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Effect Of Discrete Heater Placed At Bottom Section Of A Porous Annulus

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ABSTRACT

The current work is carried out to investigate the effect of length of isothermal heater in a porous medium fixed between inner and outer radius of the vertical cylinder. The heater is placed at the bottom section of inner radius which maintains the isothermal temperature T_h and outer radius is cooled to isothermal temperature T_∞ . The problem is modeled using Darcy law to characterize the flow behaviour inside the porous medium. It is assumed that the thermal non-equilibrium condition exists between the fluid and solid phase of the porous medium thus separate equations are incorporated for each phase of porous medium to account for the thermal non-equilibrium condition. The study is conducted for different lengths of heater corresponding to the 20%, 35% and 50% of the total height of the cylinder.

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INTRODUCTION

Heat transfer in porous medium is one of the important topics for scientific community that lead to enormous amount of work addressing plenty of issues of porous medium. The ample amount of research dedicated to porous medium is very much justifiable in the light of its importance in various applications in scientific and technological fields such as geothermal heat extractions, nuclear reactor waste disposal, heat exchangers, electronic components, solar energy storage technology, exothermic reactions in packed bed reactors, storage of grains, food processing, and the spread of pollutants underground etc. There are good books Nield, D. and A. Bejan (2006), Vafai (2000), Pop and Ingham, (2001), which explain the various phenomenon and issues related to porous medium. The heat transfer in porous medium is generally studied with respect to various geometries such as porous medium adjacent to vertical plate or inside square cavity or cylindrical geometry etc. Porous medium fixed in an annular cylinder is another area that has grabbed the attention of researchers in recent times. Among the earliest work in this area includes that of Prasad, and Kulacki(1984), Rajamani *et al.* (1995) and in last decade Badruddin *et al.* (2006a, 2006b, 2007) as well as Ahmed *et al.* (2009,) covering the thermal equilibrium as well as non-equilibrium conditions in porous vertical annulus. In many cases there exists a discrepancy in the temperature of solid matrix and the fluid flowing inside the porous medium which in turn gives rise to a condition commonly referred as thermal non equilibrium. In this case the energy equation is modeled separately for fluid as well as solid phase of porous medium. The study of porous medium using two temperature model for energy equation is relatively new which is evident from the dearth of literature addressing thermal non-equilibrium approach compared to that of thermal equilibrium approach. Saeid(2006) presented the analysis of the free convection in a horizontal cylinder using the thermal non equilibrium approach. Similar approach was adopted by Badruddin *et al.*(2006b) to study the heat transfer through vertical annulus embedded with porous medium. Salman *et al* (2011) employed the thermal non equilibrium model to analyze the mixed convection in an annular vertical cylinder. In most of the applications the temperature along the length of the cylinder is not uniform or rather, particular place or particular portion of the geometry is subjected to heating. In this case the flow pattern and the heat transfer characteristics would get affected in significant manner as obvious from the work of Saha (2010), Tye-Gingras *et al.* (2010), Müftüoğlu,

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and Bilgen (2008). Some of the other works in the area of discrete or localized heating includes that of Yan and Lin (1987) Akdag (2010). Tao and Zhang (2003). Zhao *et al.* (2007), Zhao *et al.* (2008), Saeid and Pop (2004), Sivakumar *et al.* (2010). Sivasankaran *et al.* (2011), Sankar and Do (2010). It is observed that there is no literature addressing the issue of different sizes of heater being placed at lower part of vertical annulus to the best of Authors' knowledge. Therefore an attempt has been made in the present study to determine the effects of the location and the length of the heater on the fluid flow and heat transfer characteristics by considering the various physical parameters.

Analysis:

The present problem focuses to study the heat transfer in a vertical annular cylinder subjected to discrete heating. Consider an annulus with inner radius r_i and outer radius r_o having porous medium fixed in between inner and outer radii as illustrated in fig.1. The coordinate system is chosen in such a way that the r and z axis points towards the radial and vertical direction of the annulus. A section of the inner surface of the annulus is heated to constant temperature T_w and the outer surface is maintained at constant temperature T_∞ such that $T_w > T_\infty$. The following assumptions are applied:

- The fluid follows Darcy law.
- The convective fluid and the porous medium are not in local thermal equilibrium.
- There is no phase change of the fluid in the medium.
- The properties of the fluid and those of the porous medium are homogeneous and isotropic.
- Fluid properties are constant except the variation of density with temperature.
- The fluid is transparent to radiation.
- The radiative heat flux in the z -direction is negligible in comparison to that in the r -direction.

