General Fault Admittance Method Solution of a Balanced Line-to-Line-to-Line Fault

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ABSTRACT

Background: The general fault admittance method solution of a faulted power system does not require prior knowledge of how the positive, negative and zero sequence networks are interconnected to find the symmetrical component currents and voltages at the faults and in all the branches of the network for the fault. It therefore differs from the symmetrical components method of unbalanced faults analysis that is based on interconnection of sequence networks. Solution of a balanced line-to-line-to-line short circuit or metallic fault by the general fault admittance method requires simulation of the short circuits because the fault admittances are infinite, being the inverses of fault impedances. The paper presents a technique of simulating the short circuits. Objective: The general fault admittance method is used to solve a balanced line-to-line-to-line fault by representing the short circuits by low impedance values instead of zeros. The solution presents the equations and the short circuit impedances that are used to simulate the fault to obtain the solution by the general fault admittance method. A technique for using different types and values of impedances to obtain the fault solution is presented. Results: The general fault admittance method is used to solve a balanced metallic fault on a simple power system. The faults are simulated by different types and values of impedances and the convergence of the solutions are compared. The main results show that representing the short circuits by small resistance values gives the best convergence. Conclusion: The general fault admittance method may be used to solve balanced short circuit or metallic faults, without prior knowledge that only the positive sequence network is involved in the solution.


INTRODUCTION

The paper presents a method for solving a balanced line-to-line-to-line fault using the general fault admittance method. The general fault admittance method differs from the classical approaches based on connection of symmetrical components networks in that it does not require prior knowledge of how the sequence components of currents and voltages are related. In the classical approach, knowledge of how the sequence components are related is required because the sequence networks have to be connected in a prescribed way for a particular fault. Then the sequence currents and voltages at the fault are determined, after which symmetrical component currents and voltages in the rest of the network are calculated. Phase currents and voltages are found by transforming the respective symmetrical component values (Elgerd, O.I., 1971; Sadat, H., 2004; Das, J.C., 2002; El-Hawary, M., 1995; Zhu, J., 2004; Wolter, M., Oswald, B.R., 2007; Anderson, P.M., 1995; Oswald, B.R., Panosyan, A., 2006).

The fault admittance method is general in the sense that any fault impedances can be represented, provided the special case of a zero impedance fault is catered for. This paper discusses a procedure for simulating short circuits for the balanced line-to-line-to-line fault.

Background:

A line-to-line-to-line fault presents low value impedances, with zero value for a direct short circuits or metallic faults, between three phases at the point of fault in the network. In general, a fault may be represented as in Figure 1.
In Figure 1, a fault at a busbar is represented by fault admittances in each phase, i.e. the inverse of the fault impedance in the phase, and the admittance in the ground path. Note that the fault admittance for a short-circuited phase is represented by an infinite value, while that for an open-circuited phase is a zero value. In a line-to-line-to-line fault the fault is assumed to be between phases $a$, $b$, and $c$. Thus for a line-to-line-to-line short circuit fault the admittances $Y_{af}$, $Y_{bf}$ and $Y_{cf}$ are infinite.

![Fig. 1: General Fault Representation.](image-url)

A systematic approach for using a fault admittance matrix in the general fault admittance method is given by (Sakala J.D., Daka J.S.J., 2013, 2012, 2007). The method is based on the work by (Elgerd, O.I., 1971). The method is summarized in this paper to give the reader a comprehensive view of some of the key equations and the methodology.

The general fault admittance matrix is given by

$$Y_f = \frac{1}{Y_{af} + Y_{bf} + Y_{cf}} \begin{bmatrix}
Y_{af} & -Y_{af} & -Y_{af} \\
-Y_{af} & Y_{af} + Y_{bf} + Y_{cf} & -Y_{af} \\
-Y_{af} & -Y_{af} & Y_{af} + Y_{bf} + Y_{cf}
\end{bmatrix}$$

(1)

Equation (1) is transformed using the symmetrical component transformation matrix be $T$, and its inverse $T^{-1}$, where

$$T = \begin{bmatrix}
1 & 1 & 1 \\
\alpha & \alpha & 1 \\
\alpha^2 & \alpha & 1
\end{bmatrix} \quad \text{and} \quad T^{-1} = \frac{1}{3} \begin{bmatrix}
1 & \alpha & \alpha^2 \\
1 & \alpha^2 & \alpha \\
1 & 1 & 1
\end{bmatrix},$$

in which $\alpha = e^{j120^\circ}$ is a complex operator.

