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Estimation of P[Y < X] for Burr Type XII Distribution under Several Modifications for Ranked Set Sampling

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ABSTRACT

Ranked set sampling (RSS) approach is considered a cost efficient alternative to simple random sampling (SRS) when observations are costly or time-consuming but the ranking of the observations without actual measurement can be done relatively easily. Many authors suggested different modifications for RSS to come up with new sampling techniques. Median ranked set sampling (MRSS), extreme ranked set sampling (ERSS) and percentile ranked set sampling (PRSS) are some modifications for RSS. In the current paper, the estimation of R = P[Y < X] when Y and X are two independent Burr type XII distributions with the same known shape parameter c is considered. Maximum likelihood method is proposed to estimate R based on MRSS, ERSS and PRSS data. These estimators are compared with known estimators based on SRS and RSS data in terms of their mean square errors (MSEs) and efficiencies.

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INTRODUCTION

Stress-strength reliability is one of the main tools of reliability analysis of structures. A stress-strength system fails as soon as the applied stress Y is at least as large as its strength X.Due to the practical point of view of reliability stress strength model, the estimation problem of$P(Y < X)$ has attracted the attention of many authors. This model first considered by Birnbaum (1956) and developed by Birnbaum and McCarty (1958). The latter paper for the first time included $P(Y < X)$ in its title, but the formal term stress-strength appeared in the title of Church and Harris (1970). The theoretical and practical results on the theory and applications of the stress-strength relationships in industrial and economic systems during the last decades are collected and digested in Kotz *et al*. (2003).

RSS is a sampling technique that was proposed by McIntrye (1952) for estimating the mean of pasture and forage yields. In situations when the variable of interest is costly or time-consuming, but the ranking of items according to the variable is relatively easy without actual measurement, the use of RSS is highly powerful and much superior to the standard SRS for estimating some of the population parameters. The RSS procedure can be summarized as follows. Randomly select n^2 units from the target population and rank the units within each set with respect to a variable of interest by visual inspection or by any cheap method. Then select for actual measurement the smallest ranked unit from the first set. From the second set, select for the second actual measurement the second smallest unit. The process is continued in this way until the largest ranked unit is selected from the last set.The cycle may be repeated r times to obtain a sample of size nr units from the RSS data.

Takahashi and Wakimoto(1968) established a very important mathematical theory of RSS. They showed that the mean of the RSS is an unbiased estimator of the population mean and has a smaller variance than the mean of a SRS. Dell and Clutter (1972) showed that the mean of the RSS is an unbiased estimator of the population mean whether the ranking is perfect or not.Muttlak(1997) proposed the median ranked set sampling as a modification to RSS. The MRSS procedure can be summarized as follows.Select n random samples of size n units from the target population.Rank the units within each sample with respect to a variable of interest.If the sample size n is odd, from each sample select for measurement the $(n+1/2)^{th}$ smallest ranked unit (i.e., the median of the sample). If the sample size n is even, select for the measurement from the first $\frac{n}{2}$ samplesthe

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$(n/2)^{th}$ smallest ranked unit and from the second $n/2$samples the $(n/2+1)^{th}$ smallest ranked unit.The cycle may be repeated *r* times to get *nr* units. These *nr* units form the MRSS data

Samawi *et al*. (1996) used the extreme ranked set sampling to estimate the population mean. They showed that the ERSS estimator is more efficient than the SRS estimator. In the ERSS procedure,select n random samples of size n unitsfrom the populationunder consideration and rank the units within each sample with respect to a variable of interest. If the sample size n is odd, select from $n-1/2$ samples the smallest unit, from the other$n-1/2$ the largest unit and fromthe remaining sample, the median of the sample for actual measurement. If the sample size is even, select from $n/2$ samples the smallest unit and from the other $n/2$ samples the largest unit for actual measurement**.**The cycle may be repeated r times to get nr units from ERSS data.

Muttlak (2003) introduced the PRSS approach as a modification to RSS. In the PRSS procedure, select n random samples of size n units from the population and rank the units within each sample with respect to a variable of interest. If the sample size is even, select for measurement from the first $n/2$ samples the $O(n+1)^{th}$ smallest ranked unit and from the second $n/2$ samples the $t(n+1)^{th}$smallest ranked unit, where $t=1-O$ and $0<O\leq 0.5$. If the sample size is odd, select from the first $n-1/2$ samples the $O(n+1)^{th}$ smallest ranked unit and from the other$n-1/2$ samples the $t(n+1)^{th}$ smallest ranked unit and select from the remaining sample the median for the sample for actual measurement. The cycle may be repeated r times if needed to get nr units from PRSS.

Recently, interest has been shown in estimating R using RSS by several investigators.Sengupta and Mukhuti (2008) considered an unbiased estimation of Rusing RSS for exponential populations.Muttlak *et al*. (2010) proposed three estimators of Rusing RSS when $X$and Y areindependent one-parameter exponential populations. Hussian (2014) discussed the estimation problem of stress strength model for generalized inverted exponential distribution based on RSS and SRS. Maximum likelihood method is used to estimate R using both approaches. Hassan *et al*. (2014) discussed the estimation of R when Y and X are two independent Burr type XII distributions with common known shape parameter c. These estimators compared in terms of their biases, mean square errors and efficiencies with known estimators based on SRS data.

Burr (1942) introduced twelve different forms of cumulative distribution functions which might be useful for fitting data, among those distributions Burr type XII;it has been widely used in reliability analysis. The two parameters Burr-XII distribution denoted by Burr (c,b) has the following probability density function (PDF)

$$f(x;c,b)=\mathrm{b}cx^{c-1}(1+x^{c})^{-(b+1)}, x>0, c>0, b>0. \quad (1)$$

Thecorresponding cumulative distribution function (CDF) is given as

$$F(x;c,b)=1-(1+x^{c})^{-b}, x>0, c>0, b>0. \quad (2)$$

Here,c and b are shape parameters.

The main aim of this study is to focus on the estimation problem of$R=P(Y<X)$,where $X\sim$Burr $(c,b)$and $Y\sim$Burr $(c,a)$and they are independently distributedbased on different sampling schemes. Maximum likelihood estimators (MLEs)of R using SRS and RSS will be considered. MLEs based on MRSS, ERSS and PRSStechniques will be derived. Simulation study is performed to compare different estimators.

Estimation of $R=P(Y<X)$Based on SRS and RSS data:

This section concern with the MLE and UMVUE of R based on SRS data and the MLE of R based on RSS. Therefore, firstly MLEs of theunknown parameters must be derived.

Let $X\sim$Burr $(c,b)$and $Y\sim$Burr $(c,a)$are two independent Burr type XII random variables,then according to Panahi and Aasdi (2010), it can be easily seen that:

$$R=P(Y<X)=\int_0^\infty\int_0^x bcx^{c-1}(1+x^{c})^{-(b+1)}acy^{c-1}(1+y^{c})^{-(a+1)}dydx=\frac{1}{1+\rho} \quad , \quad \rho=\frac{b}{a} \ . \quad (3)$$

Now to compute the MLE of R based on SRS, first MLEs of b and a are obtained.Let $X_1,\dots,X_p$ be a SRS from Burr $(c,b)$and $Y_1,\dots,Y_q$ be a SRS from Burr (c,a),therefore the log-likelihood function denoted by l for the observed sample is given by:

$$l=plnb+qlna+(p+q)lnc+(c-1)\left[\sum_{i=1}^{p}lnx_i+\sum_{j=1}^{q}lny_j\right]-(b+1)\sum_{i=1}^{p}ln(1+x_i^c) \\ -(a+1)\sum_{j=1}^{q}ln(1+y_j^c).$$

Differentiating the log-likelihood and equating by zero with respect to b and a. Hence, MLEs of b and a, when c is known take the following forms:

$$\hat{b}_{MLE} = \frac{p}{\sum_{i=1}^{p} ln\left(1+x_i^c\right)} \; and \; \hat{a}_{MLE} = \frac{q}{\sum_{j=1}^{q} ln\left(1+y_j^c\right)}, \tag{4}$$

where $\hat{b}_{MLE}$ and $\hat{a}_{MLE}$ are MLEs for b and a. Once the MLEs of b and a are obtained from Equation (4), then, the MLE of R using SRS is given by

$$\hat{R}_{MLE} = \frac{1}{1+\hat{\rho}_{MLE}}, \quad \text{where } \hat{\rho}_{MLE} = \frac{\hat{b}_{MLE}}{\hat{a}_{MLE}}. \tag{5}$$

To get UMVUE of R, let $\left(\sum_{i=1}^{p} ln(1+x_i^c), \sum_{j=1}^{q} ln\left(1+y_j^c\right)\right)$ is a jointly sufficient statistic for (b,a), therefore according to Panahi and Asadi (2010), the UMVUE of R based on SRS will be as follows:

$$\hat{R}_{UMVUE} = \sum_{i=0}^{p-1} (-1)^i \frac{(q-1)!(p-1)!}{(q+i-1)!(p-i-1)!} \left(\frac{T_2}{T_1}\right)^i \; if T_2 \leq T_1, \tag{6}$$

or

$$\hat{R}_{UMVUE} = 1 - \sum_{i=0}^{q-1} (-1)^i \frac{(q-1)!(p-1)!}{(q-i-1)!(p+i-1)!} \left(\frac{T_1}{T_2}\right)^i \; if T_2 > T_1, \tag{7}$$

where, $T_1 = \sum_{i=1}^{p} ln(1+x_i^c)$ and $T_2 = \sum_{j=1}^{q} ln\left(1+y_j^c\right)$.