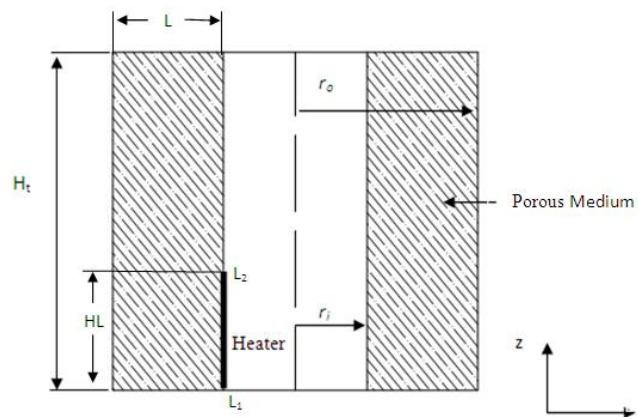


Fig. 1: Schematic diagram of porous annulus.

The governing equations that describe the flow behavior can be written as:

$$\frac{\partial(ru)}{\partial r} + \frac{\partial(rw)}{\partial z} = 0 \quad (1)$$

$$\frac{\partial w}{\partial r} - \frac{\partial u}{\partial z} = \frac{gK\beta}{\nu} \frac{\partial T}{\partial r} \quad (2)$$

$$(\rho c_p)_f \left(u \frac{\partial T_f}{\partial r} + w \frac{\partial T_f}{\partial z} \right) = \phi k_f \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T_f}{\partial r} \right) + \frac{\partial^2 T_f}{\partial z^2} \right) + h(T_s - T_f) \quad (3)$$

$$(1 - \phi) k_s \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T_s}{\partial r} \right) + \frac{\partial^2 T_s}{\partial z^2} \right) = h(T_s - T_f) + (1 - \phi) \frac{1}{r} \frac{\partial}{\partial r} (r q_r) \quad (4)$$

Corresponding boundary conditions are:

$$\text{at } r = r_i \text{ and } L_1 \leq z \leq L_2, \quad T_f = T_s = T_w, \quad u = 0 \quad (5a)$$

$$\text{at } r = r_o, \quad T_f = T_s = T_\infty, \quad u = 0 \quad (5b)$$

where u and w are Darcy's velocities in the r and z directions respectively,

The continuity equation (1) can be satisfied automatically by introducing the stream function ψ as:

$$u = -\frac{1}{r} \frac{\partial \psi}{\partial z} \quad (7a)$$

$$w = \frac{1}{r} \frac{\partial \psi}{\partial r} \tag{7b}$$

Radiation can be represented using Rosseland approximation as Raptis (1998), Badruddin *et al.* (2006c, 2007, 2011, 2006d)

$$q_r = -\frac{4n^2\sigma}{3\beta_R} \frac{\partial T^4}{\partial r} \tag{8}$$

The term T^4 in equation (8) can be expanded in Taylor series. Expanding T^4 about T_∞ Raptis (1998), Badruddin *et al* (2006c, 2006d, 2007, 2011,) and neglecting higher order terms results as:

$$T^4 \approx 4TT_\infty^3 - 3T_\infty^4 \tag{9}$$

The following parameters have been used for non-dimensionalisation of the governing equations.

$$\begin{aligned} \bar{r} &= \frac{r}{L_{ref}}, & \bar{z} &= \frac{z}{L_{ref}}, & \bar{L}_1 &= \frac{L_1}{H_t}, & \bar{L}_2 &= \frac{L_2}{H_t}, & \bar{\psi} &= \frac{\psi}{\alpha \phi L_{ref}}, & \bar{T} &= \frac{(T - T_o)}{(T_w - T_\infty)}, & T_o &= \frac{(T_w + T_\infty)}{2} \\ R_d &= \frac{4\sigma T_c^3}{\beta_R k_s}, & Ra &= \frac{g\beta\Delta TKL_{ref}}{\phi\nu\alpha_f}, & H &= \frac{hL_{ref}^2}{\phi k_f}, & Kr &= \frac{\phi k_f}{(1-\phi)k_s} \end{aligned} \tag{10}$$

