Optimal Multi-level Image Thresholding using Lévy Flight driven Algorithms – A Study with Bat, Cuckoo Search and Firefly Algorithm

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ABSTRACT

In recent years, a plethora of soft computing methods are proposed by the researchers to find optimal solutions for constrained and unconstrained optimization problems. Among them, Lévy Flight (LF) driven algorithms are widely adopted by most of the researchers. In this work, we considered the recent LF based algorithms, such as the Bat Algorithm (BA), Cuckoo Search (CS) and Firefly Algorithm (FA) to solve optimal multi-level image thresholding problem using Otsu. Optimal thresholds for the gray scale images are reached by analyzing histogram of the image. Maximization of Otsu’s between class variance function is chosen as the objective function. The performances of the LF based algorithms are assessed by considering eight standard (512 x 512) test images. The performance appraisal between the LF algorithms are carried using the well known quality measures, such as maximal objective function, Root Mean Squared Error, Peak Signal to Noise Ratio, and convergence of optimization search. The result evident that, for most of the test images, the LF driven FA offers better overall result compared with the BA and CS.

INTRODUCTION

In image processing application, segmentation acts as a primary and important procedure in the analysis and examination of gray and color images. For gray scale images, segmentation process helps to separate an image into non-overlapping, homogenous regions having similar objects (Akey, 2013). In general, the gray level histogram of the image is bi-model. Hence, the image objects are clearly distinguishable from the background by simply choosing a threshold value by considering the valley between two peaks of the histogram. But, in the real cases, the gray level histograms of the real time images are mostly multimodal and identifying the accurate position of distinct valleys in multimodal histograms is quite difficult (Sathy and Kayalvizhi, 2010). Due to these reasons, solving the multi-level thresholding problem is emerged as the significant area of research in recent years (Sathy and Kayalvizhi, 2011; 2012).

Over the years, several image segmentation methods have been proposed and executed in the literature (Lee et al., 1990; Pal and Pal, 1993; Sezgin and Sankar, 2004). In segmentation, the input image is separated into object and background. Based on the performance evaluation procedure, the segmentation methods are classified as supervised and unsupervised procedures. Unsupervised methods are preferable in real-time processing because they do not require a manually segmented image (Raja et al., 2014).

Image thresholding is considered as the most preferred practice out of all the existing image segmentation methods, because of its simplicity, robustness, exactness and capability (Rajnikanth et al., 2014a; 2014b). Traditional segmentation methods work well for a bi-level thresholding problem, when the number of threshold level increases, complexity of the thresholding problem will also increase and the traditional method requires more computational time because of multi-level complexity. Hence, in recent years, heuristic and metaheuristic algorithms based bi-level and multi-level image thresholding procedure is widely proposed by the researchers.

Horng (2011) proposed Artificial Bee Colony (ABC) algorithm based heuristic approach to solve the multi-level image thresholding problem for gray scale images using Otsu. Sathy and Kayalvizhi (2010; 2011) implemented Particle Swarm Optimization (PSO) and Bacterial Foraging Algorithm (BFA) based Otsu and validated the proposed method with the Genetic Algorithm. Further, they proposed a Modified Bacterial
Foraging (MBF) approach to solve the gray scale thresholding problem. Akay (2013) presented a detailed study on multi-level image thresholding using Kapur’s entropy and Otsu’s between class variance function using most successful heuristic approaches, such as PSO and ABC. SriMadhava Raja et al. (2014) presented a detailed study on gray scale image thresholding using Otsu and Firefly Algorithm. Recently, Rajinikanth et al. (2014) presented a detailed study about, PSO, BFO and Bat Algorithm (BA) based image thresholding using Otsu. Similar approaches are widely available in the image segmentation literature (Agarwal et al., 2013; Alhodzic and Tuba, 2014; Charansiriphasan et al., 2013; Ghamisi et al., 2012; Panda et al., 2013).