According to Hassan *et al*. (2014) the MLE of R is obtained as follows:

Let $\left\{X_{i(i)s},\ i = 1,2,\dots n; s = 1,2,\dots,r\right\}$ be a ranked set sample with sample size $p = nr$, where n and r are the set size and the number of cycles from Burr (c,b). Then the PDF of $X_{i(i)s}$ is given by:

$$f_i\left(x_{i(i)s}\right) = \frac{n!}{(i-1)!(n-i)!}\left[F\left(x_{i(i)s}\right)\right]^{i-1}\left[1-F\left(x_{i(i)s}\right)\right]^{n-i} f\left(x_{i(i)s}\right), \tag{8}$$

$$f_i\left(x_{i(i)s}\right) = \frac{n!}{(i-1)!\,(n-i)!} bcx_{i(i)s}^{c-1}\left(1+x_{i(i)s}^c\right)^{-[b(n-i+1)+1]}\left(1-\left(1+x_{i(i)s}^c\right)^{-b}\right)^{i-1}, \qquad x_{i(i)s} > 0.$$

By similar way, let $\{Y_{j(j)s}, j = 1,\dots,m; s = 1,\dots,r\}$ denote the ranked set sample of size $q = mr$ from Burr (c,a). Then the PDF of $Y_{j(j)s}$ is given by:

$$f_j\left(y_{j(j)s}\right) = \frac{m!}{(j-1)!(m-j)!}\left[F\left(y_{j(j)s}\right)\right]^{j-1}\left[1-F\left(y_{j(j)s}\right)\right]^{m-j} f\left(y_{j(j)s}\right), \tag{9}$$

$$f_j\left(y_{j(j)s}\right) = \frac{m!}{(j-1)!\,(m-j)!} acy_{j(j)s}^{c-1}\left(1+y_{j(j)s}^c\right)^{-[a(m-j+1)+1]}\left(1-\left(1+y_{j(j)s}^c\right)^{-a}\right)^{j-1}, \quad y_{j(j)s} > 0\,.$$

The likelihood function L_{RSS} of observed data will be as follows:

$$L_{RSS} = \prod_{s=1}^{r}\left[\prod_{i=1}^{n} f_i\left(x_{i(i)s}\right)\prod_{j=1}^{m} f_j\left(y_{j(j)s}\right)\right].$$

The log-likelihood function of L_{RSS} denoted by l_{RSS} will be as follows:

$$l_{RSS} = \ln\, D_1 + plnb + qlna + (c-1)\sum_{s=1}^{r}\left\{\sum_{i=1}^{n} lnx_{i(i)s} + \sum_{j=1}^{m} lny_{j(j)s}\right\}$$
$$-\sum_{s=1}^{r}\left\{\sum_{i=1}^{n}[b(n-i+1)+1]ln\left(1+x_{i(i)s}^c\right) + \sum_{j=1}^{m}[a(m-j+1)+1]ln\left(1+y_{j(j)s}^c\right)\right\}$$
$$+\sum_{s=1}^{r}\left\{\sum_{i=1}^{n}(i-1)\, ln\left(1-(1+x_{i(i)s}^c)^{-b}\right) + \sum_{j=1}^{m}(j-1) ln\left(1-(1+y_{j(j)s}^c)^{-a}\right)\right\},$$

where D_1 is a constant. The first partial derivatives of log-likelihood function with respect to b and a are given by:

$$\frac{\partial l_{RSS}}{\partial b}=\frac{p}{\tilde{b}}-\sum_{s=1}^{r}\sum_{i=1}^{n}\left[(n-i+1)\ln\left(1+x_{i(i)s}^{c}\right)-(i-1)\frac{\ln\left(1+x_{i(i)s}^{c}\right)}{\left(1+x_{i(i)s}^{c}\right)^{\tilde{b}}-1}\right]=0, \quad (10)$$

$$\frac{\partial l_{RSS}}{\partial a}=\frac{q}{\tilde{a}}-\sum_{s=1}^{r}\sum_{j=1}^{m}\left[(m-j+1)\ln\left(1+y_{j(j)s}^{c}\right)-(j-1)\frac{\ln\left(1+y_{j(j)s}^{c}\right)}{\left(1+y_{j(j)s}^{c}\right)^{\tilde{a}}-1}\right]=0. \quad (11)$$

Clearly, it is not easy to obtain a closed form solution to system ofEquations (10) and (11). Therefore, an iterative technique must be applied to solve these equations numerically to obtain an estimates of b and a. MLEs of b and a denoted by $\tilde{b}$ and $\tilde{a}$ are the solution of Equations (10) and (11). Then $\tilde{R}_{MLE}$ will beobtained by substituting $\tilde{b}$ and $\tilde{a}$ inEquation (3).

Estimation of $R=P(Y<X)$ Based on MRSSData:

Muttlak(1997)investigatedMRSS as a sampling technique to estimate the population mean. MRSS procedure depends on two cases, the first case for odd set size and the second case for even set size. This procedure can be summarizedas follows:

Case (1):for odd set size,the median value is selected from each of the n ordered sets. If the set$\{X_{1(g)s}, X_{2(g)s} \dots, X_{n(g)s}\}$is quantified, then this will be a MRSS for odd set size,where$g=n+1/2$ and$s=1,2,..,r$.

Case(2): for even set sizethe $(u)^{th}$ smallest element is chosen from the first u ordered sets, while the $(u+1)^{th}$smallest unit is chosen from each of the remaining u sets. MRSS foreven set size will be the set $\{X_{1(u)s}, \dots, X_{u(u)s}, X_{u+1(u+1)s}, \dots, X_{n(u+1)s}\}$ where $u=n/2$ and$s=1,2,..,r$.

In the following subsections maximum likelihood method of estimation will be considered to estimate R for Burr XII distribution based on MRSS technique for odd and even set sizes. To derive the MLE of R, firstly the MLEs of unknown parameters b and a must be obtained.

MLEof $R=P(Y<X)$with Odd Set Size Based on MRSS data:

Let $x_{1(g)s}, \dots, x_{n(g)s}$ is a MRSS from Burr (c,b) with sample size $p=nr$, where n is the set size, r is the number of cycles. Then, using Equation (8) the PDF of $X_{i(g)s}$will be as follows:

$$f_g\left(x_{i(g)s}\right)=\frac{n!}{[(g-1)!]^2}bcx_{i(g)s}^{c-1}\left[1+x_{i(g)s}^{c}\right]^{-(bg+1)}\left[1-\left(1+x_{i(g)s}^{c}\right)^{-b}\right]^{g-1},\ x_{i(g)s}>0. \quad (12)$$

Similarly, Let $y_{1(h)s}, \dots, y_{m(h)s}$ is a MRSS from Burr (c,a) with sample size $q=mr$, where m is the set size, r is the number of cycles. Then, using Equation (9) the PDF of $Y_{j(h)s}$will be as follows:

$$f_h\left(y_{j(h)s}\right)=\frac{m!}{[(h-1)!]^2}acy_{j(h)s}^{c-1}\left[1+y_{j(h)s}^{c}\right]^{-(ah+1)}\left[1-\left(1+y_{j(h)s}^{c}\right)^{-a}\right]^{h-1}, y_{j(h)s}>0. \quad (13)$$

The likelihood function for the observed sample based on MRSS in case of odd set size denoted by L_{MRSS1}^{*} is given as follows:

$$L_{MRSS1}^{*}=\prod_{s=1}^{r}\left[\prod_{i=1}^{n}f_g\left(x_{i(g)s}\right)\prod_{j=1}^{m}f_h\left(y_{j(h)s}\right)\right].$$

The log-likelihood function for the observed sample based on MRSS procedure in case of odd set size denoted by l_{MRSS1}^{*}will be given as follows:

$$l_{MRSS1}^{*}=\ln D_2+pln b+qln a+(c-1)\sum_{s=1}^{r}\left[\sum_{i=1}^{n}lnx_{i(g)s}+\sum_{j=1}^{m}lny_{j(h)s}\right]-\sum_{s=1}^{r}[(bg+1)$$
$$\times\sum_{i=1}^{n}\ln(1+x_{i(g)s}^{c})+(ah+1)\sum_{j=1}^{m}\ln\left(1+y_{j(h)s}^{c}\right)\Bigg]+\sum_{s=1}^{r}\left[(g-1)\sum_{i=1}^{n}\ln\left[1-\left(1+x_{i(g)s}^{c}\right)^{-b}\right]\right.$$

$$+(h-1)\sum_{j=1}^{m} ln\left[1-\left(1+y_{j(h)s}^{c}\right)^{-a}\right]\Bigg],$$

whereD_2 is a constant.The first partial derivatives of log-likelihood function with respect to b and a are given by:

$$\frac{\partial l_{MRSS\ 1}^{*}}{\partial b}=\frac{p}{b_1^*}-\sum_{s=1}^{r}\sum_{i=1}^{n}\left[g\ln\left(1+x_{i(g)s}^{c}\right)-(g-1)\frac{\ln(1+x_{i(g)s}^{c})}{\left(1+x_{i(g)s}^{c}\right)^{b_1^*}-1}\right]=0, \qquad (14)$$

$$\frac{\partial l_{MRSS\ 1}^{*}}{\partial a}=\frac{q}{a_1^*}-\sum_{s=1}^{r}\sum_{j=1}^{m}\left[h\ln\left(1+y_{j(h)s}^{c}\right)-(h-1)\frac{\ln(1+y_{j(h)s}^{c})}{\left(1+y_{j(h)s}^{c}\right)^{a_1^*}-1}\right]=0. \qquad (15)$$

It is clear that the MLEs of b and a denoted by b_1^* and a_1^* cannot be obtained in a closed form. Thus, an iterative technique must be applied to solve these equations numerically to obtain an estimates of b and a. The MLE of R denoted by R_{MRSS1}^{*}based on MRSS approach with odd set size is obtained by substituting $\rho_{MRSS1}^{*}=\frac{b_1^*}{a_1^*}$ in Equation (3).