Substitution of equations (6)-(10) into equations (2)-(4) results into:

$$\frac{\partial^2 \bar{\psi}}{\partial \bar{z}^2} + \bar{r} \frac{\partial}{\partial \bar{r}} \left(\frac{1}{\bar{r}} \frac{\partial \bar{\psi}}{\partial \bar{r}} \right) = \bar{r} Ra \frac{\partial \bar{T}_f}{\partial \bar{r}} \tag{11}$$

$$\frac{1}{\bar{r}} \left[\frac{\partial \bar{\psi}}{\partial \bar{r}} \frac{\partial \bar{T}_f}{\partial \bar{z}} - \frac{\partial \bar{\psi}}{\partial \bar{z}} \frac{\partial \bar{T}_f}{\partial \bar{r}} \right] = \left(\frac{1}{\bar{r}} \frac{\partial}{\partial \bar{r}} \left[\left(1 + \frac{4R_d}{3} \right) \bar{r} \frac{\partial \bar{T}_f}{\partial \bar{r}} \right] + \frac{\partial^2 \bar{T}_f}{\partial \bar{z}^2} \right) + H(\bar{T}_s - \bar{T}_f) \tag{12}$$

$$\left(\frac{1}{\bar{r}} \frac{\partial}{\partial \bar{r}} \left[\left(1 + \frac{4R_d}{3} \right) \bar{r} \frac{\partial \bar{T}_s}{\partial \bar{r}} \right] + \frac{\partial^2 \bar{T}_s}{\partial \bar{z}^2} \right) = HKr(\bar{T}_s - \bar{T}_f) \tag{13}$$

The boundary conditions take the form as:

at $\bar{r} = r_i$ and $\bar{L}_1 \geq \bar{z} \geq \bar{L}_2$, $\bar{\psi} = 0$, $\bar{T}_f = \bar{T}_s = \frac{1}{2}$ (14a)

at $\bar{r} = r_o$, $\bar{\psi} = 0$, $\bar{T}_f = \bar{T}_s = -\frac{1}{2}$ (14b)

The Nusselt number is calculated using following expressions:

For fluid $\bar{Nu}_f = -\frac{1}{(\bar{L}_2 - \bar{L}_1)} \int_{\bar{L}_1}^{\bar{L}_2} \left(\frac{\partial \bar{T}_f}{\partial \bar{r}} \right)_{\bar{r}=\bar{r}_i} d\bar{z}$ (16)

For solid $\bar{Nu}_s = -\frac{1}{(\bar{L}_2 - \bar{L}_1)} \int_{\bar{L}_1}^{\bar{L}_2} \left[\left(1 + \frac{4}{3} R_d \right) \frac{\partial \bar{T}_s}{\partial \bar{r}} \right]_{\bar{r}=\bar{r}_i} d\bar{z}$ (17)

The total heat transfer rate for the present problem can be expressed as:

$$q_t = \left\{ \phi k_f \left(\frac{\partial T_f}{\partial r} \right)_{r=r_i, r_o} + (1-\phi)k_s \left(1 + \frac{4}{3} R_d \right) \left(\frac{\partial T_s}{\partial r} \right)_{r=r_i} \right\} \tag{18a}$$

Using equation (17a), it can be shown that the average total Nusselt number is:

$$\bar{Nu}_t = \left(\frac{-1}{Kr+1} \right) \frac{1}{(\bar{L}_2 - \bar{L}_1)} \int_{\bar{L}_1}^{\bar{L}_2} \left\{ Kr \left(\frac{\partial \bar{T}_f}{\partial \bar{r}} \right)_{\bar{r}=\bar{r}_i} + \left(1 + \frac{4}{3} R_d \right) \left(\frac{\partial \bar{T}_s}{\partial \bar{r}} \right)_{\bar{r}=\bar{r}_i} \right\} d\bar{z} \tag{18b}$$

RESULTS AND DISCUSSION

In the present study, finite element method is used to solve the governing coupled partial differential equations (11)-(13), subjected to boundary conditions (14)–(15). The equations are coupled thus the change in one equation would affect the outcome of other two equations and vice-versa. A simple 3 noded triangular element is used to mesh the domain. Care is taken to make the results mesh independent by choosing

sufficiently dense mesh for the domain. The equations are solved in an iterative manner by setting the tolerance level of 10^{-5} , 10^{-5} and 10^{-7} for \bar{T}_f , \bar{T}_s and \bar{w} respectively. The above mentioned tolerance level indicates the difference in the values of each of the solution variables at all nodes in the domain between two successive iterations. The present method is verified by comparing the results with previously published data [30] for the case of thermal equilibrium modeling. The data for comparison with the thermal equilibrium model is obtained by setting the variables $H = 1000$ & $\gamma = 1000$ at which the thermal equilibrium condition is reached. Table-1 shows the comparative results. It is obvious from table 1 that the present method has good accuracy in predicting the heat transfer behavior of the porous medium filled in a vertical annulus.