In this paper, the heuristic algorithms proposed by Yang (2008) are considered. The most successful recent algorithms, such as Bat Algorithm (Yang, 2010), Cuckoo Search (Yang and Deb, 2009) and Firefly Algorithm (Yang, 2009; 2010) are considered in the proposed gray scale image thresholding problem. The most victorious Lévy Flight (LF) search pattern is considered in this work. The proposed procedure is tested on standard test images (512 x 512) and its performance is evaluated using measures, such as algorithm convergence, maximized objective function value, Root Mean Squared Error (RMSE), and Peak Signal to Noise Ratio (PSNR) in dB.

The paper is structured as follows: Section 2 presents the Otsu based bi-level and multi-level thresholding problem. Section 3 presents the overview of the heuristic algorithms in this study and the implementation is discussed in Section 4. Experimental results are assessed and presented in Section 5. Conclusion of the research work is specified in section 6.

**Methodology:**

Otsu’s image thresholding procedure was initially proposed in 1979 (Otsu, 1079). Due to its simplicity and better segmentation capability, it is widely adopted by most of the researchers to segment the gray scale and colour images (Ghamisi et al., 2012; 2013; 2014). In this method, maximization of Otsu’s between class variance function is used to obtain the best possible threshold values for both the bi-level and multi-level segmentation operations.

A detailed description of Otsu’s method could be found in the literatures and this process is defined as follows:

For a given image, let there be L intensity levels in the range \{0,1,2,\ldots, L-1\}. Then, it can be defined as;

\[
p_i^C = \frac{h_i^C}{N} \sum_{i=0}^{L-1} p_i^C = 1
\]

The overall mean of each element of the image is calculated as;

\[
\mu^C = \sum_{i=0}^{L-1} i p_i^C = 1
\]

The multi-level thresholding presents m-1 threshold levels \(t_j^C\), where \(j = 1,2,\ldots,m-1\), and the action is executed as:

\[
F^C(x,y) = \begin{cases} 
0, & f^C(x,y) \leq t_1^C \\
\frac{1}{2} (t_1^C + t_2^C), & t_1^C < f^C(x,y) \leq t_2^C \\
\vdots & \vdots \\
\frac{1}{2} (t_{m-2}^C + t_{m-1}^C), & t_{m-2}^C < f^C(x,y) \leq t_{m-1}^C \\
L-1, & f^C(x,y) > t_{m-1}^C 
\end{cases}
\]

The probabilities of occurrence \(w_j^C\) of classes \(D_1^C, \ldots, D_m^C\) are given by;

\[
w_j^C = \begin{cases} 
\sum_{i=0}^{j} p_i^C, & j = 1 \\
\sum_{i=j}^{j+1} p_i^C, & 1 < j < m \\
\sum_{i=j}^{L-1} p_i^C, & j = m
\end{cases}
\]

The mean of each class \(\mu_j^C\) can then be calculated as;
Heuristic Algorithms:
Lévy Flight Strategy:
In heuristic algorithms, convergence speed and optimization accuracy mainly depends on the guiding parameters which help to update the agent values. Most of the preliminary heuristic algorithms are guided by the randomization operator. Due to the randomization parameter, the optimization accuracy and the convergence will not be in expected level in most of the search cases. Hence, in recent years Lévy Flight (LF) and Brownian Distribution (BD) based search pattern is widely adopted to update the agent positions (Sri Madhava Raja et al, 2014). In the proposed work, LF is considered to update the agent positions in the heuristic algorithms.

