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Process Debt Payment Trough the Ordinary Annuities and Gradient Series: Its Theoretical and Practical Implications

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ABSTRACT

The main objective of this paper is to develop the procedure to debt payment, trough the ordinary annuities and geometrics or arithmetic's gradients: Its theoretical and practical implications.

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INTRODUCTION

The money over time has a natural change, all this derived from the inflationary effects and by cause from the capitalization (anatocism) on the operations of savings and investment, as well as, by credit or financing transactions. In commercial operations, in the purchase of goods or services to run, i.e., when it is given a deadline for settlement, is made present the effect of charging interest on a loan or financing granted. Same is the case in operations with institutions of the financial system (in the case of Mexico, the Mexican financial system), such as savings, investment, or loans. In all cases, the money will move in a timeline, from present value to future value, according to the case (Ayres, 1991; García-Santillán, 2009, Villalobos, 2012).

To understand the effect of the valuation of money over time, within the hard sciences is found the mathematics and specifically, one of its branches is called financial mathematics or financial engineering, as it is also called. This math is responsible for providing the theoretical elements and methodological to carry out that purpose. On the following lines it will be explained behavior of money over time. We will use scenarios: debt amortization due and anticipated, with the same scenario, we calculate by using arithmetic and geometric gradient, considering hypothetical values at any moment, only to financial modeling scenarios listed above and to discuss the results.

Preliminary Notes And Notation:

The annuities which are used most frequently in commercial and financial activity are annuities ordinary or past due. Also they are known as annuities certain and simple (Ayres, 1991; Portus, 1997).

The characteristics about these kinds of annuities are: The payments are made at the end of each payment interval. It is known since the signing of agreement, the start date and end of the period of the annuity. The capitalization coincides with the interval payment. The time limit begins with the signing of the agreement (Cissell, 1987; Higland, 1987; Zima, 2005; Villalobos, 2012).

Theorem:
Ordinary Annuities:

The annuities o periodical rent is given by:

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$$Rp = \frac{VF}{\left[\frac{(1+i/m)^n - 1}{i/m}\right]} \quad or \quad A = \frac{M}{\left[\frac{(1+i/m)^n - 1}{i/m}\right]} \tag{1}$$

Where:
Rp or A: periodical rent or annuities
i/m= capitalization interest rate
VF or M= Future value
n= time
VPN = Net present value

The Net Present Value (*VPN*) is given by:

$$VPN = Rp\frac{1-(1+\frac{i}{m})^{-n}}{i/m} \tag{2}$$

It is extracted

$$Rp = \frac{VPN}{\frac{1-(1+i/m)^{-n}}{i/m}} \tag{3}$$

So, in order to calculate *"n"* in future value we have:

$$VF = Rp\frac{(1+\frac{i}{m})^n - 1}{i/m} \qquad Rp\frac{(1+\frac{i}{m})^n - 1}{i/m} = VF \tag{4}$$

"*Rp*" passes to the other side by dividing

$$\frac{(1+\frac{i}{m})^n - 1}{i/m} = \frac{VF}{Rp} \tag{5}$$

"*i/m"* passes by multiplying

$$(1+i/m)^n - 1 = \left[\left(VF/Rp\right) * i/m\right] \tag{6}$$

And the unit (1) passes by summing

$$(1+i/m)^n = \left[\left(VF/Rp\right) * i/m\right] + 1 \tag{7}$$

Now we apply logarithms

$$\log((1+i/m)^n) = \log\left[\left(VF/Rp\right) * i/m\right] + 1 \tag{8}$$

Then, we extracted "*n*"

$$n = \frac{Log\left|\left(VF/Rp\right) * i\right| + 1}{Log(1+\frac{i}{m})} \tag{9}$$