MLE of $R=P(Y<X)$ with Even Set Size Based on MRSS Data:

For even set sizes, the $(u)^{th}$ smallest element is chosen from the first u ordered sets, while the $(u+1)^{th}$smallest unit is chosen from each of the remaining u sets. Let the set$\{\{X_{i(u)s}, i=1,2,\dots,u; s=$ *1,2,…,r}∪{Xiu+1s,i=u+1,…,n;s=1,2,…,r}*, be a MRSS drawn from Burr*c,b*with even set sizes where $u=n/2$.Then $X_{i(u)s}$ and $X_{i(u+1)s}$ are the$(u)^{th}$and the $(u+1)^{th}$ smallest units from the i^{th} set of the s^{th} cycle. By direct substitution into Equation (8) the PDFs of $(u)^{th}$and $(u+1)^{th}$order statistics when n is even are given as follows:

$$f_u\left(x_{i(u)s}\right)=\frac{n!}{(u-1)!\,(u)!}bcx_{i(u)s}^{c-1}\left[1+x_{i(u)s}^{c}\right]^{-(b(u+1)+1)}\left[1-\left(1+x_{i(u)s}^{c}\right)^{-b}\right]^{u-1}, \qquad x_{i(u)s}>0,$$

and

$$f_{u+1}\left(x_{i(u+1)s}\right)=\frac{n!}{(u)!\,(u-1)!}bcx_{i(u+1)s}^{c-1}\left[1+x_{i(u+1)s}^{c}\right]^{-(bu+1)}\left[1-\left(1+x_{i(u+1)s}^{c}\right)^{-b}\right]^{u}, \qquad x_{i(u+1)s}>0.$$

Similarly, let the set$\left\{\{Y_{j(v)s}, j=1,2,\dots,v; s=1,2,\dots,r\}\cup\{Y_{j(v+1)s}, j=v+1,\dots,m; s=1,2,\dots,r\}\right\}$, be the MRSS from Burr (c,a), where $v=m/2$. Then $y_{j(v)s}$and $y_{j(v+1)s}$are the $(v)^{th}$ and the $(v+1)^{th}$ smallest units from the j^{th} set of the s^{th} cycle.By direct substitution into Equation (9) the PDFs of $(v)^{th}$and $(v+1)^{th}$order statistics when$n$ is even are given as follows:

$$f_v\left(y_{j(v)s}\right)=\frac{m!}{(v-1)!\,(v)!}acy_{j(v)s}^{c-1}\left[1+y_{j(v)s}^{c}\right]^{-(a(v+1)+1)}\left[1-\left(1+y_{j(v)s}^{c}\right)^{-a}\right]^{v-1}, \qquad y_{j(v)s}>0,$$

and

$$f_{v+1}\left(y_{j(v+1)s}\right)=\frac{m!}{(v)!\,(v-1)!}acy_{j(v+1)s}^{c-1}\left[1+y_{j(v+1)s}^{c}\right]^{-(av+1)}\left[1-\left(1+y_{j(v+1)s}^{c}\right)^{-a}\right]^{v}, \qquad y_{j(v+1)s}>0.$$

The likelihood function for the observed sample with even set size based on MRSS,denoted by L_{MRSS2}^{*}, is given as follows:

$$L_{MRSS2}^{*}=\prod_{s=1}^{r}\left(\prod_{i=1}^{u}f_u\left(x_{i(u)s}\right)\prod_{i=u+1}^{n}f_{u+1}\left(x_{i(u+1)s}\right)\prod_{j=1}^{v}f_v\left(y_{j(v)s}\right)\prod_{j=v+1}^{m}f_{v+1}\left(y_{j(v+1)s}\right)\right).$$

Then, the log-likelihood function of L_{MRSS2}^{*}denoted by l_{MRSS2}^{*}will be as follows:

$$l_{MRSS2}^{*}=\ln\ D_3+plnb+qlna+(c-1)\sum_{s=1}^{r}\left[\sum_{i=1}^{u}lnx_{i(u)s}+\sum_{i=u+1}^{n}lnx_{i(u+1)s}+\sum_{j=1}^{v}lny_{j(v)s}\right.$$

$$+\sum_{j=v+1}^{m} ln y_{j(v+1)s}\Bigg] - \sum_{s=1}^{r}\{(b(u+1)+1)\sum_{i=1}^{u} ln\left[1+x_{i(u)s}^{c}\right] + (bu+1)\sum_{i=u+1}^{n} ln\left[1+x_{i(u+1)s}^{c}\right]$$

$$+(a(v+1)+1)\sum_{j=1}^{v} ln\left[1+y_{j(v)s}^{c}\right] + (av+1)\sum_{j=v+1}^{m} ln\left[1+y_{j(v+1)s}^{c}\right]\}$$

$$+\sum_{s=1}^{r}\{(u-1)\sum_{i=1}^{u} ln\left[1-\left(1+x_{i(u)s}^{c}\right)^{-b}\right] + u\sum_{i=u+1}^{n} ln\left[1-\left(1+x_{i(u+1)s}^{c}\right)^{-b}\right]$$

$$+(v-1)\sum_{j=1}^{v} ln\left[1-\left(1+y_{j(v)s}^{c}\right)^{-a}\right] + v\sum_{j=v+1}^{m} ln\left[1-\left(1+y_{j(v+1)s}^{c}\right)^{-a}\right]\}$$

whereD_3 is a constant. The first partial derivatives of log-likelihood function l_{MRSS2}^{*}withrespect to b and a are given by:

$$\frac{\partial l_{MRSS2}^{*}}{\partial b} = \frac{p}{b_2^{*}} - \sum_{s=1}^{r}\left[(u+1)\sum_{i=1}^{u} ln\left[1+x_{i(u)s}^{c}\right] + u\sum_{i=u+1}^{n} ln\left[1+x_{i(u+1)s}^{c}\right]\right]$$

$$+\sum_{s=1}^{r}\left[(u-1)\sum_{i=1}^{u}\frac{ln\left(1+x_{i(u)s}^{c}\right)}{\left(1+x_{i(u)s}^{c}\right)^{b_2^{*}}-1} + u\sum_{i=u+1}^{n}\frac{ln\left(1+x_{i(u+1)s}^{c}\right)}{\left(1+x_{i(u+1)s}^{c}\right)^{b_2^{*}}-1}\right] = 0, \qquad (16)$$

$$\frac{\partial l_{MRSS2}^{*}}{\partial a} = \frac{q}{a_2^{*}} - \sum_{s=1}^{r}\left[(v+1)\sum_{j=1}^{v} ln\left[1+y_{j(v)s}^{c}\right] + v\sum_{j=v+1}^{m} ln\left[1+y_{j(v+1)s}^{c}\right]\right]$$

$$+\sum_{s=1}^{r}\left[(v-1)\sum_{j=1}^{v}\frac{ln\left(1+y_{j(v)s}^{c}\right)}{\left(1+y_{j(v)s}^{c}\right)^{a_2^{*}}-1} + v\sum_{j=v+1}^{m}\frac{ln\left(1+y_{j(v+1)s}^{c}\right)}{\left(1+y_{j(v+1)s}^{c}\right)^{a_2^{*}}-1}\right] = 0. \qquad (17)$$

MLEs of b and a denoted by b_2^{*} and $a_2^{*}$are obtained by iteratively solving Equations (16) and (17). Thus, the MLE of R denoted by R_{MRSS2}^{*} is obtained by substituting b_2^{*} and a_2^{*} in Equation (3).

Estimation of $R = P(Y < X)$Based on ERSSData:

Samawi *et al*. (1996) introduced another modification of RSS called ERSS. This approach to data collection depends on two cases, the first case when the set size is odd and the second case when the set size is even. ERSS procedure can be designed as follows:

Case (1): for odd set size n, the largest and smallest units are selected from the first random sample to the $(n-1)^{st}$ random sample. From the n^{th} random sample select the median of the set. If the set $\{X_{1(1)s},\ldots,X_{n-1(n)s},X_{n(g)s}\}$ is quantified, then this will be ERSS for odd set size, where $g = n + 1/2$ and $s = 1,2,\ldots,r$.

Case (2):foreven set size n, the largest and smallest units are alternately taken from the first to the n^{th} random sample. If the set$\{X_{1(1)s},\ldots,X_{n-1(n)s},X_{n(n)s}\}$ is quantified, then this will be ERSS for even set size, where $s = 1,2,\ldots,r$.

In the following subsections, MLE of $R = P(Y < X)$ is derived under ERSS technique for both odd and even set sizes.