The symmetrical component fault admittance matrix is given by the product

$$Y_{fs} = T^{-1} Y_f T$$

The general expression (Sakala J.D., Daka J.S.J, 2013, 2012, 2007; Elgerd, O.I., 1971) for $Y_{fs}$ is given by:

$$Y_{fs} = \frac{1}{Y_{af} + Y_{bf} + Y_{cf}} \begin{bmatrix}
Y_{f11} & Y_{f12} & Y_{f13} \\
Y_{f21} & Y_{f22} & Y_{f23} \\
Y_{f31} & Y_{f32} & Y_{f33}
\end{bmatrix}$$

(2)

where

$$Y_{f11} = Y_{f22} = \frac{1}{3} Y_{af} \left(V_{af} Y_{af} + Y_{bf} + Y_{cf}\right) + Y_{af} Y_{bf} Y_{af} + Y_{bf} Y_{cf} + Y_{bf} Y_{cf}$$

$$Y_{f33} = \frac{1}{3} Y_{af} \left(V_{af} + Y_{bf} + Y_{cf}\right)$$

$$Y_{f12} = \frac{1}{3} Y_{af} \left(Y_{bf} Y_{af} + \alpha^2 Y_{bf} + \alpha Y_{bf}\right) - Y_{bf} Y_{cf} + \alpha Y_{bf} Y_{af} + \alpha Y_{af} Y_{bf} + \alpha^2 Y_{af} Y_{bf}$$

$$Y_{f21} = \frac{1}{3} Y_{af} \left(Y_{bf} Y_{af} + \alpha Y_{bf} + \alpha^2 Y_{bf}\right) - Y_{bf} Y_{cf} + \alpha^2 Y_{bf} Y_{af} + \alpha Y_{af} Y_{bf}$$
The expressions do simplify considerably depending on the type of fault. For example, consider a balanced three-phase fault with $Y_{af} = Y_{bf} = Y_{cf} = Y$ and the ground path admittance equal to $Y_{gf}$.

There is no coupling between the positive, negative and zero sequence networks. Since there are no negative and zero sequence voltages before the fault there will be no corresponding currents during and after the fault.

Note that in the case that the ground is not involved $Y_{gf} = 0$ and the symmetrical component fault admittance matrix reduces to

$Y_{fs} = \begin{bmatrix} Y & 0 & 0 \\ 0 & Y & 0 \\ 0 & 0 & 0 \end{bmatrix}$

(3b)

**Currents in the Fault:**

At the faulted busbar, say busbar $j$, the symmetrical component currents in the fault are given by:

$I_{sf} = Y_{fs} \left( U + Z_{sj} Y_{fs} \right)^{-1} V_{sj}^0$

(4)

where $U$ is the unit matrix

$U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

and $Z_{sj}$ is the $j^{th}$ component of the symmetrical component bus impedance matrix

$Z_{sj} = \begin{bmatrix} Z_{sj+} & 0 & 0 \\ 0 & Z_{sj-} & 0 \\ 0 & 0 & Z_{sj0} \end{bmatrix}$

The element $Z_{sj+}$ is the Thevenin’s positive sequence impedance at the faulted busbar, $Z_{sj-}$ is the Thevenin’s negative sequence impedance at the faulted busbar, and $Z_{sj0}$ is the Thevenin’s zero sequence impedance at the faulted busbar.

Note that as the network is balanced the mutual terms are all zero.

In equation (4) $V_{sj}^0$ is the prefault symmetrical component voltage at busbar $j$ the faulted busbar:

$V_{sj}^0 = \begin{bmatrix} V_{s+}^0 \\ V_{s-}^0 \\ V_{s0}^0 \end{bmatrix}$

where $V_s$ is the positive sequence voltage before the fault. The negative and zero sequence voltages are zero because the system is balanced prior to the fault.