In ordinary annuities we can establish to evaluate Net Future Value *NFV* as:

$$NFV = Pp\frac{(1+\frac{i}{m})^n - 1}{i/m} \tag{10}$$

When the interest rate changes during the time, it will seek the Future value (VF) of each annuity as follow: Calculating VF_1, VF_2, VF_n, i.e., how as many times *"i"* changes the formula is modified as follows terms:

To the first interest rate:

$$VF_1 = Rp\frac{(1+\frac{i}{m})^n - 1}{i/m} \tag{10.1}$$

To the second interest rate

$$VF_2 = VF_1(1+ i/m)^n + Rp\frac{(1+\frac{i}{m})^n - 1}{i/m} \tag{10.2}$$

And so successively

$$VF_n = VF_n(1+ i/m)^n + Rp\frac{(1+\frac{i}{m})^n - 1}{i/m} \tag{10.3}$$

Ordinary Annuities With Gradient Geometric And Arithmetic:

In a particular way the arithmetic gradient (*Ga*) or uniform, is a series of regular payments or cash flows that increase or decrease uniformly. Cash flows (*fees*) change in the same amount among each period. This is called arithmetic gradient. The other type of gradient is the geometric gradient (*Gg*), a series of periodic dues (*rents*) or cash flows that in constant percentages increase or decrease, whether, in consecutive pay periods, or constant increases of money. The cash flows (*fees*) on the same percentage change among each period. This is called geometric gradient (Felgueres, 1973; Portus, 1997; Garcia, 2000; Zima, 2005; García-Santillán, 2009; Villalobos, 2012).

The formulas generally accepted to calculate annuities with expired arithmetic gradient (post payments) or anticipated (pre-payment) are:

1.- In order to know future value, we have:

M_{ga} pre-payment

$$M_{ga} = (Rp_1 + \frac{g_a}{i/m})\left[(1+ i/m)\frac{(1+ i/m)^n - 1}{i/m}\right] - \frac{n * g_a}{i/m} \tag{11}$$

M_{ga} post-payment

$$M_{ga} = (Rp_1 + \frac{g_a}{i/m})\left[\frac{(1+ i/m)^n - 1}{i/m}\right] - \frac{n * g_a}{i/m} \tag{11.1}$$

And the actually value is given by:

VA_{ga} pre-payment

$$VA_{ga} = \left[\left(Rp_1 + \frac{g_a}{i/m}\right)\left[(1+i/m)\frac{(1+ i/m)^n - 1}{i/m}\right] - \frac{n * g_a}{i/m}\right](1+ i/m)^{-n} \tag{12}$$

VA_{ga} post-payment

$$VA_{ga} = \left[\left(Rp_1 + \frac{g_a}{i/m}\right)\left[\frac{(1+ i/m)^n - 1}{i/m}\right] - \frac{n * g_a}{i/m}\right](1+ i/m)^{-n} \tag{12.1}$$

Where:

Rp: periodical rent or annuities

M_{ga} =Amount (future value) – arithmetic gradient

M_{gg} =Amount – geometric gradient

VA_{ga} =actually value – arithmetic gradient

g_a = arithmetic gradient

g_g = geometric gradient

i/m= capitalization interest rate

n= time

And the formulas generally accepted to calculate annuities with expired geometric gradient (post payments) or anticipated (pre-payment) are:

1.- In order to know future value, we have:

M_{gg} pre-payment

$$Si \quad (1+{}^{i}/_{m})^{1}\, Gg: \qquad Mg_{g} = Rp_{1}(1+{}^{i}/_{m})\left[\frac{(1+{}^{i}/_{m})^{n} - (1+Gg)^{n}}{{}^{i}/_{m} - Gg}\right], \tag{13}$$

M_{gg} post-payment

$$Si \quad (1+{}^{i}/_{m})^{1}\, Gg: \qquad Mg_{g} = Rp_{1}\left[\frac{(1+{}^{i}/_{m})^{n} - (1+Gg)^{n}}{{}^{i}/_{m} - Gg}\right], \tag{13.1}$$

With the above consideration and since the theoretical theorems have been explained, now we can develop a calculus of hypothetic scenarios.