MLE of $R = P(Y < X)$ with Odd Set Size Based on ERSS Data:

Let $\{X_{i(1)s}, i = 1,\ldots,g-1; s = 1,2,\ldots,r\}$, $\{X_{i(n)s}, i = g,\ldots,n-1; s = 1,2,\ldots,r\}$ are the smallest and largest order statistics from Burr (c,b), where n is the set size, r is the number of cycles. The PDFs of $X_{i(1)s}$ and $X_{i(n)s}$ using Equation (8), will be as follows

$$f_1\left(x_{i(1)s}\right) = nbc\, x_{i(1)s}^{c-1}\left(1+x_{i(1)s}^{c}\right)^{-(bn+1)},\ x_{i(1)s} > 0, \qquad (18)$$

and

$$f_n\left(x_{i(n)s}\right) = nbc\, x_{i(n)s}^{c-1}\left(1+x_{i(n)s}^{c}\right)^{-(b+1)}\left[1-\left(1+x_{i(n)s}^{c}\right)^{-b}\right]^{n-1}, x_{i(n)s} > 0. \qquad (19)$$

Similarly, let $\{Y_{j(1)s}, j = 1, \dots, h-1; s = 1,2, \dots, r\}$, $\{Y_{j(m)s}, j = h, \dots, m-1; s = 1,2, \dots, r\}$ are the smallest and largest order statistics from Burr (c, a), where m is the set size, r is the number of cycles. Using Equation (9) thePDFs of $Y_{j(1)s}$ and $Y_{j(m)s}$ respectively will be as follows:

$$f_1(y_{j(1)s}) = mac\, y_{j(1)s}^{c-1}(1 + y_{j(1)s}^{c})^{-(am+1)}, y_{j(1)s} > 0, \quad (20)$$

and

$$f_m(y_{j(m)s}) = mac\, y_{j(m)s}^{c-1}(1 + y_{j(m)s}^{c})^{-(a+1)}\left[1 - (1 + y_{j(m)s}^{c})^{-a}\right]^{m-1}, y_{j(m)s} > 0. \quad (21)$$

Let$\{X_{i(g)s}, s = 1,2, \dots, r\}$ is the g^{th} order statistics from Burr (c, b) , and let $\{Y_{j(h)s}, s = 1,2, \dots, r\}$ is the h^{th} order statistics from Burr (c, a). Then, the PDFs of g^{th} and h^{th} order statistics are obtained in (12) and (13) respectively.

The likelihood function of observed sample based on ERSS in case of odd set size, denoted by L_{ERSS1}^{**}, is given as follows:

$$L_{ERSS1}^{**} = \prod_{s=1}^{r}\left[\prod_{i=1}^{g-1} f_1(x_{i(1)s}) \prod_{i=g}^{n-1} f_n(x_{i(n)s}) \prod_{j=1}^{h-1} f_1(y_{j(1)s}) \prod_{j=h}^{m-1} f_m(y_{j(m)s})\right]\left[f_g(x_{n(g)s}) f_h(y_{m(h)s})\right].$$

The log-likelihood function denoted by l_{ERSS1}^{**} is given as follows:

$$l_{ERSS1}^{**} = \ln\, D_4 + plnb + qlna + (c-1)\sum_{s=1}^{r}\left[\sum_{i=1}^{g-1} lnx_{i(1)s} + \sum_{i=g}^{n-1} lnx_{i(n)s} + \sum_{j=1}^{h-1} lny_{j(1)s}\right.$$
$$\left. + \sum_{j=h}^{m-1} ln\, y_{j(m)s}\right] - \sum_{s=1}^{r}\left[(bn+1)\sum_{i=1}^{g-1} \ln(1 + x_{i(1)s}^{c}) + (b+1)\sum_{i=g}^{n-1} \ln(1 + x_{i(n)s}^{c}) + (am+1)\right.$$
$$\left.\times \sum_{j=1}^{h-1} ln(1 + y_{j(1)s}^{c}) + (a+1)\sum_{j=h}^{m-1} ln(1 + y_{j(m)s}^{c})\right] + \sum_{s=1}^{r}\left[(n-1)\sum_{i=g}^{n-1} \ln[1 - (1 + x_{i(n)s}^{c})^{-b}]\right.$$
$$\left. + (m-1)\sum_{j=h}^{m-1} \ln[1 - (1 + y_{j(m)s}^{c})^{-a}]\right] + \sum_{s=1}^{r}\left[(c-1)(lnx_{n(g)s} + lny_{m(h)s})\right.$$
$$-(bg+1)\ln(1 + x_{n(g)s}^{c}) - (ah+1)ln(1 + y_{m(h)s}^{c}) + (g-1)\ln[1 - (1 + x_{n(g)s}^{c})^{-b}]$$
$$\left. + (h-1)\ln[1 - (1 + y_{m(h)s}^{c})^{-a}]\right],$$

whereD_4 is a constant. The first partial derivatives of log-likelihood function with respect to b and a are given by:

$$\frac{\partial l_{ERSS1}^{**}}{\partial b} = \frac{p}{b_1^{**}} - \sum_{s=1}^{r}\left[n\sum_{i=1}^{g-1} \ln(1 + x_{i(1)s}^{c}) + \sum_{i=g}^{n-1} \ln(1 + x_{i(n)s}^{c}) + gln(1 + x_{n(g)s}^{c})\right]$$
$$+ \sum_{s=1}^{r}\left[\sum_{i=g}^{n-1}(n-1)\frac{\ln(1 + x_{i(n)s}^{c})}{(1 + x_{i(n)s}^{c})^{b_1^{**}} - 1} + (g-1)\frac{\ln(1 + x_{n(g)s}^{c})}{(1 + x_{n(g)s}^{c})^{b_1^{**}} - 1}\right] = 0, \quad (22)$$

$$\frac{\partial l_{ERSS1}^{**}}{\partial a} = \frac{q}{a_1^{**}} - \sum_{s=1}^{r}\left[m\sum_{j=1}^{h-1} \ln(1 + y_{j(1)s}^{c}) + \sum_{j=h}^{m-1} \ln(1 + y_{j(m)s}^{c}) + hln(1 + y_{m(h)s}^{c})\right]$$
$$+ \sum_{s=1}^{r}\left[\sum_{j=h}^{m-1}(m-1)\frac{\ln(1 + y_{j(m)s}^{c})}{(1 + y_{j(m)s}^{c})^{a_1^{**}} - 1} + (h-1)\frac{\ln(1 + y_{m(h)s}^{c})}{(1 + y_{m(h)s}^{c})^{a_1^{**}} - 1}\right] = 0. \quad (23)$$

MLEs of b and a denoted by b_1^{**} and a_1^{**} are obtained by solving numerically Equations (22) and (23) using iterative technique. ThenR_{ERSS1}^{*} will be obtained by substituting b_1^{**} and a_1^{**} in Equation (3).

MLE of $R = P(Y < X)$ with Even Set Size Based on ERSS Data:

To obtain MLEs of R based on ERSS in case of even set size, let $X_{i(1)s}, i = 1,2,\dots,u; s = 1,2,\dots,r$ and $X_{i(n)s}, i = u+1,\dots,n; s = 1,2,\dots,r$ are the smallest and largest order statistics from Burr $(c,b)$with PDFs(18) and (19).

Similarly, let $Y_{j(1)s}, j = 1,2,\dots,v; s = 1,2,\dots,r$ and $Y_{j(m)s}, j = v+1,\dots,m; s = 1,2,\dots,r$ are the smallest and largest order statistics from Burr (c,a) with PDFs(20) and (21) respectively.

The likelihood function of observed sample using ERSS in case of even set size denoted by L_{ERSS2}^{**} is given as follows:

$$L_{ERSS2}^{**} = \prod_{s=1}^{r}\left[\prod_{i=1}^{u} f_1\left(x_{i(1)s}\right)\prod_{i=u+1}^{n} f_n\left(x_{i(n)s}\right)\prod_{j=1}^{v} f_1\left(y_{j(1)s}\right)\prod_{j=v+1}^{m} f_m\left(y_{j(m)s}\right)\right].$$

Then the log-likelihood function of L_{ERSS2}^{**} denoted by l_{ERSS2}^{**}will be as follows:

$$\begin{aligned} l_{ERSS2}^{**} &= \ln D_5 + p\ln b + q\ln a + (c-1)\sum_{s=1}^{r}\left[\sum_{i=1}^{u} \ln x_{i(1)s} + \sum_{i=u+1}^{n} \ln x_{i(n)s} + \sum_{j=1}^{v} \ln y_{j(1)s}\right. \\ &\left. + \sum_{j=v+1}^{m} \ln y_{j(m)s}\right] - \sum_{s=1}^{r}\left[(bn+1)\sum_{i=1}^{u} \ln\left(1+x_{i(1)s}^{c}\right) + (b+1)\sum_{i=u+1}^{n} \ln\left(1+x_{i(n)s}^{c}\right)\right. \\ &\left. + (am+1)\sum_{j=1}^{v} \ln\left(1+y_{j(1)s}^{c}\right) + (a+1)\sum_{j=v+1}^{m} \ln\left(1+y_{j(m)s}^{c}\right)\right] + \sum_{s=1}^{r}\left[\sum_{i=u+1}^{n}(n-1)\right. \\ &\left. \times \ln\left[1-\left(1+x_{i(n)s}^{c}\right)^{-b}\right] + \sum_{j=v+1}^{m}(m-1)\ln\left[1-\left(1+y_{j(m)s}^{c}\right)^{-a}\right]\right], \end{aligned}$$

where$D_5$is a constant. The first partial derivatives of log-likelihood function with respect to b and a are given by:

$$\frac{\partial l_{ERSS2}^{**}}{\partial b} = \frac{p}{b_2^{**}} - \sum_{s=1}^{r}\left[n\sum_{i=1}^{u} \ln\left(1+x_{i(1)s}^{c}\right) + \sum_{i=u+1}^{n} \ln\left(1+x_{i(n)s}^{c}\right) - (n-1)\sum_{i=u+1}^{n}\frac{\ln\left(1+x_{i(n)s}^{c}\right)}{\left(1+x_{i(n)s}^{c}\right)^{b_2^{**}}-1}\right] = 0, \tag{24}$$

$$\frac{\partial l_{ERSS\,2}^{**}}{\partial a} = \frac{q}{a_2^{**}} - \sum_{s=1}^{r}\left[m\sum_{j=1}^{v} \ln\left(1+y_{j(1)s}^{c}\right) + \sum_{j=v+1}^{m} \ln\left(1+y_{j(m)s}^{c}\right) - (m-1)\sum_{j=v+1}^{m}\frac{\ln\left(1+y_{j(m)s}^{c}\right)}{\left(1+y_{j(m)s}^{c}\right)^{a_2^{**}}-1}\right] = 0. \tag{25}$$

As it seems the likelihood equations have no closed form solutions in b and a. Therefore, numerical technique method is used to get the solution. MLEs of b and a will be denoted by b_2^{**} and a_2^{**},then R_{ERSS2}^{**} will be obtained by substituting b_2^{**} and a_2^{**}in Equation (3).

