Investigation of MHD Flow and Heat Transfer of a Newtonian Fluid Passing through Parallel Porous Plates in Presence of an Inclined Magnetic Field

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Abstract

The magnetohydrodynamic (MHD) flow of a Newtonian fluid in a horizontal channel bounded by two parallel porous plates in the presence of inclined magnetic field has been investigated with the purpose of determining the effect of variation of Hartmann number, suction parameter and the magnetic inclination angle on velocity and temperature distribution. The flow is driven by the movement of the upper plate and a constant pressure gradient suddenly imposed in a direction parallel to the plates. The two plates are kept at different but constant temperature while joule and viscous dissipation are taken into consideration. Numerical solutions for the governing momentum and energy equations are obtained using finite difference approximation with appropriate initial and boundary conditions. The effect of change of the Hartmann number, suction parameter and the magnetic inclination angle on temperature and velocity profiles is then discussed. Accruing from the results is the fact the fluid velocity and the temperature distribution is influenced significantly by the changes in the above mentioned parameters. Firstly it was found that increase in Hartmann number led to decrease in both velocity and the temperature of the fluid. Secondly increasing the suction parameter led to reduction of flow velocity and the maximum temperature was found to move towards the moving plate. Lastly increasing the magnetic inclination angle was found to increase both velocity and the temperature of the fluid.

Keywords:

Introduction and Literature Review:

The magnetohydrodynamic flow between two parallel plates known as Hartmann flow is a classical problem that has many applications in MHD power generators, MHD pumps, accelerators, aerodynamic heating, polymer technology, petroleum industry and purification of crude oil. (Kuranov et al., 2003) noted that many energy production or conservation processes along with propulsion system involve the use and control of conducting fluids and plasma. MHD generators and accelerators manipulate kinetic energy to generate electricity or accelerate flows. MHD pumps are used to perfect steel casting method producing alloys with fewer defects. Electric plasma propulsion systems require plasma flow control processes to produce thrust and enhance performance. The interest in the outer magnetic field effect on heat-physical process appeared seventy years ago. Research in MHD grew rapidly during the late 1950s as a result of extensive studies of ionized gases for a number of applications. Blum et al (1967) carried out one of the first works in the field of heat and mass transfer in the presence of magnetic field. Recently many research have been carried out related to this field of study. Attia (2004) developed a model that was used to investigate Hall Effect on unsteady MHD couette flow and heat transfer of a Bingham fluid with suction and injection. He studied an electrically conducting viscous incompressible non-Newtonian Bingham fluid bounded by two non-conducting parallel porous plates with heat transfer. The fluid was subjected to a uniform suction and injection. The governing momentum and energy equation were solved numerically using finite difference approximation. The effect of hall current, suction and injection parameter and the non-Newtonian fluid characteristic on velocity and temperature field was investigated. The study revealed that the hall term affects the main velocity component u in the x-direction and gives rise to another velocity w in the z-direction.