Scenarios And Development:

In business operations with suppliers (credits) and in the field of the financial system (bank funding), the term "amortization" is linked to debt, which is the progressive payment to settle a debt coming from some loan. In financing activities it is common for businesses and the people who need financing, whether in order to capitalize or to acquire capital goods (assets). Therefore, the funding obtained, must be paid within a period that has previously been established: Periodically payments (expired) or a payment to be increased proportionally, whether in periodical fees: expired or anticipated, or fees that will increase proportionally in amount or by percentage, being the case of payments with gradients geometric and arithmetic (Moore, 1963; Garcia, 2000; García-Santillán, 2009; Zima, 2005 and Villalobos, 2012).

With the previously argument, now we carry out the calculus on two scenarios:

a).- amortization with a periodical expired rent (annuities ordinary) and a periodical anticipated rent, and

b).- amortization with a periodical expired rent and an anticipated with geometric and arithmetic gradients, all this, according as follow:

Firstly, in order to calculate the periodical rents (*Rp*) we must use the net present value formula of one expired payment (Rp_1).

So, we have:

$$NPV = Rp\frac{1-(1+i/m)^{-n}}{i/m} \tag{14}$$

Now, to know the Rp_1 value of debt, NPV passes dividing and the result is:

$$Rp = \frac{NPV}{\frac{1-(1+i/m)^{-n}}{i/m}} \tag{14.1}$$

Also, if we need to know the R_{p1} value of an anticipated payment, we need to include in the formula a coefficient (1+i/m), therefore, it is equal:

$$Rp = \frac{NPV}{(1+i/m)\frac{1-(1+i/m)^{-n}}{i/m}} \tag{14.2}$$

Recalling, the expressions *i/m*, were utilized for this case in which it has to be calculate the interest rate that will be capitalized, for example: when we have a nominal annual interest rate of 12% and its capitalizations are monthly, thus, they must be dividing (12/12).

Develop A Hypothetic Case With Annuities Ordinary:

We suppose the next data: $250,000.00 debt, which must be **pay in 10 equal expired payments**, considering a nominal interest rate of 12%, with monthly capitalizations.

From the formula (14.1): $$Rp = \frac{NPV}{\frac{1-(1+i/m)^{-n}}{i/m}}$$

Where:

NPV = Net present value
Rp_1= periodical rent
i = nominal interest rate
m = capitalization
$-n$= “time” or number of payment

So, we obtained:

$$Rp = \frac{\$250,000.00}{\frac{1-(1+.12/12)^{-10}}{.12/12}} \quad Rp = \frac{\$250,000.00}{\frac{1-(1.01)^{-10}}{.01}} \quad Rp = \frac{\$250,000.00}{\frac{1-(0.90528695)}{.01}}$$

$$Rp = \frac{\$250,000.00}{9.47130453} \qquad Rp = \$26,395.52 \quad \ldots.. \qquad (14.3)$$

To verify this result, now we shall proceed to design an amortization chart in order to obtain the amounts from each concept:

Table 1: Amortization chart

n	Monthly payment	Payment to capital	Payment of interest	Remaining capital
1	$26,395.52	$23,895.52	$2,500.00	$226,104.48
2	$26,395.52	$24,134.47	$2,261.04	$201,970.01
3	$26,395.52	$24,375.82	$2,019.70	$177,594.19
4	$26,395.52	$24,619.58	$1,775.94	$152,974.61
5	$26,395.52	$24,865.77	$1,529.75	$128,108.84
6	$26,395.52	$25,114.43	$1,281.09	$102,994.41
7	$26,395.52	$25,365.58	$1,029.94	$77,628.83
8	$26,395.52	$25,619.23	$776.29	$52,009.60
9	$26,395.52	$25,875.42	$520.10	$26,134.18
10	$26,395.52	$26,134.18	$261.34	$0.00
TOTAL	$263,955.19	$250,000.00	$13,955.19	

Source: own

With the same data, now we calculate with the anticipated payments.