Estimation of $R = P(Y < X)$Based on PRSS Data:

In this section, MLE of $R = P(Y < X)$is derived under PRSS technique. Muttlak (2003) introduced PRSS procedure depending on two cases, the first case when the set size is odd and the second case when the set size is even. The two cases are separately considered in the following subsections.

MLE of $R = P(Y < X)$ with Odd SetSize Based on PRSS Data:

The main aim in this subsection is to obtain MLE of R for Burr XII distribution based onPRSSin case of odd set size. To derive the MLE of R, firstly the MLE of unknown parameters b and a will be obtained.

Let n_1 and n_2 be the nearest integer values of $O(n+1)$ and $t(n+1)$ respectively, where $0 < O \le 0.5$ and $t = 1 - O$.

Then for odd set size, the PRSS is the set:$\left\{X_{i(n_1)s}, i = 1,\dots,g-1; s = 1,\dots,r\right\} \cup \left\{X_{i(n_2)s}, i = g,\dots,n-1; s =\right.$

$1,\dots,r\} \cup \{X_{n(g)s}, s=1,2,\dots,r\}$,where$g = n+1/2$.Let $X_{1(n_1)s},\dots,X_{g-1(n_1)s},X_{g(n_2)s},\dots,X_{n-1(n_2)s},X_{n(g)s}$ is a PRSS from Burr (c,b) with sample size $p=nr$, where n is the set size and r is the number of cycles. Then using Equation (8) the PDFs of $X_{i(n_1)s}$ and $X_{i(n_2)s}$ will be as follows:

$$f_{n_1}(x_{i(n_1)s}) = \frac{n!}{(n-n_1)!(n_1-1)!} bcx_{i(n_1)s}^{c-1}\left(1+x_{i(n_1)s}^c\right)^{-[b(n-n_1+1)+1]}\left(1-\left(1+x_{i(n_1)s}^c\right)^{-b}\right)^{n_1-1}, \quad x_{i(n_1)s}>0, \tag{26}$$

and

$$f_{n_2}(x_{i(n_2)s}) = \frac{n!}{(n-n_2)!(n_2-1)!} bcx_{i(n_2)s}^{c-1}\left(1+x_{i(n_2)s}^c\right)^{-[b(n-n_2+1)+1]}\left(1-\left(1+x_{i(n_2)s}^c\right)^{-b}\right)^{n_2-1}. \quad x_{i(n_2)s}>0. \tag{27}$$

While the PDF of $X_{i(g)s}$ was defined in Equation (12).

Similarly, let m_1 and m_2 be the nearest integer values of $O(m+1)$ and $t(m+1)$ respectively. Then for odd set size, the PRSS is the set $\{Y_{j(m_1)s}, j=1,..,h-1; s=1,\dots,r\} \cup \{Y_{j(m_2)s}, j=h,..,m-1; s=1,\dots,r\} \cup \{Y_{m(h)s}, s=1,2,\dots,r\}$,where $h=m+1/2$.Let $Y_{1(m_1)s},\dots,Y_{h-1(m_1)s},Y_{h(m_2)s},\dots,Y_{m-1(m_2)s},Y_{m(h)s}$ is a PRSS from Burr (c,a) with sample size $q=mr$, where m is the set size and r is the number of cycles. Then using Equation (9) the PDFs of $Y_{j(m_1)s}$ and $Y_{j(m_2)s}$ will be as follows:

$$f_{m_1}(y_{j(m_1)s}) = \frac{m!}{(m-m_1)!(m_1-1)!} acy_{j(m_1)s}^{c-1}\left(1+y_{j(m_1)s}^c\right)^{-[a(m-m_1+1)+1]}\left(1-\left(1+y_{j(m_1)s}^c\right)^{-a}\right)^{m_1-1}, \quad y_{j(m_1)s}>0, \tag{28}$$

and

$$f_{m_2}(y_{j(m_2)s}) = \frac{m!}{(m-m_2)!(m_2-1)!} acy_{j(m_2)s}^{c-1}\left(1+y_{j(m_2)s}^c\right)^{-[a(m-m_2+1)+1]}\left(1-\left(1+y_{j(m_2)s}^c\right)^{-a}\right)^{m_2-1}, y_{j(m_2)s}>0. \tag{29}$$

The PDF of $Y_{j(h)s}$ was defined in Equation (13).

The likelihood function of theobserved PRSS in case of oddset size denoted by $\grave{L}_{PRSS1}$is given by:

$$\grave{L}_{PRSS1} = \prod_{s=1}^{r}\left[f_g(X_{n(g)s})f_h(Y_{m(h)s})\right]\left[\prod_{i=1}^{g-1} f_{n_1}(x_{i(n_1)s})\prod_{i=g}^{n-1} f_{n_2}(x_{i(n_2)s})\prod_{j=1}^{h-1} f_{m_1}(y_{j(m_1)s})\prod_{j=h}^{m-1} f_{m_2}(y_{j(m_2)s})\right].$$

The log-likelihood function of $\grave{L}_{PRSS1}$denoted by $\grave{l}_{PRSS1}$ will be as follows:

$$\begin{aligned}
\grave{l}_{PRSS1} &= lnD_6 + plnb + qlna + (c-1)\sum_{s=1}^{r}\left[\sum_{i=1}^{g-1} ln(x_{i(n_1)s}) + \sum_{i=g}^{n-1} ln(x_{i(n_2)s}) + \ln(x_{n(g)s})\right.\\
&\left.+\sum_{j=1}^{h-1} ln(y_{j(m_1)s}) + \sum_{j=h}^{m-1} ln(y_{j(m_2)s}) + \ln(y_{m(h)s})\right] - \sum_{s=1}^{r}\left[(b(n-n_1+1)+1)\sum_{i=1}^{g-1} ln\left(1+x_{i(n_1)s}^c\right)\right.\\
&+(b(n-n_2+1)+1)\sum_{i=g}^{n-1}\ln(1+x_{i(n_2)s}^c) + (bg+1)ln(1+x_{n(g)s}^c) + (a(m-m_1+1)+1)\\
&\left.\times\sum_{j=1}^{h-1} ln(1+y_{j(m_1)s}^c) + (a(m-m_2+1)+1)\sum_{j=h}^{m-1} ln(1+y_{j(m_2)s}^c) + (ah+1)ln(1+y_{m(h)s}^c)\right]\\
&+\sum_{s=1}^{r}\left[(n_1-1)\sum_{i=1}^{g-1} ln\left(1-\left(1+x_{i(n_1)s}^c\right)^{-b}\right) + (m_1-1)\sum_{j=1}^{h-1} ln\left(1-\left(1+y_{j(m_1)s}^c\right)^{-a}\right)\right.\\
&+(n_2-1)\sum_{i=g}^{n-1} ln\left(1-\left(1+x_{i(n_2)s}^c\right)^{-b}\right) + (m_2-1)\sum_{j=h}^{m-1} ln\left(1-\left(1+y_{j(m_2)s}^c\right)^{-a}\right)\\
&\left.+(g-1)ln\left(1-\left(1+x_{n(g)s}^c\right)^{-b}\right) + (h-1)ln\left(1-\left(1+y_{m(h)s}^c\right)^{-a}\right)\right],
\end{aligned}$$

whereD_6 is a constant. The first partial derivatives of b and a are given by:

$$\frac{\partial \grave{l}_{PRSS1}}{\partial b} = \frac{p}{\grave{b}_1} - \sum_{s=1}^{r}\left[(n-n_1+1)\sum_{i=1}^{g-1}\ln\left(1+x_{i(n_1)s}^c\right) + (n-n_2+1)\sum_{i=g}^{n-1} ln\left(1+x_{i(n_2)s}^c\right) + g ln\left(1+x_{n(g)s}^c\right)\right]$$
$$+\sum_{s=1}^{r}\left[(n_1-1)\sum_{i=1}^{g-1}\frac{ln\left(1+x_{i(n_1)s}^c\right)}{\left(1+x_{i(n_1)s}^c\right)^{\grave{b}_1}-1} + (n_2-1)\sum_{i=g}^{n-1}\frac{ln\left(1+x_{i(n_2)s}^c\right)}{\left(1+x_{i(n_2)s}^c\right)^{\grave{b}_1}-1} + (g-1)\frac{ln\left(1+x_{n(g)s}^c\right)}{\left(1+x_{n(g)s}^c\right)^{\grave{b}_1}-1}\right] = 0, \tag{30}$$

$$\frac{\partial \grave{l}_{PRSS1}}{\partial a} = \frac{q}{\grave{a}_1} - \sum_{s=1}^{r}\left[(m-m_1+1)\sum_{j=1}^{h-1}\ln\left(1+y_{j(m_1)s}^c\right) + (m-m_2+1)\sum_{j=h}^{m-1} ln\left(1+y_{j(m_2)s}^c\right) + \right.$$
$$\left. h ln(1+y_{m(h)s}^c\right] + \sum_{s=1}^{r}\left[(m_1-1)\sum_{j=1}^{h-1}\frac{\ln\left(1+y_{j(m_1)s}^c\right)}{\left(1+y_{j(m_1)s}^c\right)^{\grave{a}_1}-1} + (m_2-1)\sum_{j=h}^{m-1}\frac{ln\left(1+y_{j(m_2)s}^c\right)}{\left(1+y_{j(m_2)s}^c\right)^{\grave{a}_1}-1} + \frac{(h-1)ln\left(1+y_{m(h)s}^c\right)}{\left(1+y_{m(h)s}^c\right)^{\grave{a}_1}-1}\right] = 0 \tag{31}$$

MLEs of b and a denoted by $\grave{b}_1$ and $\grave{a}_1$can be found by solving the system of Equations (30) and (31). Although the proposed estimators cannot be expressed in closed forms, they can easily be obtained through the use of an appropriate numerical technique. Then $\grave{R}_{PRSS1}$ will be obtained by substituting $\grave{b}_1$ and$\grave{a}_1$in Equation (3).

