Australian Journal of Basic and Applied Sciences, 7(4): 457-464, 2013
ISSN 1991-8178

Design of Reduced Order Digital Filters Using Pso and Routh Schemes

[1]P. Gomathi, [2]T. Manigandan

[1]Research Scholar, PA College of Engineering and Technology, Pollachi, India.
[2]Principal PA College of Engineering and Technology, Pollachi, India.

Abstract: A new scheme for the design of complex higher order digital filter is introduced in this paper. When the order of the filter is increased, the number of filter coefficients to be designed is also increased which leads to the higher complexity in design and this problem is simplified by using the proposed scheme of model order which uses the advantages of Routh method and Particle Swarm Optimisation(PSO). The reduction is done based on the minimization of cumulative error index between original and reduced models. The reduction scheme always produces a stable reduced model if the higher order system is stable. The illustrative examples are given to support the proposed scheme.

Key words: Digital Filter, Filter Coefficients, Model order reduction, Particle Swarm Optimisation(PSO), Routh array, Cumulative error index.

INTRODUCTION

The Infinite Impulse Response (IIR) filter is a discrete time system that is designed to pass spectral content of the input signal in a specified band of frequencies by considering all the infinite samples of impulse response. The design of a higher order filter will be definitely a highly complicated task and it takes a longer tile for the design of larger number of filter coefficients. Since the mathematical models obtained for a real time system results in a higher order model, the order reduction is applied in almost all fields of engineering which uses reduced models for the simulation and design of complex systems. Model order reduction is an area of research and many techniques have been proposed.

The Routh Approximation method introduced by Hutton and Friedland, 1975; has proven to be the most successful tools for getting the reduced order models due to its simplicity in computation and efficiency. Novel scheme is proposed to obtain a second order reduced model for stable LTICS is presented (Manigandan et al., 2005) which provides good approximation and preserves the stability of the higher order system. Order reduction of linear-time invariant systems is illustrated by Panda et al., 2009; employing two methods, one using the advantages of Routh approximation and other by particle Swarm optimization. A combined method making use of advantages of the Mihalov criterion and factor division was proposed in Prasad et al., 2003. An error minimization technique (Mittal et al., 2004) based model reduction is discussed.

Particle swarm optimization (PSO) is a population based stochastic optimization technique developed by Eberhart and Kennedy, 2001; inspired by social behavior of bird flocking or fish schooling. The PSO method is a member of the wide category of Swarm Intelligence methods [Kennady and Eberhart, 1995]. Model order reduction using Mihailav criterion and Routh Approximation is given by Karanthikumar et al, 2011. The reduction method on three point expansion scheme is presented by Palaniswami and Sivanandan, the effectiveness is shown through illustrations. Mittal et al, 2009 also proposed a routh based model reduction method with the effectiveness of time moments and Markov parameters.

Many authors uses the method of model reduction for the design of filters(Sreeram et al, 1992, Ramesh et al., 2009, Nalin Harischandra, 2011, Manish Kansal, 2011) and proved that the complexity of the design is reduced. In this paper a simplified approach for the design of reduced order filter is proposed by using Routh array (Nagrath et al, 2006) and Particle Swarm Optimisation

Problem Definition:

Consider an n^{th} order stable linear time invariant discrete filter described by the transfer function (1)

$$G(z) = \frac{N(z)}{D(z)} = \frac{B_0 z^m + B_1 z^{m-1} + B_2 z^{m-2} + B_3 z^{m-3} + \dots\dots\dots + B_m}{A_0 z^n + A_1 z^{n-1} + A_2 z^{n-2} + A_3 z^{n-3} + \dots\dots\dots + A_n} \tag{1}$$

where, A_i $(0 \le i \le n)$ and B_i $(0 \le i \le m)$ are scalar constants and $m \le n$.
The corresponding reduced second order filter model is of the form (2)

$$R_2(z) = \frac{D_1 z + D_0}{E_2 z^2 + E_1 z + E_0} \tag{2}$$

Corresponding Author: P. Gomathi, Research Scholar, PA College of Engineering and Technology, Pollachi, India.

The objective is to find a reduced second order filter model in the form of equation (2) for the original system described by equation (1), such that the reduced order filter model retains the important characteristics of the original higher order filter. The second order filter is stable if the higher order filter is stable.

Performance Specifications Selected for the Comparison:

The performance of filters defined in (1) and (2) are compared using the following responses

- Magnitude response
- Phase response
- Group delay
- Phase delay
- Pole zero plot

Proposed Method of Model Order Reduction:

The original higher order system is reduced into second order using the algorithm explained by Gomathi and Manigandan, 2011; the pseudo code for the reduction is given below.

