus (40,50,60)

FIS_{A5}=(50,70,90) \oplus (40,50,60)=(0,0)

FIS_{Adum}=Earliest or Min\{\text{ES} (ES_{Succ})\} \oplus \text{Latest or max} \{\text{EF} (EF_{Pred})\} \oplus \text{F}_{Adum}

FIS_{Adum}=(90,120,150) \oplus (80,110,140) \oplus (10,10,10)

FIS_{A4}=Earliest or Min\{\text{ES} (ES_{Succ})\} \oplus \text{Latest or max} \{\text{EF} (EF_{Pred})\} \oplus \text{F}_{A4}

FIS_{A4}=(70,100,130) \oplus (50,70,90) \oplus (20,30,40)=(0,0)

FIS_{A3}=Earliest or Min\{\text{ES} (ES_{Succ})\} \oplus \text{Latest or max} \{\text{EF} (EF_{Pred})\} \oplus \text{F}_{A3}

FIS_{A3}=(80,110,140) \oplus (50,70,90) \oplus (30,40,50)=(0,0)

FIS_{A2}=Earliest or Min\{\text{ES} (ES_{Succ})\} \oplus \text{Latest or max} \{\text{EF} (EF_{Pred})\} \oplus \text{F}_{A2}

FIS_{A2}=(50,70,90) \oplus (30,40,50) \oplus (20,30,40)=(0,0)

FIS_{A1}=Earliest or Min\{\text{ES} (ES_{Succ})\} \oplus \text{Latest or max} \{\text{EF} (EF_{Pred})\} \oplus \text{F}_{A1}

FIS_{A1}=(30,40,50) \oplus (0,0,0) \oplus (30,40,50)=(0,0)

Fuzzy independent slack represents the attribute that an activity start or finish may be deferred without reducing the ability or fuzzy slack of any other activity’s start or finish to be deferred. To some extent, it is a more reliable indicator of when an activity is “needed” than fuzzy free slack. However, as noted previously, when fuzzy independent slack was first defined, there did not appear a practical use for its calculation. In Fig.1, the only three activities to have fuzzy independent slack would be Activities (A_{dum}, A_{8}, A_{11}). So fuzzy independent slack of the three activities are [{10,10,10}, (40,40,40), (40,60,80)] respectively.

Conclusions:

In this paper a new method based on the fuzzy theory has been developed to solve the project scheduling problem under the fuzzy environment. In this method, duration of activities are considered as triangular fuzzy numbers. A new method proposed to use ranking function for triangular fuzzy numbers to find fuzzy critical path in a fuzzy project network by calculating fuzzy earliest starting (ES), earliest finishing (EF), fuzzy latest finish (LF) and the latest start (LS). A new method is also proposed to find fuzzy total slack (FTS), fuzzy
free slack (FFS), fuzzy independent slack (FIS) for each activity in a fuzzy project network. Through a numerical real life project, calculations involved in this method have been illustrated. The method proposed in this paper has shown more effective in schedule and monitor and control the project in the fuzzy environment.

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<th>Activity Name</th>
<th>On Fuzzy Critical Path</th>
<th>Fuzzy Normal time/fn</th>
<th>ES</th>
<th>LS</th>
<th>EF</th>
<th>LF</th>
<th>FTS</th>
<th>FFS</th>
<th>FIS</th>
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REFERENCES


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