Design of Reduced Order Digital Filters Using Pso and Routh Schemes

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Abstract: A new scheme for the design of complex higher order digital filter is introduced in this paper. When the order of the filter is increased, the number of filter coefficients to be designed is also increased which leads to the higher complexity in design and this problem is simplified by using the proposed scheme of model order which uses the advantages of Routh method and Particle Swarm Optimisation (PSO). The reduction is done based on the minimization of cumulative error index between original and reduced models. The reduction scheme always produces a stable reduced model if the higher order system is stable. The illustrative examples are given to support the proposed scheme.

Key words: Digital Filter, Filter Coefficients, Model order reduction, Particle Swarm Optimisation (PSO), Routh array, Cumulative error index.

INTRODUCTION

The Infinite Impulse Response (IIR) filter is a discrete time system that is designed to pass spectral content of the input signal in a specified band of frequencies by considering all the infinite samples of impulse response. The design of a higher order filter will be definitely a highly complicated task and it takes a longer tile for the design of larger number of filter coefficients. Since the mathematical models obtained for a real time system results in a higher order model, the order reduction is applied in almost all fields of engineering which uses reduced models for the simulation and design of complex systems. Model order reduction is an area of research and many techniques have been proposed.

The Routh Approximation method introduced by Hutton and Friedland, 1975; has proven to be the most successful tools for getting the reduced order models due to its simplicity in computation and efficiency. Novel scheme is proposed to obtain a second order reduced model for stable LTICS presented (Manigandan et al., 2005) which provides good approximation and preserves the stability of the higher order system. Order reduction of linear-time invariant systems is illustrated by Panda et al., 2009, employing two methods, one using the advantages of Routh approximation and other by particle Swarm optimization. A combined method making use of advantages of the Mihalov criterion and factor division was proposed in Prasad et al., 2003. An error minimization technique (Mittal et al., 2004) based model reduction is discussed.

Particle swarm optimization (PSO) is a population based stochastic optimization technique developed by Eberhart and Kennedy, 2001; inspired by social behavior of bird flocking or fish schooling. The PSO method is a member of the wide category of Swarm Intelligence methods [Kennady and Eberhart, 1995]. Model order reduction using Mihailov criterion and Routh Approximation is given by Karanthikumar et al, 2011. The reduction method on three point expansion scheme is presented by Palaniswami and Sivanandan, the effectiveness is shown through illustrations. Mittal et al, 2009 also proposed a routh based model reduction method with the effectiveness of time moments and Markov parameters.

Many authors uses the method of model reduction for the design of filters(Sreeram et al, 1992, Ramesh et al., 2009, Nalin Harischandra, 2011, Manish Kansal, 2011) and proved that the complexity of the design is reduced. In this paper a simplified approach for the design of reduced order filter is proposed by using Routh array (Nagrath et al, 2006) and Particle Swarm Optimisation

Problem Definition:

Consider an n oorder stable linear time invariant discrete filter described by the transfer function (1)

\[ G(z) = \frac{N(z)}{D(z)} = \frac{B_0 z^n + B_1 z^{n-1} + B_2 z^{n-2} + B_3 z^{n-3} + \ldots}{A_0 z^n + A_1 z^{n-1} + A_2 z^{n-2} + A_3 z^{n-3} + \ldots} \]

where, \( A_i \) (0 \( \leq \) i \( \leq \) n) and \( B_i \) (0 \( \leq \) i \( \leq \) m) are scalar constants and m\( \leq \)n.

The corresponding reduced second order filter model is of the form (2)

\[ R_2(z) = \frac{D_0 z^2 + D_0}{E_2 z^2 + E_2 z + E_0} \]

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The objective is to find a reduced second order filter model in the form of equation (2) for the original system described by equation (1), such that the reduced order filter model retains the important characteristics of the original higher order filter. The second order filter is stable if the higher order filter is stable.

**Performance Specifications Selected for the Comparison:**

The performance of filters defined in (1) and (2) are compared using the following responses

- Magnitude response
- Phase response
- Group delay
- Phase delay
- Pole zero plot

**Proposed Method of Model Order Reduction:**

The original higher order system is reduced into second order using the algorithm explained by Gomathi and Manigandan, 2011; the pseudo code for the reduction is given below.

Get the order n for the discrete higher order transfer function

If (Order of the system > 2)

1. Convert LTIDS into LTICS
2. Construct Routh stability array for the numerator of the higher order system
3. Select $e_0$ as constant of denominator, $d_0$ as constant of numerator and $d_1$ as s row from Routh array
4. Construct numerator polynomial of reduced system as $d_1s + d_0$
5. Equate (1) & (2) and cross multiply
6. Equate the coefficients on both sides form (n+2) equations.
7. By substituting the values of $e_0$, $d_0$ and $d_1$, solve the equations and get the values of $e_2$ and $e_1$.
8. Optimise the values using PSO
9. Construct transfer function of reduced model by substituting optimized values in (2)
10. Convert LTICS into LTIDS
11. Verify the characteristics of original higher order filter and reduced filter by applying step input.

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1. Design the filter coefficients using the reduced model $R(z)$
2. Verify the effectiveness of the reduced model by comparing its responses with the original filter.
3. Represent the filter in direct form structure.