MLE of $R = P(Y < X)$ with Even Set Size Based on PRSSData:

To derive the MLE of $R = P(Y < X)$ for Burr XII distribution based onPRSS sampling scheme in case of even set size, firstly the MLEs of unknown parameters b and a will be obtained.

Letn_1, n_2, O and t be defined, as in previous subsection, then for even set size, the PRSS is the set: $\{X_{i(n_1)s}, i = 1, \dots, u; s = 1, \dots, r\} \cup \{X_{i(n_2)s}, i = u+1, \dots, n; s = 1, \dots, r\}$, where $u = n/2$.

Let $X_{1(n_1)s}, \dots, X_{u(n_1)s}$, $X_{u+1(n_2)s}, \dots, X_{n(n_2)s}$isPRSS from Burr (c, b) with sample size $p = nr$, where n is the set size, r is the number of cycleswith PDFs(26) and (27).

Similarly, let the set $\{Y_{j(m_1)s}, j = 1, .., v; s = 1, \dots, r\} \cup \{Y_{j(m_2)s}, j = v+1, .., m; s = 1, \dots, r\}$is the PRSS for even set size, where $v = m/2$.Let $Y_{1(m_1)s}, \dots, Y_{v(m_1)s}, Y_{v+1(m_2)s}, \dots, Y_{m(m_2)s}$ is a PRSS from Burr (c, a) with sample size $q = mr$, where m is the set size ,r is the number of cycleswithPDFs (28) and (29).

The likelihood function of observedPRSS in case of even set size denoted by $\grave{L}_{PRSS2}$is given by:

$$\grave{L}_{PRSS2} = \prod_{s=1}^{r}\left[\prod_{i=1}^{u} f_{n_1}\left(x_{i(n_1)s}\right)\prod_{i=u+1}^{n} f_{n_2}\left(x_{i(n_2)s}\right)\prod_{j=1}^{v} f_{m_1}\left(y_{j(m_1)s}\right)\prod_{j=v+1}^{m} f_{m_2}\left(y_{j(m_2)s}\right)\right].$$

The log-likelihood function of PRSS data in case of even set size denoted by $\grave{l}_{PRSS2}$ is given by:

$$\grave{l}_{PRSS2} = lnD_7 + p ln b + q ln a + (c-1)\sum_{s=1}^{r}\left[\sum_{i=1}^{u} ln\, x_{i(n_1)s} + \sum_{i=u+1}^{n} ln x_{i(n_2)s} + \sum_{j=1}^{v} ln\, y_{j(m_1)s}\right.$$
$$\left. + \sum_{j=v+1}^{m} ln\, y_{j(m_2)s}\right] + \sum_{s=1}^{r}\left[(n_1-1)\sum_{i=1}^{u} ln\left[1-\left(1+x_{i(n_1)s}^c\right)^{-b}\right] + (n_2-1)\sum_{i=u+1}^{n} ln\left[1-\left(1+x_{i(n_2)s}^c\right)^{-b}\right]\right.$$
$$\left. +(m_1-1)\sum_{j=1}^{v} ln\left[1-\left(1+y_{j(m_1)s}^c\right)^{-a}\right] + (m_2-1)\sum_{j=v+1}^{m} ln\left[1-\left(1+y_{j(m_2)s}^c\right)^{-a}\right]\right]$$
$$-\sum_{s=1}^{r}\left[[b(n-n_1+1)+1]\sum_{i=1}^{u} ln\left(1+x_{i(n_1)s}^c\right) + [b(n-n_2+1)+1]\sum_{i=u+1}^{n} ln\left(1+x_{i(n_2)s}^c\right)\right.$$
$$\left. +[a(m-m_1+1)+1]\sum_{j=1}^{v} ln\left(1+y_{j(m_1)s}^c\right) + [a(m-m_2+1)+1]\sum_{j=v+1}^{m} \ln\left(1+y_{j(m_2)s}^c\right)\right],$$

where D_7 is a constant. The first partial derivatives of log-likelihood function with respect to b and a are given by:

$$\frac{\partial \grave{l}_{PRSS2}}{\partial b}=\frac{p}{\grave{b}_2}+\sum_{s=1}^{r}\left[(n_1-1)\sum_{i=1}^{u}\frac{ln\left(1+x_{i(n_1)s}^c\right)}{\left(1+x_{i(n_1)s}^c\right)^{\grave{b}_2}-1}+(n_2-1)\sum_{i=u+1}^{n}\frac{ln\left(1+x_{i(n_2)s}^c\right)}{\left(1+x_{i(n_2)s}^c\right)^{\grave{b}_2}-1}\right]$$

$$-\sum_{s=1}^{r}\left[(n-n_1+1)\sum_{i=1}^{u}ln\left(1+x_{i(n_1)s}^c\right)+(n-n_2+1)\sum_{i=u+1}^{n}ln\left(1+x_{i(n_2)s}^c\right)\right]=0, \quad (32)$$

$$\frac{\partial \grave{l}_{PRSS2}}{\partial a}=\frac{q}{\grave{a}_2}+\sum_{s=1}^{r}\left[(m_1-1)\sum_{j=1}^{v}\frac{ln\left(1+y_{j(m_1)s}^c\right)}{\left(1+y_{j(m_1)s}^c\right)^{\grave{a}_2}-1}+(m_2-1)\sum_{j=v+1}^{m}\frac{ln\left(1+y_{j(m_2)s}^c\right)}{\left(1+y_{j(m_2)s}^c\right)^{\grave{a}_2}-1}\right]$$

$$-\sum_{s=1}^{r}\left[(m-m_1+1)\sum_{j=1}^{v}ln\left(1+y_{j(m_1)s}^c\right)+(m-m_2+1)\sum_{j=v+1}^{m}ln\left(1+y_{j(m_2)s}^c\right)\right]=0. \quad (33)$$

Equations (32) and (33) do not have explicit solutions and they have to be obtained numerically.Let the MLEs of b and a are denoted by $\grave{b}_2$ and $\grave{a}_2$ respectively. Once $\grave{b}_2$ and $\grave{a}_2$ are obtained, the MLE of R, say $\grave{R}_{PRSS2}$, can be obtained by substituting $\grave{b}_2$ and $\grave{a}_2$ in Equation (3).

Numerical Illustration:

In this study, a computer simulation is conducted to study the efficiency of estimating R, using SRS, RSS, MRSS, ERSS and PRSS techniques for Burr XII distribution. Without loss of generality the common shape parameter c will be assumed to be one in all the experiments. Two cases will be considered separately to draw inference on R. The first case (i) for odd set size and the second case (ii) for even set size. The proposed estimators depend on the two cases will be calculated. Comparison between the proposed estimators for R using MRSS, ERSS and PRSS with known estimators based on SRS and RSS approaches will be carried out. 1000 random samplesfrom Burr (c,b) and Burr (c,a) distributionsare generated with sample sizes $p=nr$ and $q=mr$ respectively, where $n=3,4,5,6,7,8$ and $m=3,4,5,6,7,8$ and number of cycles $r=10$. The ratio $\rho=\frac{b}{a}$ is selected as $0.1, 0.5, 1, 2$ and 6, then MSEs and efficiencies of all estimators of R based on SRS, RSS, MRSS, ERSS and PRSS will be computed.

The simulation results are summarized in Tables$(1-4)$and represented through Figures $(1-5)$. The performance of the estimated parameters is evaluated using MSEs and the efficiencies criteria.The results of MSEs for different estimators $\hat{R}_{MLE}, \hat{R}_{UMVUE}, \tilde{R}_{MLE}, R^*_{MRSS1}, R^*_{MRSS2}, R^{**}_{ERSS1}, R^{**}_{ERSS2}, \grave{R}_{PRSS1}$ and $\grave{R}_{PRSS2}$ are listedin Tables$(1-2)$. Also, the results of the efficiencies for different estimators are reported in Tables $(3-4)$.

Table 1: MSEs of the estimators $\hat{R}_{MLE}, \hat{R}_{UMVUE}, \tilde{R}_{MLE}, R^*_{MRSS1}, R^{**}_{ERSS1}$ and $\grave{R}_{PRSS1}$ for odd set size.