Get the order n for the discrete higher order transfer function

If (Order of the system >2)

{

1. Convert LTIDS into LTICS
2. Construct Routh stability array for the numerator of the higher order system
3. Select e_0 as constant of denominator, d_0 as constant of numerator and d_1 as s row from Routh array
4. Construct numerator polynomial of reduced system as d_1s+d_0
5. Equate (1) & (2) and cross multiply
6. Equate the coefficients on both sides form (n+2) equations.
7. By substituting the values of e_0, d_0 and d_1, solve the equations and get the values of e_2 and e_1.
8. Optimise the values using PSO
9. Construct transfer function of reduced model by substituting optimized values in (2)
10. Convert LTICS into LTIDS
11. Verify the characteristics of original higher order filter and reduced filter by applying step input.

}

1. Design the filter coefficients using the reduced model R(z)
2. Verify the effectiveness of the reduced model by comparing its responses with the original filter.
3. Represent the filter in direct form structure.

Numerical Examples:

Numerical Example 1:

Consider digital IIR filter with transfer function defined by Ramesh et al.(2009)

$$H(z)=\frac{1+2z^{-1}+2z^{-2}+z^{-3}}{1+0.5416z^{-1}+0.625z^{-2}+0.333z^{-3}} \tag{3}$$

(3) is rewritten as in (4)

$$H(z)=\frac{z^3+2z^2+2z+1}{z^3+0.54167z^2+0.625z+0.333} \tag{4}$$

(4) is transformed into continuous domain using bilinear transformation as in (5)

$$H(s)=\frac{5.331s^2+63.97}{s^3+7.55s^2+10.22s+26.65} \tag{5}$$

The reduced model (6) using proposed method is obtained as

$$R(s)=\frac{1.5322s+63.97}{7.8099s^2+10.252s+26.65} \tag{6}$$

The discrete equivalent of (6) is obtained using inverse bilinear transformation as in (7)

$$R(z)=\frac{0.8551z^2+1.632z+0.7769}{z^2-0.1171z+0.4769} \tag{7}$$

The comparison of step responses of original and reduced systems is given in Figure 1.

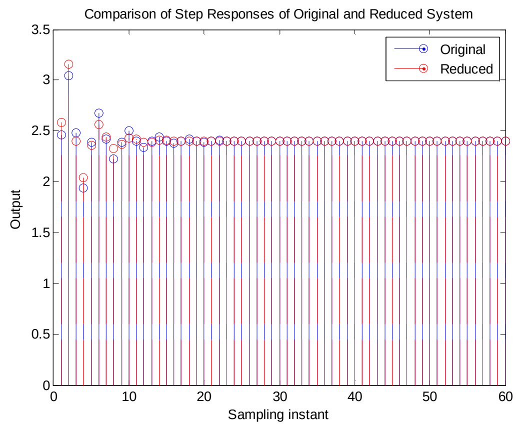

Fig. 1: Comparison of step responses of original and reduced systems.

The performance comparisons are given in Figure 2 to Figure 6.

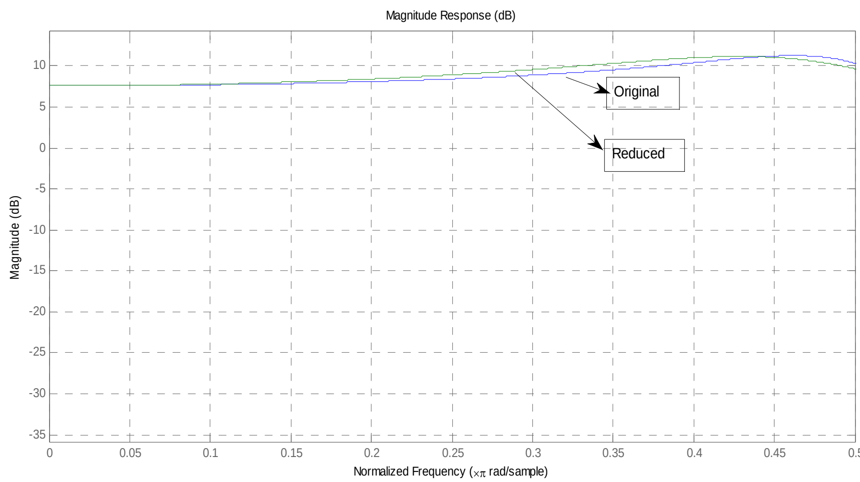

Fig. 2: Comparison of magnitude responses of original and reduced in example 1.