In addition increasing the suction parameter was noted to decrease the main velocity u at the center of the channel due to the convection of the fluid from the region of the lower half to the center which has higher fluid speed. It also decreased the temperature at the center due to the influence of the convection in the pumping the fluids from the colder lower half towards the center of the channel. Hazim (2005) studied the effect of suction...
and injection on the unsteady flow between two parallel plates with variable properties. The viscosity of the fluid was assumed to vary exponentially with temperature while the thermal conductivity was assumed to depend linearly with the temperature. The viscous dissipation was taken into consideration while the governing momentum and energy equation were solved using Crank-Nicolson implicit method of finite difference approximation. The results showed that the effect of suction on the velocity depends greatly on the viscosity parameter. Guria and Jana (2006) displayed a model meant to investigate three-dimensional fluctuating couettes flow through the porous plates with heat transfer. The flow considered was unsteady and the viscous incompressible fluid was passed through two horizontal plates. They subjected the stationary plate to a periodic suction and the moving plate to a uniform injection. Approximate solutions were obtained for the velocity and the temperature field. The governing equations for the flow and heat transfer were solved numerically using perturbation technique. The effect of increasing the frequency parameter in periodic suction was found to decrease main flow velocity while on the other hand increasing the parameter increased the cross flow velocity. It was also observed that the temperature increased with the increase in frequency parameter. Ganesh (2007) studied unsteady MHD stokes flow of electrically conducting, viscous and incompressible fluid between two parallel porous plates in presence of transverse magnetic field. They considered a fluid being withdrawn through both walls of the channel at the same rate. Exact solution was obtained for all values of suction Reynolds number and Hartmann number. It was concluded that when Hartmann number was increased the magnitude of axial velocity profiles decreased while that of the radial velocity increased marginally. Das (2009) analyzed the effect of suction and injection on MHD three dimension couettes flow and heat transfer. The model composed of a viscous incompressible electrically conducting fluid passed through a porous medium bounded by two infinite horizontal parallel porous plates in presence of transverse magnetic field. He subjected the upper plate to a constant suction velocity while the lower plate was subjected to a transverse sinusoidal injection velocity. On the basis of certain assumption of the fluid the equation of continuity, momentum and energy were obtained and solved using series expansion method. The effect of flow parameter on the velocity field, temperature field, skin friction, and Nusselt number were studied and analyzed with the help of figures and tables. The study demonstrated that the growing magnetic parameter retarded the main velocity of the flow and accelerated the cross flow however the suction/injection parameter reversed the effect. Moreover increasing both Prandtl number (Pr) and suction parameter had a retarding effect on the temperature field. Attia et al. (2010) examined unsteady MHD couettes flow with heat transfer of a viscoelastic fluid under exponential decaying pressure gradient. An electrically, incompressible non Newtonian viscoelastic fluid was considered. A sudden uniform and an exponential decaying pressure gradient, an external magnetic field perpendicular to the plate and uniform suction and injection through the surface of the plate was applied. The effect of magnetic field, the parameter describing the non-Newtonian behavior and the suction parameter on both the velocity and the temperature was investigated. It was found that increasing Hartmann number (Ha) decreased the velocity at its steady time. This was attributed to the fact that there was magnetic damping force on velocity. On the other hand temperature (T) increased with increase in Hartman number (Ha) at small times but at large times it was found to decrease. This was due to increasing Joule dissipation at small times and decreasing Joule dissipation at large times. The increase in suction parameter lowered the velocity due to convection of the fluid from regions in the lower half to the center which has high fluid velocity. The parameter also lowered the temperature at the center of the channel when it was increased and this was again due to influence of convection in pumping the fluid from cold lower half towards the center of the channel. Attia et al. (2011) investigated MHD flow of an electrically conducting, viscous and incompressible fluid bounded by two parallel non-conducting porous plates with heat transfer. An external magnetic field and a uniform suction and injection are applied perpendicular to the plate while the fluid motion was subjected to an exponential decaying pressure gradient. Increasing Hartmann number was found to decrease velocity due to increasing magnetic damping force. On the other hand its increase caused temperature rise at small times and temperature drop at large times. Again at small times velocity was small and hence joule dissipation increased. For large times there was low velocity and in turn the joule and viscous dissipation was decreased. This consequently reduced temperature. The increase in suction parameter was found to reduce the velocity at the center and its steady state time. This was again attributed to the convection of fluid from regions in the lower half to the center which has higher fluid speed. Iyaya et al. (2012) considered a steady Poiseuille flow between two infinite porous plates in an inclined magnetic field. They examined an electrically conducting viscous and incompressible fluid moving between two infinite parallel plates both kept at a constant distance between them. They were interested on finding out the effect of Hartmann number on the velocity of the fluid. The partial differential equations were obtained and solved analytically and the results were illustrated graphically. The physical aspect of the problem was discussed and the study concluded that high Hartmann flow which implies high magnetic field strength decreases the velocity of the fluid flow. These examples show the importance of knowledge of the laws governing fluid flow for proper understanding of the processes involved. Keeping in view of the wide area of practical importance of fluid flows as mentioned above the objective of the present study is to investigate the unsteady MHD flows and heat transfer of a Newtonian fluid moving between parallel porous plates in presence of an inclined magnetic field.
Mathematical Formulation:

Consider unsteady, fully developed lamina flow of an incompressible viscous fluid through an infinitely long horizontal channel extending in the $x$ and $z$ direction.

The two non-conducting plates are located at $y = \pm h$ and extend from $x = -\infty$ to $\infty$ and from $z = -\infty$ to $\infty$. The upper plate is suddenly set into motion with a velocity $U_0$ while the lower plate is stationary. The upper plate is simultaneously subjected to a step change in temperature from $T_1$ to $T_2$. The upper and the lower plates are kept at two constant temperatures $T_1$ and $T_2$ respectively. The fluid is acted upon by constant pressure gradient $\frac{\partial p}{\partial x}$ in the $x$ direction and a uniform suction from above and injection below which are applied at $t=0$. It should be noted that since the plates of the channel are assumed to be infinite, all of the physical dependent variables will only depend on $y$ and $t$ except pressure. A constant magnetic field $B_0$ is applied in the direction making an acute angle $\theta$ with the $y$ axis. The magnetic Reynolds number is assumed small so that the induced magnetic field is neglected. It is also assumed that no electric field exists and that the Hall effect is neglected. However, viscous heating is included in the model. Under these assumptions the velocity distribution is given as