LF is a random walk with a sequence of arbitrary steps and is conceptually similar to the search path of a foraging animal (Sri Madhava Raja et al, 2013). In LF, the flight span and the length between two successive changes in direction are drawn from a probability distribution. A detailed explanation of LF is discussed in the book by Yang (Yang, 2008). Lévy flight is superdiffusive markovian process, whose step length is drawn from the Lévy distribution in terms of a simple power-law formula;

\[ L(s) \sim |s|^{-1-\beta} \]  

where \( 0 < \beta \leq 2 \)

Lévy flight = \( LF(s) = A \cdot |s|^{1/\beta} \)  

where

\[ A = \beta \Gamma(\beta) \sin \left( \frac{\beta \pi}{2} \right) \frac{1}{\pi} \]  

Bat Algorithm:
The Bat Algorithm (BA) is based on the bio-sonar characteristics of microbats. BA was anticipated by modelling the navigating and hunting potential of bats (Yang, 2010). A detailed examination on the BA algorithm can be found in (Yang, 2013).

The BA was developed based on the following assumptions (Kotteeswaran and Sivakumar, 2013):
- All the bats use echolocation to sense distance, prey, and background barriers.
- The bats will fly with a velocity \( V_c \) at position \( X_c \) with an emitted frequency \( f_{min} \) varying wavelength \( \lambda \) and a loudness \( A_c \). During the search, the bats will automatically adjust the frequency, wavelength and pulse emission rate \( r \in [0,1] \) based on the target distance.
- Along with the above said parameters, the loudness also varies from a large \( A_c \) value to a constant \( A_{min} \) value.

The Traditional BA (TBA) has three mathematical discrete equations, defining the velocity update (eqn. 11), the position update (eqn. 12), and the frequency vector (eqn. 13) as given below:
\[ V_i(t+1) = V_i(t) + \left( \frac{V_i(t) - G_{best}}{\beta} \right) F_i \]
\[ X_i(t+1) = X_i(t) + V_i(t+1) \]
\[ F_i = F_{min} + (F_{max} - F_{min}) \beta \]

where \( \beta \) is a random integer in the range \([0,1]\).

From eqn.11, it is noted that, the velocity update mainly depends on the frequency vector. During the optimization search, a new solution for each bat is generated based on the following relation:
\[ X_{new} = X_{old} + \alpha A^t \]
where \( \varepsilon \) is a random numeral in the range \([-1,1]\) and \( A \) is the loudness of emitted sound by bats during the exploration of search space.

The minimum and maximum values of the loudness variable \( A \) is chosen as \( A_0 = 10 \) and \( A_{min} = 1 \) (which decay in steps of 0.01). Other related mathematical representations for loudness adjustment are presented below:
\[ A_t(t+1) = \alpha A_t(t) \]
\[ r_t(t+1) = r_t(0) [1 - \exp(-\eta \|)] \]
\[ \eta = \gamma V \]

where \( \alpha \) and \( \gamma \) are constants typically assigned with a numeral value of 0.75 (Rajinikanth et al., 2014).

**Cuckoo Search Algorithm:**

Cuckoo Search (CS) is one of the successful algorithms, proposed by Yang and Deb in 2009. This algorithm is based on the breeding tricks of parasitic cuckoos (Yang and Deb, 2009). CS algorithm is developed based on the following rules:
- Each cuckoo lays an egg and dumps in a randomly chosen nest
- The nest with high survived egg will be carried over to the next generation. Cuckoo’s egg generally hatches several days before than the host’s eggs. The cuckoo chick grows faster and expels the host’s eggs and chicks.
- In a search universe, the number of host nest is fixed. The host bird discovers the cuckoo’s egg with a probability \( p_{ab} \in [0,1] \). When the host identifies the egg, it may remove it from nest, or simply abandon the nest and build a new nest.

In CS, during the optimization search, the new solution \( X_i^{(t+1)} \) mainly depends on the old solution \( X_i^{(t)} \) and the search guiding procedure (Yang and Gandomi, 2012). In this work, the following expressions are considered to find the new solution;
\[ X_i^{(t+1)} = X_i^{(t)} + \alpha \oplus LF \]
where \( \alpha > 0 \) is the succeeding step.