Table 1: Validation of present results

A_r	Ra	R_r	\bar{Nu}	
			Rajamani <i>et al.</i> (1995)	Present
5	50	0.25	1.619	1.7117
		1	2.105	2.1800
	100	0.25	2.349	2.4596
		1	3.025	3.0859
	200	0.25	3.694	3.7034
		1	4.630	4.5618

Fig.2 shows the streamlines and the isothermal lines for fluid and solid when heater is placed at the bottom section of the inner radius. The bottom of the heater always coincides with $\bar{Z} = 0$ for three lengths i.e. HL= 20%, 35% and 50% indicating that the heater length varies from bottom to upward direction in all the three cases. The streamline and isothermal figures corresponds to constant values of $Ra = 100$, $Rd = 1$, $Ar = 2$, $Rr = 1$, $H = 15$ and $K = 1$. It is seen that the fluid movement is restricted to only a small section at the lower part of the annulus for HL=20%. However the circulation region has occupied considerable part of the porous medium when heater length is increased to 35% and beyond. It is also seen that the fluid cell moves towards the central part of the annulus when heater length is increased. It is worth mentioning that the fluid and solid isotherms tend to look alike at large values of H and K as illustrated in fig.3. This is consistent with findings that the thermal equilibrium approaches when inter-phase heat transfer coefficient (H) and thermal conductivity ratio (K) are very large (Saied, 2006, Badruddin *et al*, 2006b, 2007, Ahmed, 2011) that facilitates the exchange of energy between fluid and solid phases.. The increased heater length leads to higher amount of thermal energy penetration inside the depth of the porous medium along the radial direction in solid as well as fluid phase. It is seen that the increased heater length has higher effect on fluid than the solid phase which is reflected by greater area of porous medium being occupied by higher temperature fluid isotherms as compared to that of solid isotherms. This can be attributed to the fact that the main energy transport mode in solid is by conduction whereas convective current plays vital role in energy transfer in fluid phase.

Fig.4 shows the average Nusselt number variation with respect to the interphase heat transfer coefficient when the heater is placed at the bottom section of the annulus. It may be noted that the heater length is varied in 3 steps i.e. 20%, 35% and 50% of the total height of the cylinder. Fig.4 corresponds to the constant values of the parameters $Ra = 100$, $Rd = 1$, $Ar = 2$, $Rr = 1$ and $Kr = 10$. It is obvious from fig.3 that the average Nusselt number for fluid (Nu_f), solid (Nu_s) and total Nusselt number (Nu_t) increases with increase in the heater length. It is also evident that average Nusselt number for solid phase is influenced considerably because of change in H. It is evident from fig.4 that the heat transfer rate in solid phase initially decreases with increase in interphase heat transfer coefficient until $H \leq 0.6$ and thereafter starts increasing with increase in H. It may be noted that the Nu_s does not decrease in case when whole of the inner surface is maintained isothermally to T_w (Badruddin, 2006b). Thus the behavior of Nu_s in segmental heating is different from that of whole surface heating. Fig.4 shows the average Nusselt number variation with respect to the conductivity ratio for heater placed at the bottom section of annulus. The other parameters relevant to fig 5 are $Ra = 100$, $Rd = 1$, $Ar = 2$, $Rr = 1$, $H = 15$. It is clear that the average Nusselt number decreases initially with increase in Kr and then starts increasing at higher values of Kr for all the three lengths of heater being investigated which is again different from the observed behavior of whole surface heating (Badruddin, 2006b). It is also evident that the effect of Kr is greater if the heater is of shorter length.