The phase currents in the fault are then obtained by transformation:

$I_{sj} = Y_{fs} = \left( U + Z_{sj} Y_{fs} \right)^{-1} V_{sj}^0$

(5)

**Voltages at the Busbars:**

The symmetrical component voltage at the faulted busbar $j$ is given by:

$V_{sj} = \begin{bmatrix} V_{s+} \\ V_{s-} \\ V_{s0} \end{bmatrix} = (U + Z_{sj} Y_{fs})^{-1} V_{sj}^0$

(6)
The symmetrical component voltage at a busbar $i$ for a fault at busbar $j$ is given by:

$$V_{ji} = V_{is} - Z_{ij} Y_{j} (U + Z_{ij} Y_{j}) V_{ji}^0$$

where $Y_{j} = \begin{bmatrix} Y_{j+} & 0 & 0 \\ 0 & Y_{j-} & 0 \\ 0 & 0 & Y_{j0} \end{bmatrix}$

(7)

gives the symmetrical component prefault voltages at busbar $i$. The negative and zero sequence prefault voltages are zero.

In equation (7), $Z_{ij}$ gives the $ij$th components of the symmetrical component bus impedance matrix, the mutual terms for row $i$ and column $j$ (corresponding to busbars $i$ and $j$):

$$Z_{ij} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The phase voltages in the fault, at busbar $j$, and at busbar $i$ are then obtained by transformation:

$$V_{gj} = TV_{fj} \quad \text{and} \quad V_{gi} = TV_{fi}$$

(8)

**Currents in Lines, Transformers and Generators:**

The symmetrical component currents in a line between busbars $i$ and $j$ are given by:

$$I_{ji} = Y_{j0} (V_{fi} - V_{pj})$$

where

$$Y_{j0} = \begin{bmatrix} Y_{j+} & 0 & 0 \\ 0 & Y_{j-} & 0 \\ 0 & 0 & Y_{j0} \end{bmatrix}$$

is the symmetrical component admittance of the branch between busbars $i$ and $j$.

Equation (9) also applies to transformers, when there is no phase shift between the terminal quantities or when the phase shift is catered for when assembling the phase quantities. In the latter case the positive sequence values are phase shifted forward and the negative sequence values are phase shifted backwards by the phase shift (usually $\pm 30^\circ$). The line currents on the delta connected side of a delta star transformer should have the appropriate phase to line conversion factor.

Equation (9) also applies to a generator where the source voltage will be the prefault induced voltage and the receiving end busbar voltage is the postfault voltages at the busbar.

The phase currents in the branch are found by transformation:

$$I_{gj} = TV_{fj}$$

(10)

**Balanced Line-To-Line-To-Line Fault Simulation:**

Equation (3b) gives the symmetrical component fault admittance matrix for a balanced line-to-line-to-line fault when the fault impedances in the faulted phases are equal. It is restated here for ease of reference:

$$Y_{j} = \begin{bmatrix} Y & 0 & 0 \\ 0 & Y & 0 \\ 0 & 0 & Y \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The value $Y$ is the fault admittance in the faulted phases.

The symmetrical component fault admittance matrix may be substituted in equation (4) to obtain the simplified value of $I_{gj}$ given in equation (11), in which $V_{j}^0$ is the prefault voltage on bus bar $j$. The simplified formulation in equation (11), for the line-to-line-to-line fault, is useful for checking the accuracy of the symmetrical component currents in the fault when the general form of the solution used.
\[ I_{pu} = \frac{V^0}{Z_{pu}} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \]  

(11)

The impedances required to simulate the line-to-line-to-line fault in general terms are the impedances in the faulted phases. In the current work various impedances are considered namely; purely resistive, resistive and inductive and purely inductive. The impedances in the faulted phases are assumed equal. In practice, this is not significant as the general form allows use of different fault admittance values.

**Computation of the Balanced Line-To-Line-To-Line Fault:**

A computer program has been developed, which incorporates the equations (1) to (11), to solve unbalanced faults for a general power system using the fault admittance matrix method. The program is then applied on a simple power system comprising of three bus bars to solve for a line-to-line-to-line fault. A simple system is chosen because it is easy to check the results against those that are obtained by hand. Once the program is validated on a simple system then it can be used on large systems and ultimately on practical systems with confidence.

**Sample System:**

Figure 2 shows a simple three bus bar power system with one generator, one transformer and one transmission line. The system if configured based on the simple power system that Saadat uses (Sadat, H., 2004).