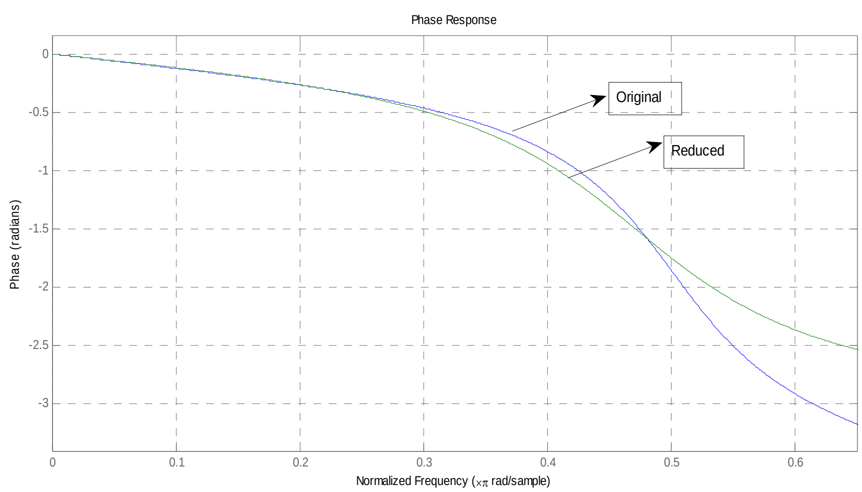

Fig. 3: Comparison of phase responses of original and reduced in example 6.1.

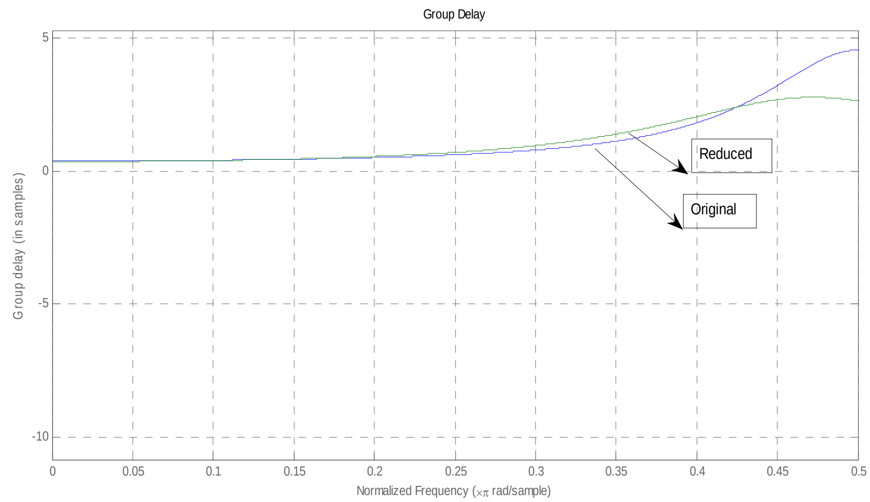

Fig. 4: Comparison of Group delay of original and reduced in example 1.

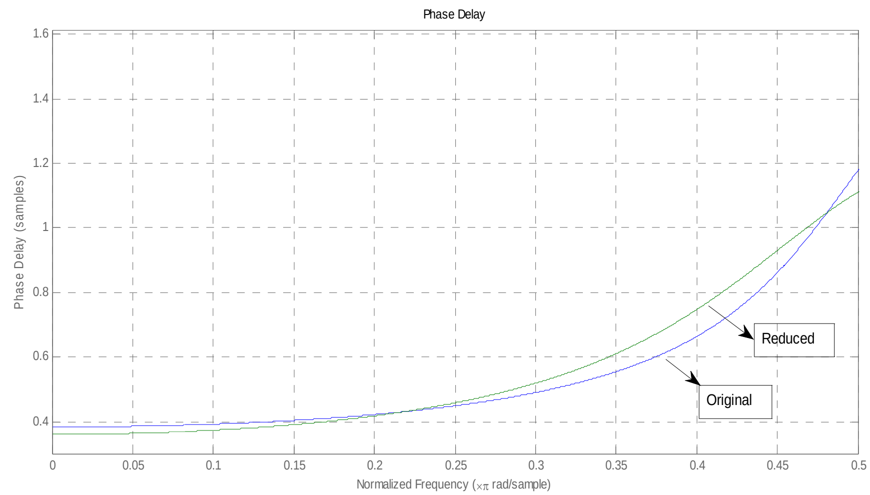

Fig. 5: Comparison of phase delay of original and reduced in example 1.