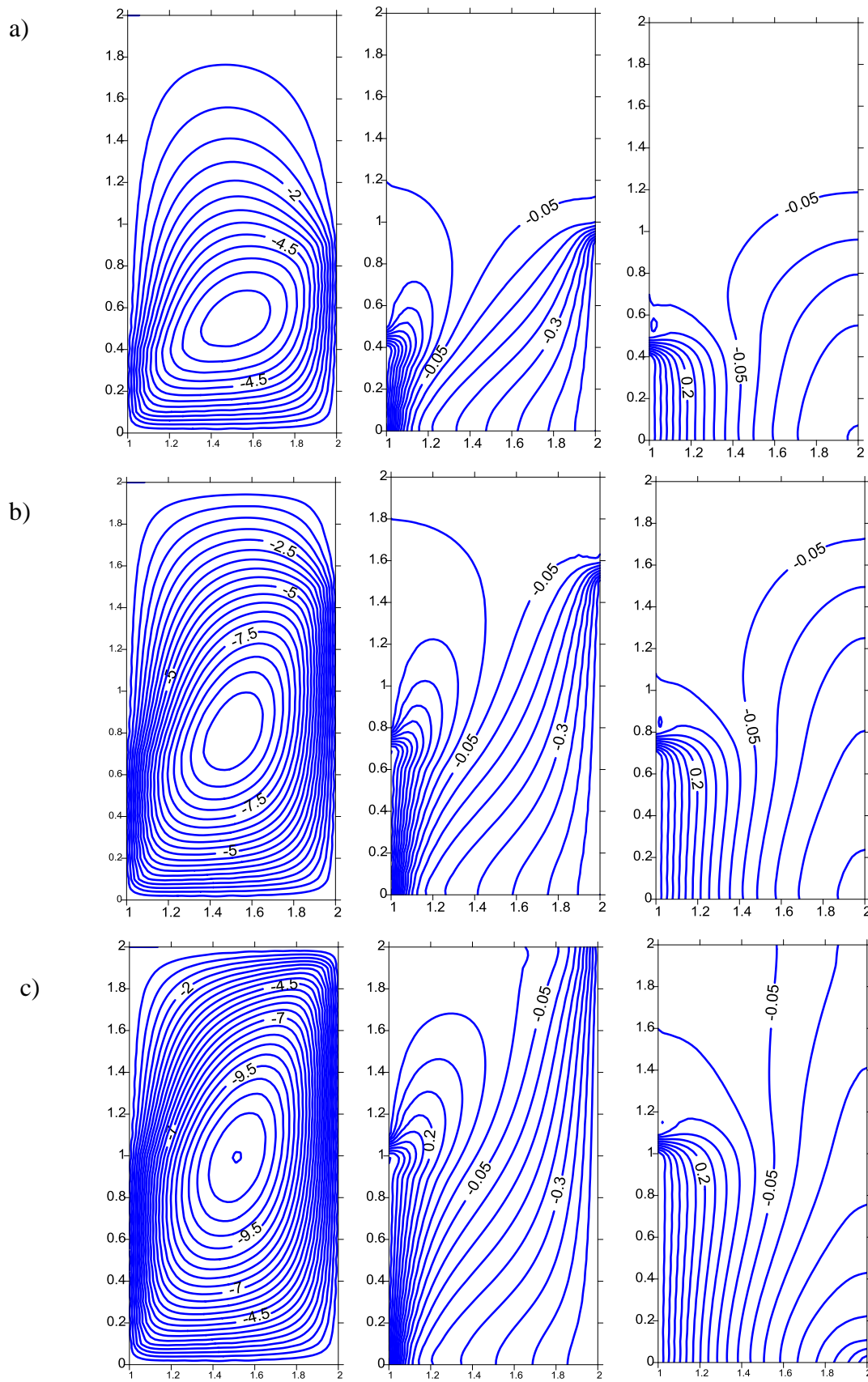


Fig. 2: Streamlines (left) Isotherms for fluid (centre) and solid (right) for heater placed at bottom section a) HL=20% b) HL=35% c) HL=50% at $Ra = 100, Rd = 1, Ar = 2, Rr = 1, H = 15$ and $K = 1$