![Sample Three Bus Bar System](image)

**Fig. 2:** Sample Three Bus Bar System.

The power system per unit data is given in Table 1, where the subscripts 1, 2, and 0 refer to the positive, negative and zero sequence values respectively. The neutral point of the generator is grounded through a zero impedance.

**Table 1:** Power System Data.

<table>
<thead>
<tr>
<th>Item</th>
<th>S\text{base} (MVA)</th>
<th>V\text{base} (kV)</th>
<th>X_1 (pu)</th>
<th>X_2 (pu)</th>
<th>X_0 (pu)</th>
</tr>
</thead>
<tbody>
<tr>
<td>G_1</td>
<td>100</td>
<td>20</td>
<td>0.15</td>
<td>0.15</td>
<td>0.05</td>
</tr>
<tr>
<td>T_1</td>
<td>100</td>
<td>20\text{220}</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>L_1</td>
<td>100</td>
<td>220</td>
<td>0.25</td>
<td>0.25</td>
<td>0.7125</td>
</tr>
</tbody>
</table>

The transformer windings are delta connected on the low voltage side and earthed-star connected on the high voltage side, with the neutral solidly grounded. The phase shift of the transformer is \(30^\circ\), i.e. from the generator side to the line side. Figure 3 shows the transformer voltages for a Yd11 connection which has a \(30^\circ\) phase shift.

The computer program incorporates an input program that calculates the sequence admittance and impedance matrices and then assembles the symmetrical component bus impedance matrix for the power system. The symmetrical component bus impedance incorporates all the sequences values and has \(3n\) rows and \(3n\) columns where \(n\) is the number of bus bars. In general, the mutual terms between sequence values are zero as a three-phase power system is, by design, balanced.

The power system is assumed to be at no load before the occurrence of a fault. In practice the pre-fault conditions, established by a load flow study may be used. For developing a computer program the assumption of no load, and therefore voltages of 1.0 per unit at the bus bars and in the generator, is adequate.

The line-to-line-to-line fault is assumed to be at busbar 1, the load busbar. Various impedances to simulate the line-to-line-to-line fault are considered; a purely resistive, a resistive and inductive combination, and a purely inductive value.

The line-to-line-to-line fault is described by the impedances in the respective phases and in the ground path. In the general fault admittance method the impedances to be input are those in the \(a\), \(b\) and \(c\) phases. The open circuit values for the ground path is not input since its respective fault admittance is zero.

The initial fault impedance values are of the order of \(10^{-3}\) \(\Omega\). The sequence fault currents is calculated for the initial value. A second value of the fault impedance is used, obtained by multiplying the initial value by a
factor of $10^{-1}$. The second value of sequence fault currents is calculated. The absolute value of the change in the positive sequence current is compared against a tolerance of $10^{-8}$, and if smaller the solution is considered converged. If the absolute value of the change is larger than the tolerance, the fault impedance is again reduced and another value calculated. The iterative process is repeated until either convergence or non-convergence. Note that the order of the initial value of the fault impedances is much smaller than any of the components positive sequence impedances.

![Diagram of Delta Star Transformer Voltages](image)

**Fig. 3:** Delta Star Transformer Voltages for Yd11.

The presence of the delta-earthed-star transformer poses a challenge in terms of its modelling. In the computer program, the transformer is modelled in one of two ways; as a normal star-star connection, for the positive and negative sequence networks or as a delta-star transformer with a phase shift. In the former model, the phase shifts are incorporated when assembling the sequence currents to obtain the phase values.

In particular on the delta connected side of the transformer the positive sequence currents’ angles are increased by the phase shift while the angle of the negative sequence currents are reduced by the same value. The zero sequence currents, if any, are not affected by the phase shifts.

Both models for the delta star transformer give same results. The $\sqrt{3}$ line current factor is used to find the line currents on the delta side of the delta star transformer.