**Firefly Algorithm:**

Firefly algorithm is also proposed by Yang in 2009. This algorithm is developed by imitating the flashing illumination patterns generated by invertebrates such as glowworm and firefly, which generates chemically produced light from their lower abdomen. This bioluminescence with various flashing patterns generated by glowworm/firefly is used to establish communication between two neighboring insects, to search for pray and also to find mates (Yang, 2009; 2010).

The classical FA is developed by taking into account the following conditions:
(i) All the fireflies are unisex and one firefly will be attracted with other nearest firefly regardless of their sex.
(ii) The attractiveness between two fireflies is proportional to the luminance.
(iii) The brightness of a firefly is somehow related with the analytical form of the objective function assigned to guide the search process.

The overall performance (exploration time, speed of convergence, and optimization accuracy) of the FA depends on the cost function, which monitors the optimization search. For a maximization problem, luminance of a firefly is considered to be proportional to the value of cost function, (ie. luminance = objective function).

The chief parameters which decide the efficiency of the FA are the variations of light intensity and attractiveness between neighboring fireflies (Kotteeswaran and Sivakumar, 2014).

Variation in luminance can be analytically expressed with the following Gaussian form:
\[ I(r) = I_0 e^{-\gamma d^2} \]
where \( I = \text{new light intensity}, I_0 = \text{original light intensity}, \) and \( \gamma = \text{light absorption coefficient}. \)

The attractiveness towards the luminance can be analytically represented as:
\[ \beta = \beta_0 e^{-\gamma d^2} \]
where $\beta$ = attractiveness coefficient, and $\beta_0$ = attractiveness at $r = 0$.

The above equation describes a characteristic distance $\Gamma = 1/\sqrt{\gamma}$ over which the attractiveness changes significantly from $\beta_0$ to $\beta_0 e^{-1}$. The attractiveness function $\beta(d)$ can be any monotonically decreasing functions such as the following form;

$$\beta(d) = \beta_0 e^{-\gamma d^m} \quad (m \geq 1) \tag{20}$$

For a fixed $\gamma$, the characteristic length becomes;

$$\Gamma = \gamma^{-1/m} \rightarrow l_m \rightarrow \infty \tag{21}$$

Conversely, for a given length scale $\Gamma$, the parameter $\gamma$ can be used as a typical initial value (that is $\gamma = 1/\Gamma m$).

The Cartesian distance between two fireflies $i$ and $j$ at $x_i$ and $x_j$ in the $n$ dimensional search space can be mathematically expressed as;

$$d_{ij}^t = \left\| X_j^t - X_i^t \right\|_2 = \sqrt{n \sum_{k=1}^{n} (X_{j,k} - X_{i,k})^2} \tag{22}$$

In FA, the light intensity at a particular distance $d$ from the light source $X_I^t$ obeys the inverse square law. The light intensity of a firefly $I$, as the distance $d$ increases in terms of $I \propto 1/d^2$. The movement of the attracted firefly $i$ towards a brighter firefly $j$ can be determined by the following position update equation;

$$X_i^{t+1} = X_i^t + \beta_0 e^{-2 \gamma d_{ij}^t} \left( X_j^t - X_i^t \right) + \alpha \cdot \text{sign}(\text{rand} \cdot \frac{1}{2}) \oplus \text{Lévy} \tag{23}$$

where, $X_i^{t+1}$ = updated position of firefly, $X_i^t$ = initial position of firefly, and $\beta_0 e^{-2 \gamma d_{ij}^t} \left( X_j^t - X_i^t \right)$ = attraction between fireflies.

In the proposed work, a comparative investigation is presented between LF driven BA, CS, and FA.

**Implementation:**

Heuristic algorithm based optimal multi-level thresholding problem is performed in this paper. The procedure deals with finding best possible threshold values within the gray scale histogram range $[0, L-1]$ by maximizing Otsu’s between class variance function. In this procedure, the dimension of the optimization search is assigned based on the number of threshold ($m$) values. This procedure is executed using the LF driven BA, CS and FA.