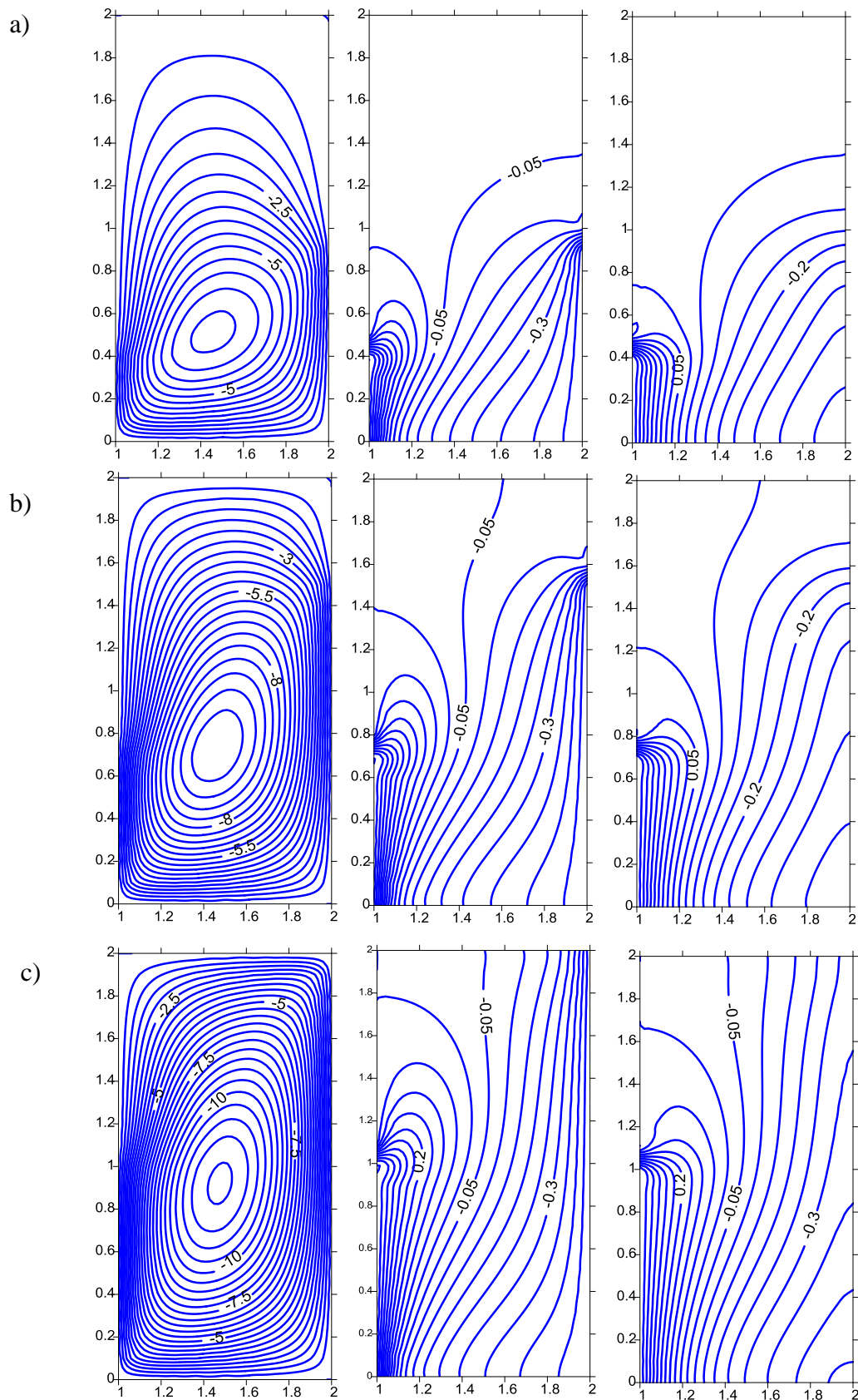


Fig. 3: Streamlines (left) Isotherms for fluid (centre) and solid (right) for heater placed at bottom section a) HL=20% b) HL=35% c) HL=50% at $Ra = 100, Rd = 1, Ar = 2, Rr = 1, H = 100$ and $K = 1$