We suppose the next data: $250,000.00 debt, which must be paid **in 10 equal anticipated payments, considering** a nominal interest rate of 12%, with monthly capitalizations.

From the formula (14.2):

$$Rp_1 = \frac{NPV}{(1+i/m)\frac{1-(1+i/m)^{-n}}{i/m}} \qquad (14.3.1)$$

$$Rp = \frac{\$250,000.00}{(1+.12/12)\frac{1-(1+.12/12)^{-10}}{.12/12}} = \frac{\$250,000.00}{(1.01)\frac{1-(1.01)^{-10}}{.01}} = \frac{\$250,000.00}{(1.01)\frac{1-(0.90528695)}{.01}}$$

$$Rp_1 = \frac{\$250,000.00}{(1.01)(9.47130453)} = \frac{\$250,000.00}{(9.566017575)} = \_\_\_\$26,134.18\_\_\_\_ \quad (14.3.2)$$

To verify this result, now we shall proceed to design an amortization chart to obtain the amounts from each of the concepts:

Table 2: Amortization chart

n	Monthly payment	Payment interest	Capital payment	Remaining capital
				$250,000.00
0	$26,134.18	0.000	$26,134.18	$223,865.82
1	$26,134.18	$2,238.66	$23,895.52	$199,970.30
2	$26,134.18	$1,999.70	$24,134.47	$175,835.83
3	$26,134.18	$1,758.36	$24,375.82	$151,460.01
4	$26,134.18	$1,514.60	$24,619.58	$126,840.43
5	$26,134.18	$1,268.40	$24,865.77	$101,974.66
6	$26,134.18	$1,019.75	$25,114.43	$76,860.23
7	$26,134.18	$768.60	$25,365.58	$51,494.65
8	$26,134.18	$514.95	$25,619.23	$25,875.42
9	$26,134.18	$258.75	$25,875.42	0.00
TOTAL	$263,955.19	$13,955.19		

Source: own

Develop A Hypothetic Case With Gradients:

Now, to develop a hypothetic case with gradients (arithmetic and geometric): We suppose the next data: $250,000.00 debt, which must be paid in 10 equal expired payments, considering a nominal interest rate of 12%, with monthly capitalizations. The first payment is increased by $791.87 (in arithmetic gradient) and 3% (in geometric gradient).

Gradient Arithmetic (amortization chart)

Table 3: Summary of calculus

VA_{ga}=	$250,000.00	Expired annuities		Anticipated annuities	
a =	$791.87	Rp_1 =	$22,897.10	Rp_1 =	$22,635.76
n =	10.00	Ga =	$791.87	Ga =	$791.87
i=	1.00%	n =	10.00	n =	10.00
Rp_1 (expired annuities)=	$22,897.10	i=	1.00%	i=	1.00%
Rp_1 (anticipated annuities)=	$22,635.76	VA_{ga}=	$250,000.00	VA_{ga}=	$250,000.00

Source: own

Table 4: Amortization chart (expired annuities)

n	Monthly payment	Payment interest	Capital payment	Remaining capital
0				$250,000.00
1	$22,897.10	$2,500.00	$20,397.10	229,602.90
2	23,688.97	2,296.03	21,392.94	208,209.96
3	24,480.84	2,082.10	22,398.74	185,811.22
4	25,272.71	1,858.11	23,414.60	162,396.63
5	26,064.58	1,623.97	24,440.61	137,956.02
6	26,856.45	1,379.56	25,476.89	112,479.13
7	27,648.32	1,124.79	26,523.53	85,955.60
8	28,440.19	859.56	27,580.63	58,374.97
9	29,232.06	583.75	28,648.31	29,726.66
10	30,023.93	297.27	29,726.66	0.00
Total	$264,605.15	$14,605.14	$250,000.01	