**RESULTS AND DISCUSSION**

**Fault Simulation Impedances:**

The Thevenin’s self sequence impedances of the network seen from the faulted bus bar are:

$$
\begin{bmatrix}
0.5 & 0 & 0 \\
0 & 0.5 & 0 \\
0 & 0 & 0.8125
\end{bmatrix}
$$

In the classical solution for the balanced line-to-line-to-line fault the negative and zero sequence currents are zero while the positive sequence current is found by inverting the positive sequence impedance at the fault point. Thus the sequence (positive, negative and zero) currents due to a line-to-line-to-line fault at the faulted bus bar are:

$$
- \begin{bmatrix} 2 \\ j0 \\ 0 \end{bmatrix}
$$

In the general fault admittance method the values of fault impedances that give accurate values of sequence currents are given in Table 2. Case 1 in the table is for a resistive fault, case two is for a resistive and inductive fault while case 3 is for an inductive fault. The resistive fault impedance gives a better convergence, since the current tolerance is met by a relatively higher fault impedance, than for the other two cases.

**Table 2:** Solution Convergence Characteristics.

<table>
<thead>
<tr>
<th>Case</th>
<th>Fault impedance</th>
<th>Ground path</th>
<th>Current tolerance</th>
<th>Current difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$r+j0$</td>
<td>$5x10^{-8}$</td>
<td>$1x10^{-8}$</td>
<td>$9.9x10^{-9}$</td>
</tr>
<tr>
<td>2</td>
<td>$(r+jx)$</td>
<td>$(5+j5)x10^{-11}$</td>
<td>$1x10^{-8}$</td>
<td>$1.8x10^{-9}$</td>
</tr>
<tr>
<td>3</td>
<td>$(0+jx)$</td>
<td>$j5x10^{-11}$</td>
<td>$j5x10^{-11}$</td>
<td>$1x10^{-8}$</td>
</tr>
</tbody>
</table>
There is a limit as to how low the fault impedance should be. When the value becomes too low the matrix 
\((U + Z_{\text{ss}}Y_{\text{f}})^{-1}\) in equation (6) may not compute, and the solution for the symmetrical component currents may 
break down. Before solution breakdown the values of the symmetrical component currents become inaccurate, 
depending on how much of the effect of the unity matrix in the equation is lost.

The computation results purely resistive fault impedance for a total fault impedance of \(10^{-9}\) \(\Omega\), are listed in 
Table 3.

**Simulation Results:**

The results obtained from the computer program are listed in Table 3. A summary of the transformer phase 
currents is shown in Figure 4.

![Transformer Currents for a Line-to-Line-to-Line Fault.](image)

**Fault admittance matrix and sequence impedances at the faulted bus bar:**

The symmetrical component fault admittance matrix obtained from the program for the line-to-line-to-line 
fault is in agreement with the theoretical value, obtained using equation (3b). The self sequence impedances at 
the faulted bus bar obtained from the program are equal to the theoretical values.

**Fault Currents:**

The symmetrical component fault currents obtained from the program using equations (4) and (11) are in 
agreement with the theoretical values. In particular, the positive sequence current is equal to the prefault voltage 
divided by the positive sequence self impedance at the faulted bus bar. In addition, the negative and zero 
sequence currents are zero. This is consistent with the classical approach that excludes the negative and zero 
sequence networks from the solution. Thus, only the positive sequence network is considered when calculating 
sequence currents and voltages.

The phase currents in the fault obtained from the program are in agreement with the theoretical values. In 
particular, the currents in the fault are equal to the prefault voltages divided by the positive sequence 
impedances.

The phase currents in the transmission line are equal to the currents in the fault. Note that the current at the 
receiving end of the line is by convention considered as flowing into the line, rather than out of it.

Figure 4 shows the transformer phase currents. The currents on the line side are equal to the currents in the 
line, after allowing for the sign change due to convention. The currents on the line side are balanced and there 
in no current flow into the ground. The currents in the transformer windings satisfy the ampere-turn balance 
requirements of the transformer. The magnitudes of the currents at the sending end of the transformer, the delta 
connected side, are \(\sqrt{3}\) times the magnitudes of the currents in the phase windings, consistent with theory.

The phase fault currents flowing from the generator are balanced and equal to the phase currents into the 
transformer.
**Fault Voltages:**

The symmetrical component voltages at the fault point obtained from the program using equation (7) are in agreement with the theoretical values. In particular, the positive, negative and zero sequence voltages for a balanced metallic line-to-line-to-line fault are zero.