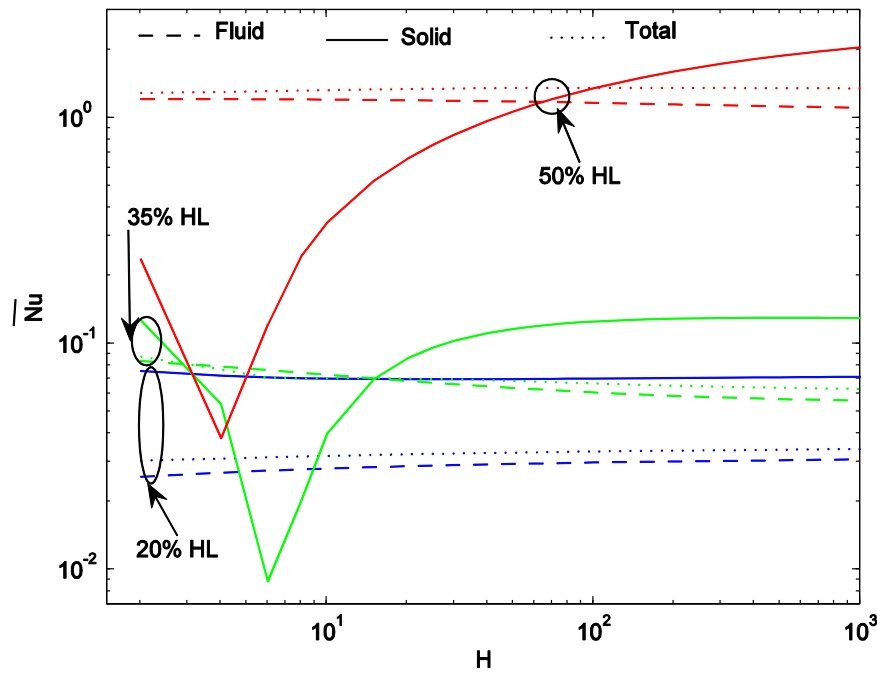


Fig. 4: Average Nusselt number variation with H and different heater length at bottom of annulus

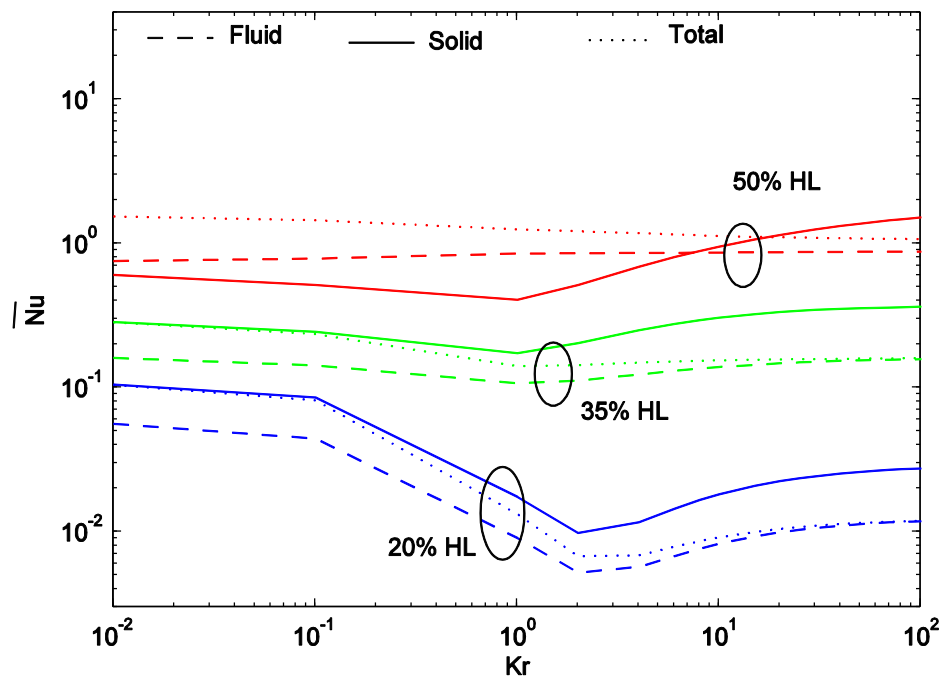


Fig. 5: Average Nusselt number variation with Kr and different heater length at Bottom of annulus

Conclusion:

The present study is carried out to investigate the effect of heater length and placement at the inner radius of a porous annulus. It can be said that the increased heater length increases the heat transfer rate in the annulus. It is found that the increased heater length at bottom section of annulus has higher effect on fluid than the solid phase. It is seen that the heat transfer rate in solid phase initially decreases with increase in interphase heat transfer coefficient until $H \leq 0.6$ and thereafter starts increasing with increase in H . It can be concluded that the heat transfer behavior of segmental heating is substantially different from that of whole wall heating.

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