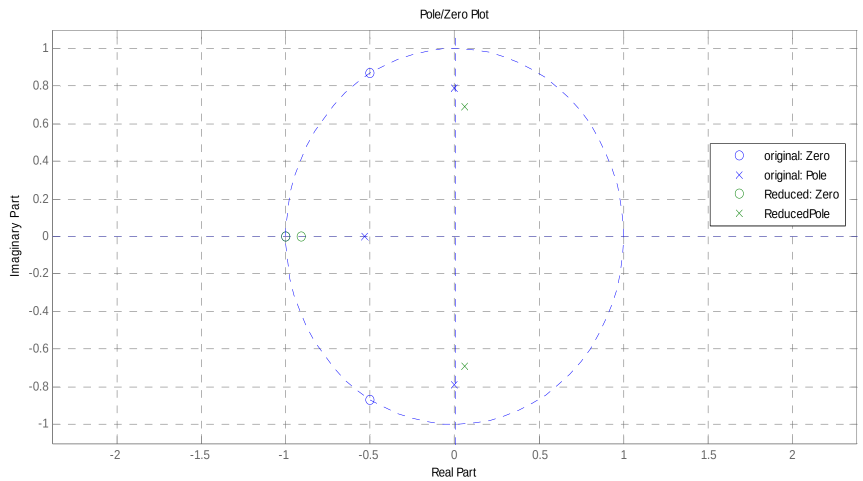

Fig. 6: Comparison of pole zero plots of original and reduced in example 1.

The different parameters of the original and reduced order filter re compared in Table 1.

Table 1: Comparisons of the parameters of original and reduced order filter.

Parameter	Original	Reduced
Numerator length	4	3
Denominator length	4	3
Stable	Yes	Yes
Implementation cost		
Number of multipliers	5	5
Number of adders	6	4
Number of states	6	4

The filter co efficient are compared in Table 2

Table 2: Comparison of the filter coefficients of original and reduced order filter.

	Filter coefficients
Numerator-Original	1,2,2,1
Denominator – Original	1, 0.5416, 0.625, 0.333
Numerator – Reduced	0.85509, 1.6319, 0.7769
Denominator - Reduced	1, -0.1171, 0.4769

Numerical Example 2:

Consider the eighth order system transfer function (8)

$$G(z) = \frac{1.682\,z^7 + 1.116\,z^6 - 0.21\,z^5 + 0.152\,z^4 - 0.516\,z^3 - 0.262\,z^2 + 0.044\,z - 0.018}{8z^8 - 5.046\,z^7 - 3.348\,z^6 + 0.63\,z^5 - 0.456\,z^4 + 1.548\,z^3 + 0.786\,z^2 - 0.132\,z + 0.018} \tag{8}$$

Using the proposed scheme of model reduction, the reduced model is obtained as in (9)

$$R_2(z) = \frac{0.08835z^2 + 0.03753z - 0.05081}{z^2 - 1.786z + 0.8614} \tag{9}$$

The step responses and other performance responses of original and reduced order system are compared in figures 7 – 11.

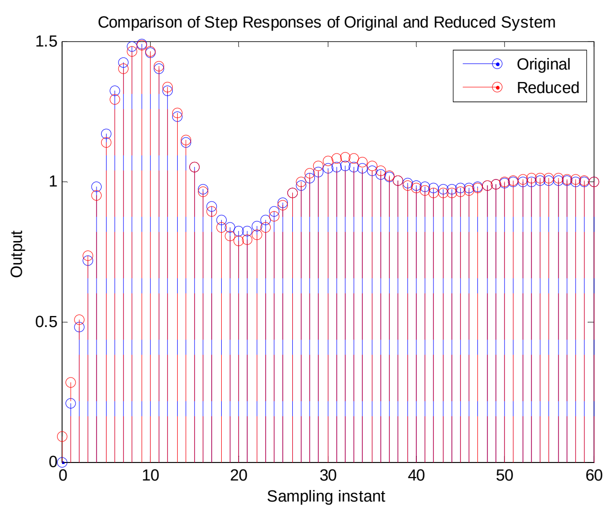

Fig. 7: Comparison of step responses of example 2.