Table 4.1: Amortization chart (anticipated annuities)

n	Monthly payment	Payment interest	Capital payment	Remaining capital
0				$250,000.00
1	$22,635.76	$0.00	$22,635.76	227,364.24
2	23,427.63	2,273.64	21,153.98	206,210.26
3	24,219.50	2,062.10	22,157.39	184,052.87
4	25,011.37	1,840.53	23,170.84	160,882.03
5	25,803.24	1,608.82	24,194.42	136,687.61
6	26,595.11	1,366.88	25,228.23	111,459.38
7	27,386.98	1,114.59	26,272.38	85,187.00
8	28,178.85	851.87	27,326.98	57,860.02
9	28,970.72	578.60	28,392.12	29,467.91
10	29,762.59	294.68	29,467.91	0.00
Total	$261,991.75	$11,991.71	$250,000.01	

Gradient Geometric (amortization chart)

Table 5: Summary of calculus

VAgg=	$250,000.00	Expired annuities		Anticipated annuities	
Gg=	3.00%	Rp_1 =	$23,080.83	Rp_1 =	$22,852.31
n =	10.00	*Gg* =	3.00%	*Gg* =	3.00%
i=	1.00%	*n* =	10.00	*n* =	10.00
Rp_1 (expired annuities)=	$23,080.83	*i*=	1.00%	*i*=	1.00%
Rp_1 (anticipated annuities)=	$22,852.31	VA_{gg}=	$250,000.00	VA_{gg}=	$250,000.00

Source: own

Table 6: Amortization chart (expired annuities)

n	Monthly payment	Payment interest	Capital payment	Remaining capital
0				250,000.00
1	23,080.83	2,500.00	20,580.83	229,419.17
2	23,773.26	2,294.19	21,479.07	207,940.10
3	24,486.46	2,079.40	22,407.05	185,533.05
4	25,221.05	1,855.33	23,365.72	162,167.33
5	25,977.68	1,621.67	24,356.01	137,811.32
6	26,757.01	1,378.11	25,378.90	112,432.42
7	27,559.72	1,124.32	26,435.40	85,997.03
8	28,386.51	859.97	27,526.54	58,470.48
9	29,238.11	584.70	28,653.40	29,817.08
10	30,115.25	298.17	29,817.08	0.00
Total	$264,595.88	$14,595.86	$250,000.00	

Table 6.1: Amortization chart (anticipated annuities)

n	Monthly payment	Payment interest	Capital payment	Remaining capital
				250,000.00
0	22,852.31	0.00	22,852.31	227,147.69
1	23,537.88	2,271.48	21,266.40	205,881.29
2	24,244.02	2,058.81	22,185.20	183,696.09
3	24,971.34	1,836.96	23,134.37	160,561.71
4	25,720.48	1,605.62	24,114.86	136,446.85
5	26,492.09	1,364.47	25,127.62	111,319.23
6	27,286.85	1,113.19	26,173.66	85,145.57
7	28,105.46	851.46	27,254.00	57,891.57
8	28,948.62	578.92	28,369.71	29,521.86
9	29,817.08	295.22	29,521.86	0.00
Total	$261,976.13	$11,976.13	$249,999.99	

Discussion:

In the resolution of the exercises, for all scenarios were used the amount of $250,000.00 dls., to be paid in 10 payments, assuming an interest rate of 12% compounded monthly. For the case of annuities simple and certain in their expired modality and anticipated, it was revealed that: The amount of each equal payments were $ 26,395.52 (14.3), which gives us an amount of $263,955.19 (table 1) corresponding $ 13,955.19 (table 1) to pay interest.

In the case that payments should be made in advance (anticipated form), then we obtained the following information: The amount of each payment is for $ 26,134.18 (14.3.1) which gives an amount of $263,955.19 (table 2) corresponding likewise $13,955.18 (table 2) to pay interest. In the last payment, the interest is lower slightly ($258.75) based on the above calculation (table 1) of $261.34 As we can be appreciated, the first payment is made when obtaining financing or credit, which assumes that each payment must be made at the beginning of each month, hence the first payment is made at month 0 and the last at the end of the ninth month, in total are 10 payments.