The phase voltages at busbar 2 are balanced and have magnitudes of 50% of the prefault value. At bus bar 3, the phase voltages are balanced with magnitudes of 70% and lead the phase voltages at bus bar 2 by $30^\circ$ in all the phases.

**Table 3: Simulation Results - Unbalanced Fault.**

<table>
<thead>
<tr>
<th>General Fault Admittance Method - Delta Star Transformer Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of busbars = 3</td>
</tr>
<tr>
<td>Number of transmission lines = 1</td>
</tr>
<tr>
<td>Number of transformers = 1</td>
</tr>
<tr>
<td>Number of generators = 1</td>
</tr>
<tr>
<td>Faulted busbar = 1</td>
</tr>
<tr>
<td>Fault type = 3</td>
</tr>
</tbody>
</table>

**Line-to-Line-to-Line Fault**

- Phase a resistance = $5.0000e-010$
- Phase a reactance = $0.0000e+000$
- Phase b resistance = $5.0000e-010$
- Phase b reactance = $0.0000e+000$
- Phase c resistance = $5.0000e-010$
- Phase c reactance = $0.0000e+000$

**Fault Admittance Matrix**

Real and imaginary parts of Fault Admittance Matrix:

<table>
<thead>
<tr>
<th>2.0000e+009 +j 0.0000e+000</th>
<th>0.0000e+000 +j 0.0000e+000</th>
<th>0.0000e+000 +j 0.0000e+000</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0000e+000 +j 0.0000e+000</td>
<td>2.0000e+009 +j 0.0000e+000</td>
<td>0.0000e+000 +j 0.0000e+000</td>
</tr>
<tr>
<td>0.0000e+000 +j 0.0000e+000</td>
<td>0.0000e+000 +j 0.0000e+000</td>
<td>0.0000e+000 +j 0.0000e+000</td>
</tr>
</tbody>
</table>

**Thevenin's Symmetrical Component Impedance Matrix of Faulted Busbar**

Real and imaginary parts of Symmetrical Component Impedance Matrix:

<table>
<thead>
<tr>
<th>0.0000 +j 0.5000</th>
<th>0.0000 +j 0.0000</th>
<th>0.0000 +j 0.0000</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0000 +j 0.0000</td>
<td>0.0000 +j 0.5000</td>
<td>0.0000 +j 0.0000</td>
</tr>
<tr>
<td>0.0000 +j 0.0000</td>
<td>0.0000 +j 0.0000</td>
<td>0.0000 +j 0.8125</td>
</tr>
</tbody>
</table>

**Fault Current in Symmetrical Components**

<table>
<thead>
<tr>
<th>Simplified Method</th>
<th>General Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>+ve 0.0000 +j -2.0000</td>
<td>0.0000 +j -2.0000</td>
</tr>
<tr>
<td>-ve 0.0000 +j 0.0000</td>
<td>0.0000 +j 0.0000</td>
</tr>
<tr>
<td>zero 0.0000 +j 0.0000</td>
<td>0.0000 +j 0.0000</td>
</tr>
</tbody>
</table>

**Magnitude and Angle**

<table>
<thead>
<tr>
<th>Simplified Method</th>
<th>General Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real +ve 2.0000</td>
<td>-90.0000</td>
</tr>
<tr>
<td>Real -ve 0.0000</td>
<td>90.0000</td>
</tr>
<tr>
<td>Real zero 0.0000</td>
<td>90.0000</td>
</tr>
</tbody>
</table>

**Fault Current in Phase Components**

<table>
<thead>
<tr>
<th>Real</th>
<th>Imag</th>
<th>Magn</th>
<th>Angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phase a</td>
<td>0.0000 +j -2.0000</td>
<td>2.0000</td>
<td>-90.0000</td>
</tr>
<tr>
<td>Phase b</td>
<td>-1.7321 +j 1.0000</td>
<td>2.0000</td>
<td>150.0000</td>
</tr>
<tr>
<td>Phase c</td>
<td>1.7321 +j 1.0000</td>
<td>2.0000</td>
<td>30.0000</td>
</tr>
</tbody>
</table>