$$\bar{q}(t) = \langle u(y), V_0, 0 \rangle \quad (1)$$

While the magnetic distribution is written as

$$\bar{B} = \langle 0, B_0 \cos \theta, 0 \rangle \quad (2)$$

Fig. 1: Flow model and coordinate sys

In this model the upper plate is impulsively started and moving with a velocity $U_0$ causing the fluid to flow in presence of a constant pressure gradient. The MHD flow problem is mathematically presented with a continuity equation

$$\nabla \cdot \bar{q} = 0 \quad (3)$$

The momentum equation

$$\rho \left( \frac{\partial \bar{q}}{\partial t} + \langle \bar{q}, \nabla \rangle \bar{q} \right) = -\nabla p + \mu \nabla^2 \bar{q} + \bar{f} \times \bar{B} \quad (4)$$

and the energy equation

$$\rho C_p \left( \frac{\partial T}{\partial t} + \langle \bar{q}, \nabla \rangle T \right) = k \nabla^2 T + \mu \phi + \frac{j^2}{\sigma} \quad (5)$$

where

$$\phi = 2 \left[ \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial u}{\partial y} \right)^2 \right] + \left( \frac{\partial v}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial y} \right)^2 + \left( \frac{\partial w}{\partial x} \right)^2 + \left( \frac{\partial w}{\partial y} \right)^2 + \left( \frac{\partial w}{\partial z} \right)^2 - \frac{2}{3} \left( \nabla \cdot \bar{q} \right)^2 \quad (6)$$

The third term on the right hand side of equation (4) is the Lorentz force and $\bar{f}$ is the current density vector defined as

$$\bar{f} = a (\bar{E} + \bar{q} \times \bar{B}) \quad (7)$$

where

$$\bar{E} = (0, 0, 0) \quad (8)$$

Using the velocity (1), magnetic field (2) and electric field (8) distribution, equation (4) and (5) becomes

$$\rho \left( \frac{\partial \bar{u}}{\partial t} + \bar{V}_0 \frac{\partial \bar{u}}{\partial y} \right) = -\nabla p + \mu \frac{\partial^2 \bar{u}}{\partial x^2} - \sigma B_0^2 \bar{u} \cos^2 \theta \quad (9)$$
\[
\rho c_p \left( \frac{\partial T}{\partial t} + V_0 \frac{\partial T}{\partial y} \right) = k \frac{\partial^2 T}{\partial y^2} + \mu \left( \frac{\partial u}{\partial y} \right)^2 + \sigma B_0^2 u^2 \cos^2 \theta \tag{10}
\]

Where \( \rho, \mu, \sigma, c, k \) are density, coefficient of viscosity, electrical conductivity, specific heat capacity and thermal conductivity of the fluid. The second and the third terms on the right hand side of equation (10) represent the viscous and joule dissipation.

The velocity \( u(y, t) \) satisfies both the initial condition \( u(y, 0) = 0 \) and the boundary conditions; \( u(h, t) = U_0 \) and \( u(-h, t) = 0 \) for \( t > 0 \). The temperature \( T(y, t) \) at any point on the fluid satisfies the initial condition \( T(y, 0) = T_1 \) and the boundary conditions; \( T(h, t) = T_1 \) and \( T(-h, t) = T_2 \) for \( t > 0 \).

Introducing the following dimensionless variables and parameters \( x^* = \frac{x}{h}, \ y^* = \frac{y}{h}, \ u^* = \frac{u}{U_0}, \ t^* = \frac{tU_0}{h} \),

\[
p^* = \frac{p}{\rho U_0^2}, \ T^* = \frac{T - T_1}{T_2 - T_1}, \ Re = \frac{\rho U_0 h}{\mu}, \ S = \frac{V_0}{U_0}, \ Ha^2 = \frac{\sigma B_0^2 h^2}{\mu}, \ Pr = \frac{\mu C_p}{k}, \ E_c = \frac{U_0^2}{C_p(T_2 - T_1)}, \]

\( \) are density, coefficient of viscosity, electrical conductivity, specific heat capacity and thermal conductivity of the fluid. The second and the third terms on the right hand side of equation (10) represent the viscous and joule dissipation.