The observed difference can be said to be minimal, since in prepayments the only thing that can save is the interest payment in the first month, not accruing this charge, contrary to the model of annuities expired.

In order to make payment in the modality of annuities with series gradient (arithmetic and geometric sequences) were considered the same data: $250,000.00 dls, value of debt, which must be paid at 10 payments (calculated with payments past due and anticipated) at a rate nominal interest of 12% monthly capitalization. The increase in every payment is: $791.87 in the case of arithmetic gradient and 3% in the case of geometric gradient.

The obtained results from the application of the repayment of principal with arithmetic gradient expired are: the first payment is by the amount of $22,897.10 which increases arithmetically in the amount of $791.87 the first and so on the following with respect to the previous one, which gives an amount of $264,605.15 (table 4) corresponding $14,605.14 (table 4) to payment of interest. Now in the modality of annuity due (anticipated) with arithmetic gradient were obtained the following results: the first payment is by the amount of $22,635.76

which increases arithmetically in the amount of $791.87 the first and so on the following regarding the above, which gives a final amount of $261,991.75 (table 4.1) corresponding $11,991.71 (table 4.1) to payment of interest.

The observed difference in the payment of interest is significant, hence in prepayments we can save interests by the amount of $2,613.43 with respect to amortization payment modality with arithmetical gradients expired.

Results obtained from the application of the amortization with geometric gradient up (expired) are: the first payment is by the amount of $23,080.83 which increases in geometric form in 3% from first to second and so on the following with respect to the previous one, which gives a total amount of $264,595.88 (table 6) corresponding $14,595.86 (table 6) to interest payments. Now in the modality of annuity due (anticipated) with geometric gradient, were obtained the following results: the first payment is by the amount of $22,852.31 which geometrically increases as well in 3%, which gives a total of $261,976.13 (table 6.1), corresponding to $11,976.13 (table 6.1) to pay interest.

Conclusion:

As we can observe on the results obtained, the payments made in the modality of annuity expired and anticipated, the only different thing is: that in the modality expired payment is made at the end of each payment period and the amount of interest is slightly greater, otherwise, when is made in anticipated payments, at the time of contracting debt, then, is made in advance first payment and so on.

In another context, it was noted -based on the results- that payments with gradients arithmetic and geometric series, it is very significant a reduction in the payment of interest, ie, conforming every payment is amortized within any of the modalities (past due or anticipated) are being increasing every payments, resulting from this that the payment to capital (outstanding principal) is greater, and this brings with it a reduction in payment of interest. In both cases, whether with an increase regarding the amount of the payment or as a percentage is attractive for the debtor and at the end will pay less interest.

Now, considering that the aim of this paper is to demonstrate through mathematical modeling that the use of arithmetical and geometrical gradients favors reducing at the payment of interest (in charge of) of who gets some type of financing (debt). It is very important that companies and people should evaluate this position, since for the monthly payment their obligations, on the cash flow budget should be considered the amount of each payment to be made in the time period which was contracted debt, considering the increase in the payment, either arithmetical or geometrical form that has been agreed for this purpose.

The practical implications would be as follows: companies which have obtained financing should be aware that the method of payment by gradient series (arithmetical and geometrical) is aimed to reducing payment of interest, which translates into a benefit for companies, therefore should focus their financial strategy in this regard.

The theoretical implications show evidence about the theorems used in the calculations of the different proposed scenarios, allow us to obtain convincing evidence on the comparison in each mode of payment (about funding), having demonstrated that the format of ordinary annuities with expired or anticipated equal payments, generates higher interest payments regarding to payments made by arithmetical and geometrical gradients series.

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Authors have declared that no competing interests exist.

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