![Block diagram representation of implementation process](image)
Step 5: If optimal values are attained, stop the search, display the segmented image, threshold values, and the objective function value.

Step 6: Compute and display the performance measure values such as RMSE and PSNR.

The performance of multi-level segmentation procedure is assessed using well known image quality measures, such as maximal between-class variance, Root Mean Squared Error (RMSE) and Peak Signal to Noise Ratio (PSNR) in dB.

PSNR is used to find the similarity of the segmented image against the original image based on the RMSE of each pixel (Akay, 2013):

\[
RMSE_{(o,s)} = \sqrt{\frac{1}{MN} \sum_{i=1}^{H} \sum_{j=1}^{W} [o(i,j) - s(i,j)]^2}
\]

\[
PSNR(o,s) = 20 \log_{10} \left( \frac{255}{\sqrt{\text{MSE}(o,s)}} \right) \text{ dB}
\]

where \( o \) and \( s \) are original and segmented images of size \( H \times W \).

![Fig. 2: Original and segmented Hunter image with LF driven FA](image-url)
### Table 1: Original image and the corresponding gray histogram

<table>
<thead>
<tr>
<th>Image</th>
<th>Original Image</th>
<th>Histogram</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gold hill</td>
<td><img src="image1" alt="Gold hill Image" /></td>
<td><img src="image2" alt="Gold hill Histogram" /></td>
</tr>
<tr>
<td>Jet</td>
<td><img src="image3" alt="Jet Image" /></td>
<td><img src="image4" alt="Jet Histogram" /></td>
</tr>
<tr>
<td>Traffic</td>
<td><img src="image5" alt="Traffic Image" /></td>
<td><img src="image6" alt="Traffic Histogram" /></td>
</tr>
<tr>
<td>Map</td>
<td><img src="image7" alt="Map Image" /></td>
<td><img src="image8" alt="Map Histogram" /></td>
</tr>
<tr>
<td>House</td>
<td><img src="image9" alt="House Image" /></td>
<td><img src="image10" alt="House Histogram" /></td>
</tr>
<tr>
<td>Couple</td>
<td><img src="image11" alt="Couple Image" /></td>
<td><img src="image12" alt="Couple Histogram" /></td>
</tr>
<tr>
<td>Butterfly</td>
<td><img src="image13" alt="Butterfly Image" /></td>
<td><img src="image14" alt="Butterfly Histogram" /></td>
</tr>
</tbody>
</table>
Table 2: Segmented dataset with LF driven BA

<table>
<thead>
<tr>
<th>Thresholded images</th>
<th>m = 2</th>
<th>m = 3</th>
<th>m = 4</th>
<th>m = 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Goldhill</td>
<td>![Image]</td>
<td>![Image]</td>
<td>![Image]</td>
<td>![Image]</td>
</tr>
<tr>
<td>Jet</td>
<td>![Image]</td>
<td>![Image]</td>
<td>![Image]</td>
<td>![Image]</td>
</tr>
<tr>
<td>Traffic</td>
<td>![Image]</td>
<td>![Image]</td>
<td>![Image]</td>
<td>![Image]</td>
</tr>
<tr>
<td>Map</td>
<td>![Image]</td>
<td>![Image]</td>
<td>![Image]</td>
<td>![Image]</td>
</tr>
<tr>
<td>House</td>
<td>![Image]</td>
<td>![Image]</td>
<td>![Image]</td>
<td>![Image]</td>
</tr>
<tr>
<td>Couple</td>
<td>![Image]</td>
<td>![Image]</td>
<td>![Image]</td>
<td>![Image]</td>
</tr>
<tr>
<td>Butterfly</td>
<td>![Image]</td>
<td>![Image]</td>
<td>![Image]</td>
<td>![Image]</td>
</tr>
</tbody>
</table>

RESULTS AND DISCUSSION

Otsu guided, Lévy Flight (LF algorithms based multi-level thresholding techniques have been tested on standard gray scale test images (512 x 512), such as Hunter, Goldhill, Jet, Traffic, Map, House, Couple, and Butterfly. From the grey histogram, it is noted that, Hunter, Goldhill, Jet, and Traffic offers a smooth histogram and the rest of the images (Map, House, Couple, and Butterfly) shows hastily altering pixel levels. The simulation work is executed on a work station with Intel Dual Core 1.6 GHz CPU with 2 GB of RAM and is equipped with Matlab R2010a software.