The initial and boundary condition for the velocity and the temperature distribution in the dimensionless form is written as

\[
u(y, 0) = 0 \quad u(-1, t) = 0 \quad u(1, t) = 1 \quad T(y, 0) = 0 \quad T(-1, t) = 1 \quad T(1, t) = 0 \tag{13}
\]

Equations (11) and (12) represents a system of partial differential equation which is solved numerically under the initial and boundary conditions (13) and (14) using the finite difference approximation. The PDE obtained are written in the form

\[
c \left( x, t, u, \frac{\partial u}{\partial x} \right) \frac{\partial u}{\partial t} = x^{-m} \left( \frac{\partial}{\partial x} \left( x^m f \left( x, t, u, \frac{\partial u}{\partial x} \right) \right) \right) + s \left( x, t, u, \frac{\partial u}{\partial x} \right)
\]

where the functions \( c, f \) and \( s \) are then determined. The boundary conditions are then passed in the form of

\[
p(x, t, u) + q(x, t) f \left( x, t, u, \frac{\partial u}{\partial x} \right) = 0 \quad \text{where } p \text{ and } q \text{ defines the left and the right boundary. The partial}
\]
differential equations obtained are then solved by using pdepe in MATLAB. The pdepe is used to solve parabolic and elliptic PDE in one spatial variable \( x = [a, b] \) and the time \( t = [t_0, \max t] \). The computations are performed using small values of \( \Delta y \). In our study we have set \( \Delta y = 0.001 \) and \( \Delta t = 0.5 \).

All calculations have been carried out for \( \Pr=1, \ E_c=0.2, \ \frac{\partial p}{\partial x} = -5 \).

RESULTS AND DISCUSSION

To study the effect of Hartmann number (Ha) suction parameter (S) and magnetic inclination angle (\( \theta \)) on unsteady MHD fluid flow and heat transfer between two parallel porous plates, the velocity and temperature profiles are shown graphically for different values of Ha, S and \( \theta \).

![Velocity distribution at t=2](image)

![Temperature distribution at t=2](image)
Velocity distribution at $t=2$

- $S=0$
- $S=1$
- $S=2$

Temperature distribution at $t=2$

- $S=0$
- $S=1$
- $S=2$
Discussion:

Figure 2a shows the effect of Hartmann number on the velocity profiles. From the results we observe that as the Hartmann number increases the velocity decreases. This implies that the greater the magnetic forces the less the velocity. Flow with higher Hartmann number will increase magnetic damping force hence reduces the fluid velocity. Figure 2b shows the effect of Hartmann number (Ha) on the temperature profiles. From the results we observe that as the Hartmann number increases the temperature decreases. Increasing Hartmann number decreases velocity greatly and in turn decreasing the joule and viscous dissipation. Figure 3a shows the effect of the suction parameter (S) on velocity distribution. Increasing the suction parameter decreases the fluid velocity due to the convection of the fluid the regions in the lower half to the center which has the higher fluid speed. Figure 3b shows the effect of suction parameter (S) on temperature distribution. The profile shows that increasing suction parameter increases the temperature. The maximum temperature tends to move towards the moving plate as the parameter (S) increases due to convection in pumping the fluid from lower hotter plate to upper colder plate. The effect of magnetic field inclination angle on velocity distribution is shown on figure 4a. The profiles predict that the velocity increases as the magnetic inclination angle increases. Application of a transverse magnetic field normal to the flow direction has a tendency to create a drag-like Lorentz force which has a decreasing effect on the flow velocity. When the magnetic field is inclined at an angle the Lorentz force reduces and consequently increasing velocity. The change in dimensionless temperature as the magnetic field inclination angle increases is shown on figure 4b. The temperature profiles obtained predicts that as the magnetic field inclination angle increases, the temperature increases.

Summary and Conclusion:

An investigation of unsteady MHD flow and heat transfer of a Newtonian fluid between two parallel porous plates in presence of an inclined magnetic has been done on this paper. The objective was to investigate the effects of Hartmann number (Ha), suction parameter (S), and magnetic inclination angle (θ) on the fluid velocity and temperature distribution. An analysis of the effect of these parameters on the flow velocity and temperature distribution has been carried out. The equations governing the flow under consideration are solved using finite difference approximation. It was observed high Hartmann number decreased both velocity and temperature of the fluid. Increase in suction parameter led to decrease in velocity while the maximum temperature moved towards the moving plate. Finally both the fluid velocity and temperature was found to increase with increase in magnetic inclination angle. It can be therefore be concluded that with suitable values of Hartmann number, suction parameter and magnetic inclination angle, the velocity and temperature can be regulated and hence these parameters are a convenient control method for heat and mass transfer process.

REFERENCES