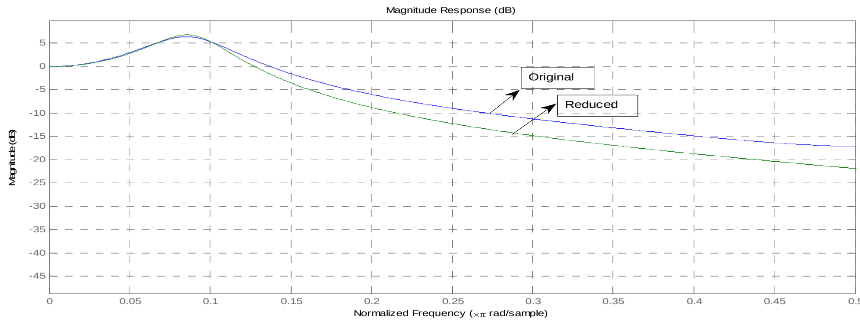

Fig. 8: Comparison of magnitude responses of original and reduced in example 2.

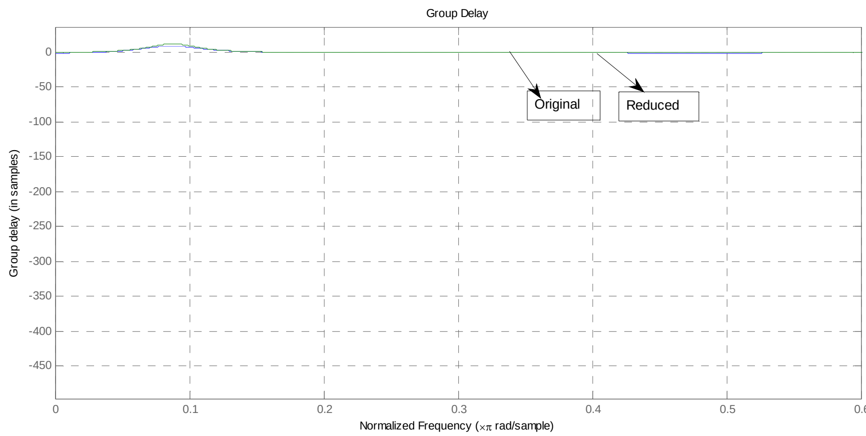

Fig. 9: Comparison of Group delay of original and reduced in example 2.

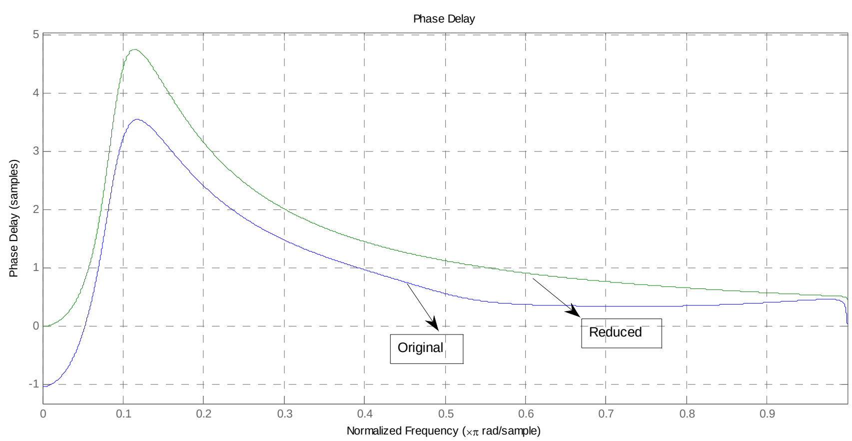

Fig. 10: Comparison of Phase delay of original and reduced in example 2.

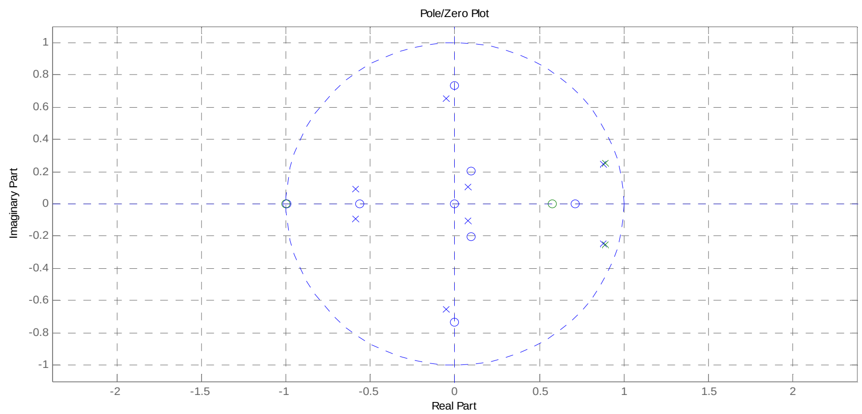

Fig. 11: Comparison of pole zero plots of original and reduced in example 2.