**Numerical Examples:**

**Numerical Example 1:**

Consider digital IIR filter with transfer function defined by Ramesh et al.(2009)

$$H(z) = \frac{1 + 2z^{-1} + 2z^{-2} + z^{-3}}{1 + 0.5416z^{-1} + 0.625z^{-2} + 0.333z^{-3}}$$

(3) is rewritten as in (4)

$$H(z) = \frac{z^3 + 2z^2 + 2z + 1}{z^3 + 0.54167z^2 + 0.625z + 0.333}$$

(4) is transformed into continuous domain using bilinear transformation as in (5)

$$H(s) = \frac{5.331s^2 + 63.97}{s^3 + 7.55s^2 + 10.22s + 26.65}$$

The reduced model (6) using proposed method is obtained as

$$R(s) = \frac{1.5322s + 63.97}{7.8099s^2 + 10.252s + 26.65}$$

(6)

The discrete equivalent of (6) is obtained using inverse bilinear transformation as in (7)

$$R(z) = \frac{0.8551z^2 + 1.632z + 0.7769}{z^2 - 0.1171z + 0.4769}$$

(7)

The comparison of step responses of original and reduced systems is given in Figure 1.
Fig. 1: Comparison of step responses of original and reduced systems.

The performance comparisons are given in Figure 2 to Figure 6.

Fig. 2: Comparison of magnitude responses of original and reduced in example 1.

Fig. 3: Comparison of phase responses of original and reduced in example 6.1.
Fig. 4: Comparison of Group delay of original and reduced in example 1.

![Group Delay Diagram](image)

Fig. 5: Comparison of phase delay of original and reduced in example 1.

![Phase Delay Diagram](image)

Fig. 6: Comparison of pole zero plots of original and reduced in example 1.

![Pole Zero Plot Diagram](image)
The different parameters of the original and reduced order filter are compared in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Original</th>
<th>Reduced</th>
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</thead>
<tbody>
<tr>
<td>Numerator length</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>Denominator length</td>
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<td>3</td>
</tr>
<tr>
<td>Stable</td>
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<td>Yes</td>
</tr>
<tr>
<td>Implementation cost</td>
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<td>5</td>
</tr>
<tr>
<td>Number of adders</td>
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<td>4</td>
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<tr>
<td>Number of states</td>
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</table>

The filter coefficients are compared in Table 2.

<table>
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<th>Filter coefficients</th>
<th>Numerator-Original</th>
<th>Denominator – Original</th>
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<tr>
<td></td>
<td>1, 2, 2, 1</td>
<td>1, 0.5416, 0.625, 0.333</td>
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<tr>
<td></td>
<td>Numerator – Reduced</td>
<td>0.85509, 1.6319, 0.7769</td>
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<tr>
<td></td>
<td>Denominator - Reduced</td>
<td>1, -0.1171, 0.4769</td>
</tr>
</tbody>
</table>

**Numerical Example 2:**

Consider the eighth order system transfer function (8)

\[
G(z) = \frac{1.682 z^7 + 1.116 z^6 - 0.21 z^5 + 0.152 z^4 - 0.516 z^3 - 0.262 z^2 + 0.044 z - 0.018}{8 z^7 - 5.046 z^6 - 3.348 z^5 + 0.63 z^4 - 0.456 z^3 + 1.548 z^2 + 0.786 z^1 - 0.132 z - 0.018}
\]

Using the proposed scheme of model reduction, the reduced model is obtained as in (9)

\[
R_{i}(z) = \frac{0.08835z^2 + 0.03753z - 0.05081}{z^1 - 1.786z + 0.8614}
\]

The step responses and other performance responses of original and reduced order system are compared in figures 7 – 11.

**Fig. 7:** Comparison of step responses of example 2.
Fig. 8: Comparison of magnitude responses of original and reduced in example 2.

Fig. 9: Comparison of Group delay of original and reduced in example 2.

Fig. 10: Comparison of Phase delay of original and reduced in example 2.
Fig. 11: Comparison of pole zero plots of original and reduced in example 2.

Table 3: Comparisons of the parameters of original and reduced order filter.

<table>
<thead>
<tr>
<th>Parameter</th>
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<th>Reduced</th>
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<tbody>
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<tr>
<td>Denominator length</td>
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<td>Stable</td>
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<td>Number of states</td>
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</table>

The parameters are compared in Table 3 and the filter coefficients are compared in Table 4.

Table 4: Comparisons of the filter coefficients of original and reduced order filter.

<table>
<thead>
<tr>
<th>Filter coefficients</th>
<th></th>
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<tbody>
<tr>
<td>Numerator-Original</td>
<td>1.682, 1.116, -0.21, 0.152, -0.516, -0.262, 0.044, -0.018</td>
</tr>
<tr>
<td>Denominator – Original</td>
<td>-8, -5.046, -3.348, 0.63, -0.456, 1.548, 0.786, -0.132, 0.018</td>
</tr>
<tr>
<td>Numerator – Reduced</td>
<td>0.08835, 0.03753, -0.05081</td>
</tr>
<tr>
<td>Denominator - Reduced</td>
<td>1, -1.786, 0.8614</td>
</tr>
</tbody>
</table>

Conclusion:
The proposed method is used for the design of digital filters. From the simulation results obtained it is shown that the reduced model retains the characteristics of the original higher order filter. The magnitude response, phase response, group delay and phase delay are compared. It is shown that the number of filter coefficients are reduced by using the reduced order model and it reduces the design complexity.

REFERENCES