**Symmetrical Component Voltages at Faulted Busbar**

<table>
<thead>
<tr>
<th>Real</th>
<th>Imag</th>
<th>Magn</th>
<th>Angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>+ve 0.0000</td>
<td>-0.0000</td>
<td>0.0000</td>
<td>-90.0000</td>
</tr>
<tr>
<td>-ve 0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>zero 0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

**Phase Voltages at Faulted Busbar**

<table>
<thead>
<tr>
<th>Real</th>
<th>Imag</th>
<th>Magn</th>
<th>Angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phase a</td>
<td>0.0000</td>
<td>-0.0000</td>
<td>-90.0000</td>
</tr>
<tr>
<td>Phase b</td>
<td>-0.0000</td>
<td>0.0000</td>
<td>150.0000</td>
</tr>
<tr>
<td>Phase c</td>
<td>0.0000</td>
<td>0.0000</td>
<td>30.0000</td>
</tr>
</tbody>
</table>
Postfault Voltages at Busbar number = 1
Phase a 0.0000 -0.0000 0.0000 -90.0000
Phase b -0.0000 0.0000 0.0000 150.0000
Phase c 0.0000 0.0000 0.0000 30.0000

Postfault Voltages at Busbar number = 2
Phase a 0.5000 -0.0000 0.5000 -0.0000
Phase b -0.2500 -0.4330 0.5000 240.0000
Phase c -0.2500 0.4330 0.5000 120.0000

Postfault Voltages at Busbar Number = 3
Phase a 0.6062 0.3500 0.7000 30.0000
Phase b 0.0000 -0.7000 0.7000 -90.0000
Phase c -0.6062 0.3500 0.7000 150.0000

Postfault Currents in Lines
<table>
<thead>
<tr>
<th>Line SE</th>
<th>RE</th>
<th>Current</th>
<th>Current</th>
<th>Current</th>
<th>Current</th>
<th>Current</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>2.0000</td>
<td>90.0000</td>
<td>2.0000</td>
<td>-30.0000</td>
<td>2.0000</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>2.0000</td>
<td>90.0000</td>
<td>2.0000</td>
<td>-30.0000</td>
<td>2.0000</td>
</tr>
</tbody>
</table>

Postfault Currents in Transformers
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<thead>
<tr>
<th>Line SE</th>
<th>RE</th>
<th>Current</th>
<th>Current</th>
<th>Current</th>
<th>Current</th>
<th>Current</th>
</tr>
</thead>
<tbody>
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<td>-60.0000</td>
<td>3.4641</td>
<td>180.0000</td>
<td>3.4641</td>
</tr>
<tr>
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<td>2</td>
<td>2.0000</td>
<td>90.0000</td>
<td>2.0000</td>
<td>-30.0000</td>
<td>2.0000</td>
</tr>
</tbody>
</table>

Postfault Currents in Generators
<table>
<thead>
<tr>
<th>Gen SE</th>
<th>RE</th>
<th>Current</th>
<th>Current</th>
<th>Current</th>
<th>Current</th>
<th>Current</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>3.4641</td>
<td>-60.0000</td>
<td>3.4641</td>
<td>180.0000</td>
<td>3.4641</td>
</tr>
</tbody>
</table>

Conclusion:
A procedure for simulating the fault impedances of a metallic line-to-line-to-line fault has been proposed and tested. The results show that a purely resistive fault impedance gives the best convergence. For the system studied, a value of the order of $10^{-9}$ Ω is found suitable. The method allows the estimated fault impedances to be reduced until the convergence to a preset tolerance is reached. In cases where convergence is not as good as for purely resistive fault impedances the tolerance is reduced and process repeated until convergence is obtained.

The line-to-line-to-line fault is interesting for studying the delta earthed star transformer arrangement. The currents and voltages are balanced, as in the prefault condition, on both sides of the transformer. Phase shifts of 30° between the voltages on the delta side to those on the star connected side are shown, consistent with theory. The results give an insight in the effect that a delta earthed star transformer has on a power system during line-to-line-to-line faults.

The main advantage of the general fault admittance method is that the user is not required to know before hand how the sequence networks should be connected at the fault point in order to obtain the sequence currents and voltages. The user can deduce the various relationships from the results. The method is therefore easier to use and teach than the classical approach in which each sequence network is solved in isolation and then the results combined to obtain the phase quantities.

REFERENCES