ρ	(n,m)	$\hat{R}_{MLE}$	$\hat{R}_{UMVUE}$	$\tilde{R}_{MLE}$	R^*_{MRSS1}	R^{**}_{ERSS1}	$\grave{R}_{PRSS1}$		
							20%	30%	40%
0.1	(3,3)	0.000503	0.000486	0.000263	0.000240	0.000268	0.000268	0.000268	0.000240
	(5,5)	0.000274	0.000269	0.000108	0.000103	0.000108	0.000108	0.000102	0.000102
	(7,7)	0.000195	0.000192	0.000052	0.000052	0.000056	0.000050	0.000050	0.000047
0.5	(3,3)	0.003285	0.003436	0.001806	0.001665	0.001840	0.001840	0.001840	0.001665
	(5,5)	0.001917	0.001940	0.000764	0.000727	0.000767	0.000767	0.000723	0.000723
	(7,7)	0.001367	0.001368	0.000373	0.000372	0.000400	0.000362	0.000362	0.000345
1	(3,3)	0.004047	0.004302	0.002283	0.002093	0.002297	0.002297	0.002297	0.002093
	(5,5)	0.002427	0.002470	0.000963	0.000927	0.000965	0.000965	0.000908	0.000908
	(7,7)	0.001724	0.001748	0.000471	0.000460	0.000506	0.000458	0.000458	0.000438
2	(3,3)	0.003216	0.003291	0.001823	0.001668	0.001823	0.001823	0.001823	0.001668
	(5,5)	0.001954	0.001972	0.000763	0.000722	0.000762	0.000762	0.000717	0.000717
	(7,7)	0.001358	0.001368	0.000427	0.000364	0.000394	0.000363	0.000363	0.000348
6	(3,3)	0.001024	0.001010	0.000573	0.000521	0.000570	0.000570	0.000570	0.000521
	(5,5)	0.000620	0.000607	0.000235	0.000225	0.000233	0.000233	0.000219	0.000219
	(7,7)	0.000432	0.000426	0.000122	0.000112	0.000122	0.000111	0.000111	0.000104

Table 2: MSEs of the estimators $\hat{R}_{MLE}, \hat{R}_{UMVUE}, \tilde{R}_{MLE}, R^*_{MRSS2}, R^{**}_{ERSS2}$ and $\grave{R}_{PRSS2}$ for even set size.

ρ	(n,m)	$\hat{R}_{MLE}$	$\hat{R}_{UMVUE}$	$\tilde{R}_{MLE}$	R^*_{MRSS2}	R^{**}_{ERSS2}	$\grave{R}_{PRSS2}$		
							20%	30%	40%
0.1	(4,4)	0.000352	0.000339	0.000153	0.000147	0.000157	0.000157	0.000133	0.000147
	(6,6)	0.000229	0.000225	0.000079	0.000072	0.000094	0.000094	0.000075	0.000072
	(8,8)	0.000185	0.000181	0.000044	0.000044	0.000050	0.000046	0.000044	0.000044
0.5	(4,4)	0.002372	0.002403	0.001073	0.001102	0.001231	0.001231	0.000944	0.001102
	(6,6)	0.001595	0.001614	0.000570	0.000515	0.000666	0.000666	0.000537	0.000515

	(8,8)	0.001287	0.001295	0.000319	0.000318	0.000420	0.000329	0.000316	0.000318
1	(4,4)	0.002938	0.003012	0.001346	0.001285	0.001552	0.001552	0.001193	0.001285
	(6,6)	0.002006	0.002040	0.000701	0.000650	0.000832	0.000832	0.000679	0.000650
	(8,8)	0.001610	0.001631	0.000405	0.000401	0.000530	0.000413	0.000399	0.000401
2	(4,4)	0.002321	0.002364	0.001066	0.001013	0.001236	0.001236	0.000949	0.001013
	(6,6)	0.001601	0.001614	0.000574	0.000515	0.000653	0.000653	0.000538	0.000515
	(8,8)	0.001274	0.001286	0.000321	0.000317	0.000420	0.000325	0.000316	0.000317
6	(4,4)	0.000723	0.000714	0.000328	0.000310	0.000384	0.000384	0.000294	0.000310
	(6,6)	0.000500	0.000492	0.000160	0.000157	0.000198	0.000198	0.000165	0.000157
	(8,8)	0.000393	0.000391	0.000098	0.000096	0.000120	0.000098	0.000096	0.000096

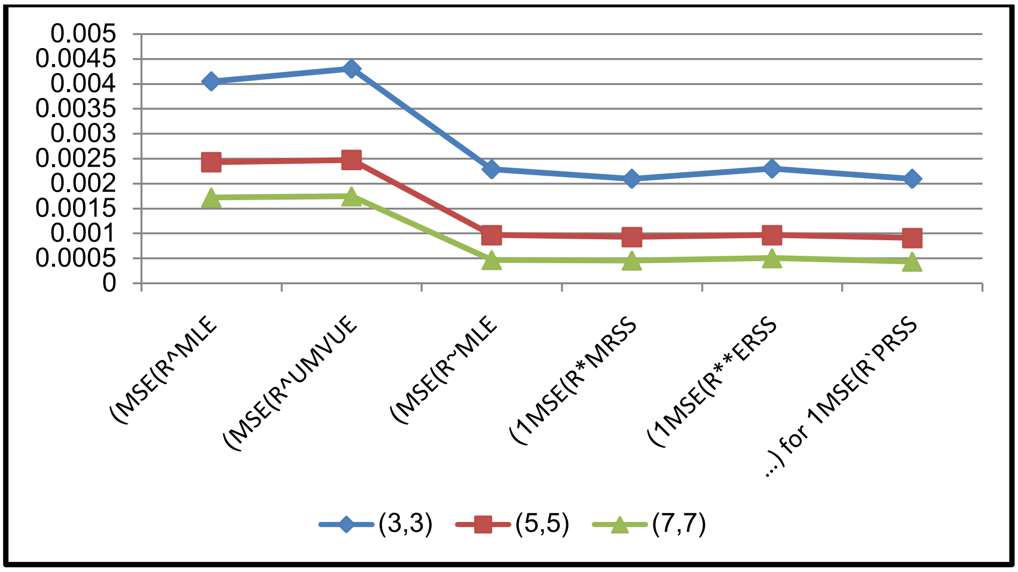

Fig. 1: MSEs of the estimators $\hat{R}_{MLE}$, $\hat{R}_{UMVUE}$, $\tilde{R}_{MLE}$, $R^{*}_{MRSS\ 1}$, $R^{**}_{ERSS\ 1}$ and $\grave{R}_{PRSS1}$ for odd set size at $\rho = 1$

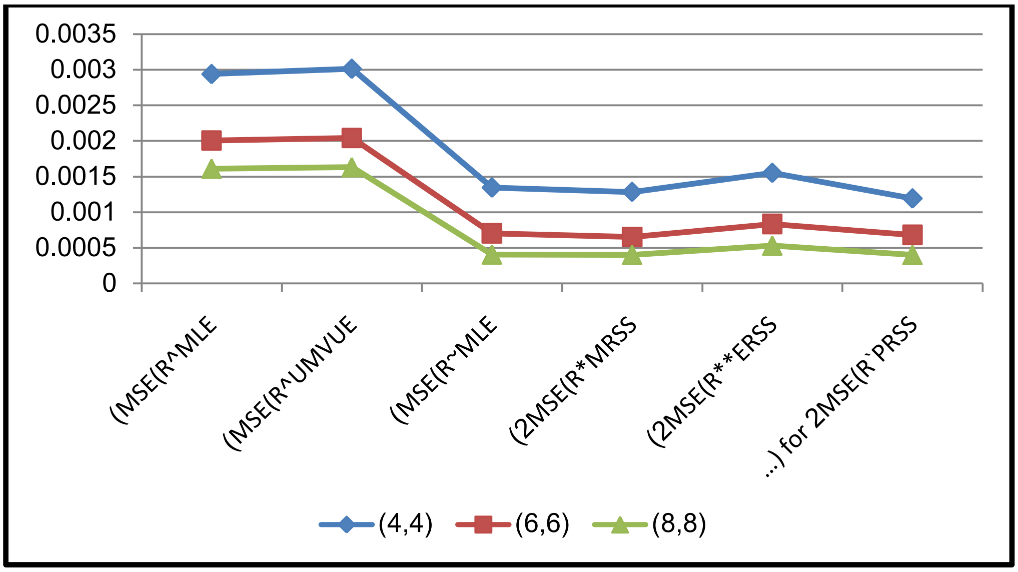

Fig. 2: MSEs of the estimators $\hat{R}_{MLE}$, $\hat{R}_{UMVUE}$, $\tilde{R}_{MLE}$, $R^{*}_{MRSS\ 2}$, $R^{**}_{ERSS\ 2}$ and $\grave{R}_{PRSS2}$ for even set size at $\rho = 1$.