During the optimization search, population of heuristic algorithms are chosen as 25, number of iteration is allotted as 500, the search dimension is assigned as ‘m’, and the stopping criterion is maximal between-class variance function. For each image, and for each m value, the segmentation process is repeated 15 times and the mean value among the trials is chosen as set of optimal thresholds and performance measures.

The Bat Algorithm parameters are assigned based on the work by Rajinikanth et al. (2014); the firefly algorithm parameters are chosen based on the work by Sri Madhava Raja et al. (2014) and the cuckoo search parameters are assigned based on Alihodzic and Tuba (2014).
Initially, the optimal thresholding process is implemented on the Hunter image for \( m = \{ 2, 3, 4, 5 \} \) using the LF based BA, CS, and FA. In Fig. 2 (a, b) shows the original image and the corresponding gray level histogram. Fig. 2 (c - f) shows the segmented image and the corresponding optimal gray threshold values obtained using the BA for \( m = 2, 3,4 \) and 5. Fig. 2g shows the convergence of optimization search and from this one can observe that, the FA offers faster convergence compared with the BA and CS.

The above said procedure is repeated for the image dataset shown in Table 1 with BA, CS and FA and the corresponding result are clearly presented in Table 2, 3 and 4. Table 2 shows the segmented gray scale image with BA for \( m = 2-5 \). Table 3 presents the maximized objective function value and the corresponding optimal threshold values for various ‘m’ values. Table 4 presents the vital details, such as convergence of algorithm, RMSE value and PSNR in dB.

In the proposed work, multi-level image thresholding process is proposed with the recent heuristic algorithms introduced by Yang (2008; 2009; 2010). From Table 3 and Table 4, it can be observed that, all the algorithm helps to achieve approximately similar objective function values and PSNR values. From Table 4, one can observe that, the Firefly algorithm provides faster convergence compared with the BA and CS.

**Conclusion:**

In this paper, bi-level and multi-level image thresholding problem is addressed using Lévy Flight (LF) driven Bat Algorithm (BA), Cuckoo Search (CS), and Firefly Algorithm (FA). Maximization of Otsu’s between-class variance function is chosen as the objective function. In this work, the segmentation process is attempted for \( m = 2,3,4 \) and 5. In order to validate the performance of considered heuristic algorithms, eight standard test images are examined. The proposed segmentation procedure is validated using both the qualitative and quantitative analysis. All the algorithms offer approximately similar values for objective function, RMSE, and PSNR for the considered ‘m’ levels. For most of the images, the convergence of FA is better compared with the BA and CS. The future work will include the implementation of the proposed method for color image datasets.
Table 4: Performance measure values for LF driven BA, CS and FA (mean value of trials)

<table>
<thead>
<tr>
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<th>No. of iterations</th>
<th>RMSE</th>
<th>PSNR (dB)</th>
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<td></td>
<td>BA</td>
<td>CS</td>
<td>FA</td>
</tr>
<tr>
<td>Hunter</td>
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<td>52</td>
<td>58</td>
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<tr>
<td></td>
<td>3</td>
<td>83</td>
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</tr>
<tr>
<td></td>
<td>5</td>
<td>186</td>
<td>177</td>
</tr>
<tr>
<td>Gold</td>
<td>2</td>
<td>38</td>
<td>44</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>95</td>
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<td></td>
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**REFERENCES**