Table 3: Comparisons of the parameters of original and reduced order filter.

Parameter	Original	Reduced
Numerator length	8	3
Denominator length	9	3
Stable	Yes	Yes
Implementation cost		
Number of multipliers	17	5
Number of adders	15	4
Number of states	15	4

The parameters are compared in Table 3 and the filter co efficient are compared in Table 4

Table 4: Comparisons of the filter coefficients of original and reduced order filter.

	Filter coefficients
Numerator-Original	1.682, 1.116, -0.21 0.152, -0.516, -0.262, 0.044, -0.018
Denominator – Original	8, -5.046, -3.348, 0.63, -0.456, 1.548, 0.786, -0.132, 0.018
Numerator – Reduced	0.08835, 0.03753, -0.05081
Denominator - Reduced	1 -1.786, 0.8614

Conclusion:

The proposed method is used for the design of digital filters. From the simulation results obtained it is shown that the reduced model retains the characteristics of the original higher order filter. The magnitude response, phase response, group delay and phase delay are compared. It is shown that the number of filter co efficient are reduced by using the reduced order model and it reduces the design complexity.

REFERENCES

Gomathi, P. and T.A. Manigandan, 2012. Noval Method for Design of PID Controller for Linear System using Reduced Order Model. European Journal of Scientific Research, 92(3): 380-389.

Kennedy, J. and R.C. Eberhart, 1995. Particle swarm optimization, IEEE Int. Conf. on Neural Networks, 6: 1942-1948.

Kennedy, J. and R.C. Eberhart, 2001. Swarm Intelligence, Morgan Kaufman Publishers, California.

Kranthikumar, D., S.K. Nagar and J.P. Tiwari, 2011. Model order reduction of Interval systems using Mihailov Criterion and Routh Approximation, International journal of Engineering Science and Technology, 3(7): 5593-5598.

Manigandan, T., N. Devarajan and S.N. Sivanandam, 2005. Design of PID controller using Reduced order model Academic Open Internet Journal, 15: 1-15.

Manish Kansal, Vijay kumar, Dinesh Arora and Hardep Singh Saini, 2011. Design and Implementation of digital filter for removal of power supply noise, International Journal of Soft Computing and Engineering, 1(4): 242-246.

Maurice F. Hutton and Bernard Friedland, 1975. Routh Approximations for reducing order of Linear, Time - Invariant Systems, IEEE Transactions on Automatic Control, 20(3): 329-337.

Mittal, A.K., R. Prasad and S.P. Sharma, 2004. Reduction of linear Dynamic Systems using an Error minimization Technique IE(I) Journal-EL, 84: 161-165.

Mittal, S.K., Dinesh Chandra and Bharti Dwivedi, 2009. The Effects of Time moments and Markov parameters on Reduced order modeling, ARPN Journal of Engineering and Applied Sciences, 4(5).

Nagrath, I.J. and M. Gopal, 2006. Control Systems Engineering, New age International publishers New Delhi.

Nalin Harischandra, 2011. Designing digital IIR filters by Model order reduction techniques, Project FEL3500(2010/11), The Royal Institute of Technology, Sweden.

Palaniswami, S. and S.N. Sivanandam, 1999. Simple and Direct methods for Model Reduction of Continuous Systems using Three point expansion scheme, International conference on Computer modeling, Simulation and Communication, Jaipur, India, 290-296.

Panda, S., S.K. Tomar, R. Prasad and C. Ardil, 2009. Reduction of linear time invariant systems using Routh-approximation and PSO, International Journal of Applied Mathematics and Computer Science, 5: 82-89.

Prasad, R., S.P. Sharma and A.K. Mittal, 2003. Linear Model Reduction Using the advantages of Mihailov Criterion and Factor Division, IE (I) Journal-EL, 84: 7-10.

Ramesh, K., A. Nirmalkumar and G. Gurusamy, 2009. Design of digital IIR filters with the advantages of Model order reduction technique International Journal of Electronic Engineering Research, 1(2): 89-100.

Sreeram, V. and P. Agathoklis, 1992. Design of Linear phase IIR filters via Impulse response grammians, IEEE transactions on Signal Processing, 40: 389-394.