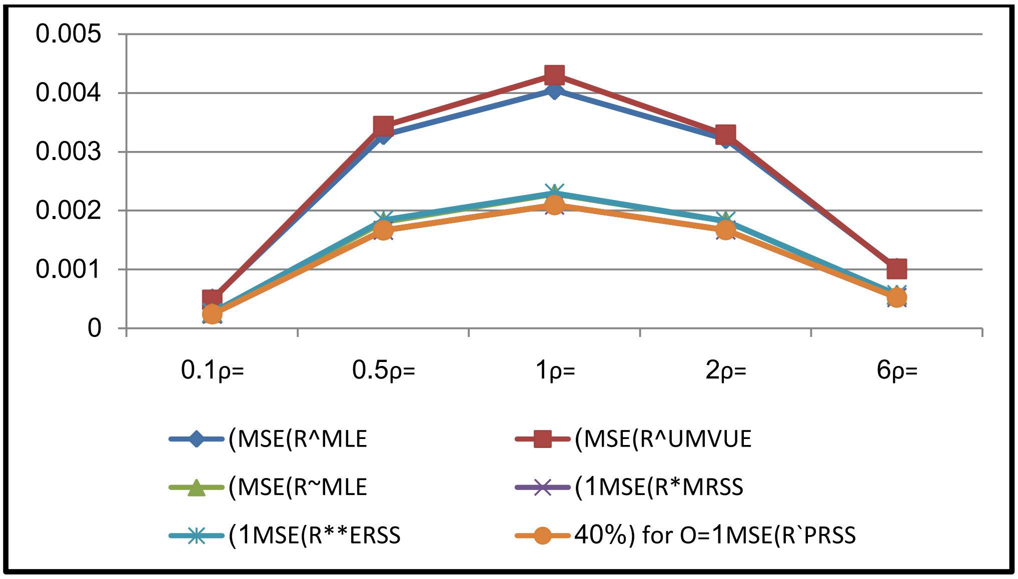

Fig. 3: MSEs of the estimators $\hat{R}_{MLE}$, $\hat{R}_{UMVUE}$, $\tilde{R}_{MLE}$, $R^*_{MRSS\,1}$, $R^{**}_{ERSS\,1}$ and $\grave{R}_{PRSS1}$ at $(n,m) = (3,3)$.

Depending on MSEs of different estimators, the following conclusions can be observed:

1. MSEs of all estimators $\hat{R}_{MLE}$, $\hat{R}_{UMVUE}$, $\tilde{R}_{MLE}$, R^*_{MRSS1}, R^*_{MRSS2}, R^{**}_{ERSS1}, R^{**}_{ERSS2}, $\grave{R}_{PRSS1}$ and $\grave{R}_{PRSS2}$ decrease as the set size increases in all cases.

2. Based on SRS data, MSEs of $\hat{R}_{MLE}$ are greater than MSEs of $\hat{R}_{UMVUE}$ at $\rho = 0.1$ and 6, otherwise MSEs of $\hat{R}_{MLE}$ are smaller than MSEs of $\hat{R}_{UMVUE}$ in all cases.

3. MSEs of $\tilde{R}_{MLE}$, R^*_{MRSS1}, R^*_{MRSS2}, R^{**}_{ERSS1}, R^{**}_{ERSS2}, $\grave{R}_{PRSS1}$ and $\grave{R}_{PRSS2}$ based on RSS, MRSS, ERSS and PRSS data are smaller than MSEs of $\hat{R}_{MLE}$ and $\hat{R}_{UMVUE}$ based on SRS data in all cases.

4. R^*_{MRSS1} and R^*_{MRSS2} based on MRSS have the smallest MSEs in all cases comparing with the other estimators. However, in almost all cases, $\grave{R}_{PRSS1}$ for $O = 0.40$ has the smallest MSEsin case of odd set size (see for example Figure(1)) and $\grave{R}_{PRSS2}$ for $O = 0.30$ has the smallest MSEs in case of even set size (see for example Figure(2)).

5. MSEs of all estimators $\hat{R}_{MLE}$, $\hat{R}_{UMVUE}$, $\tilde{R}_{MLE}$, R^*_{MRSS1}, R^*_{MRSS2}, R^{**}_{ERSS1}, R^{**}_{ERSS2}, $\grave{R}_{PRSS1}$ and $\grave{R}_{PRSS2}$ increase as the value of ρ increases up to $\rho = 1$ then MSEs decrease as the value of ρ increases in all cases (see for example Figure (3)).

Table 3: Efficiencies of the estimators $\hat{R}_{UMVUE}$, $\tilde{R}_{MLE}$, $R^*_{MRSS\,1}$, $R^{**}_{ERSS\,1}$ and $\grave{R}_{PRSS1}$ with respect to $\hat{R}_{MLE}$ for odd set size.

ρ	(n,m)	$\hat{R}_{UMVUE}$	$\tilde{R}_{MLE}$	R^*_{MRSS1}	R^{**}_{ERSS1}	$\grave{R}_{PRSS1}$		
						20%	30%	40%
0.1	(3,3)	1.038	1.912	2.094	1.875	1.875	1.875	2.094
	(5,5)	1.018	2.636	2.639	2.524	2.524	2.674	2.674
	(7,7)	1.015	3.743	3.726	3.491	3.878	3.878	4.074
0.5	(3,3)	0.978	1.806	1.973	1.785	1.785	1.785	1.973
	(5,5)	0.948	2.506	2.637	2.498	2.498	2.651	2.651
	(7,7)	0.999	3.659	3.668	3.417	3.776	3.776	3.954
1	(3,3)	0.967	1.772	1.934	1.762	1.762	1.762	1.934
	(5,5)	0.980	2.519	2.617	2.515	2.515	2.673	2.673
	(7,7)	0.986	3.654	3.679	3.405	3.762	3.762	3.931
2	(3,3)	0.997	1.764	1.928	1.764	1.764	1.764	1.928
	(5,5)	0.991	2.559	2.706	2.562	2.562	2.726	2.726
	(7,7)	0.993	3.869	3.731	3.439	3.763	3.763	3.953
6	(3,3)	1.015	1.787	1.963	1.796	1.796	1.796	1.963
	(5,5)	1.022	2.640	2.746	2.655	2.655	2.828	2.828
	(7,7)	1.014	3.780	3.844	3.530	3.884	3.884	4.036

Table 4: Efficiencies of the estimators $\hat{R}_{UMVUE}$, $\tilde{R}_{MLE}$, $R^*_{MRSS\,2}$, $R^{**}_{ERSS\,2}$ and $\grave{R}_{PRSS2}$ with respect to $\hat{R}_{MLE}$ for even set size

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ρ	(n,m)	$\hat{R}_{UMVUE}$	$\tilde{R}_{MLE}$	R^{*}_{MRSS2}	R^{**}_{ERSS2}	$\grave{R}_{PRSS2}$		
						20%	30%	40%
0.1	(4,4)	1.039	2.303	2.395	2.014	2.014	2.646	2.395
	(6,6)	1.017	2.871	3.169	2.411	2.411	3.040	3.169
	(8,8)	1.020	4.163	4.165	3.089	3.986	4.188	4.165
0.5	(4,4)	0.987	2.211	2.309	1.926	1.926	2.510	2.309
	(6,6)	0.988	2.794	3.094	2.394	2.394	2.966	3.094
	(8,8)	0.993	4.022	4.042	3.018	3.904	4.064	4.042
1	(4,4)	0.975	2.182	2.287	1.893	1.893	2.462	2.287
	(6,6)	0.984	2.776	3.084	2.410	2.410	2.953	3.084
	(8,8)	0.988	3.974	4.006	3.002	3.892	4.028	4.006
2	(4,4)	0.982	2.177	2.292	1.878	1.878	2.445	2.292
	(6,6)	0.992	2.785	3.107	2.448	2.448	2.971	3.107
	(8,8)	0.991	3.962	4.009	3.011	3.916	4.028	4.009
6	(4,4)	1.013	2.201	2.328	1.884	1.884	2.640	2.328
	(6,6)	1.017	2.829	3.173	2.520	2.520	3.027	3.173
	(8,8)	1.007	3.996	4.062	3.055	3.991	4.077	4.062

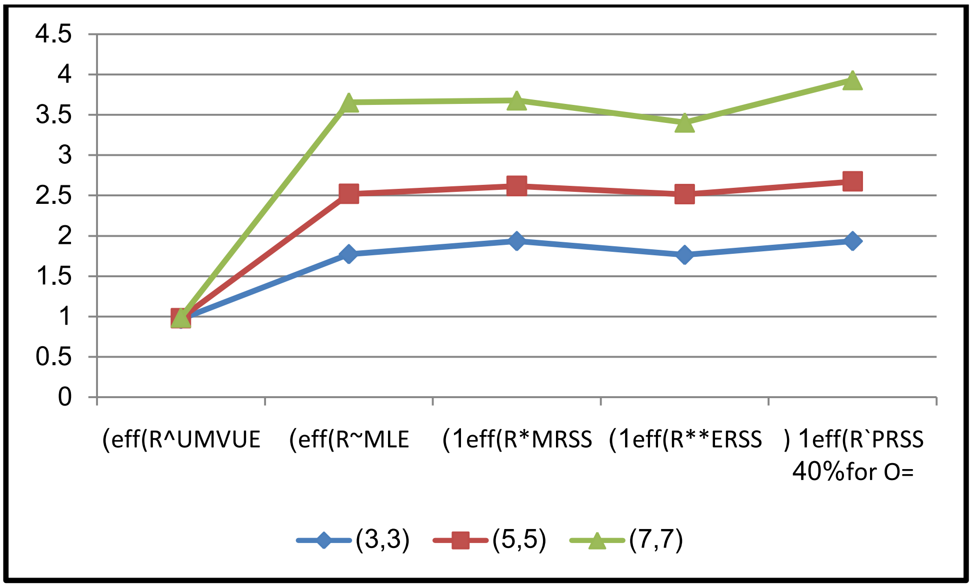

Fig. 4: The efficiencies of the estimators $\hat{R}_{UMVUE}$, $\tilde{R}_{MLE}$, $R^{*}_{MRSS\ 1}$, $R^{**}_{ERSS\ 1}$ and $\grave{R}_{PRSS1}$ with respect to $\hat{R}_{MLE}$ for odd set size at $\rho = 1$.

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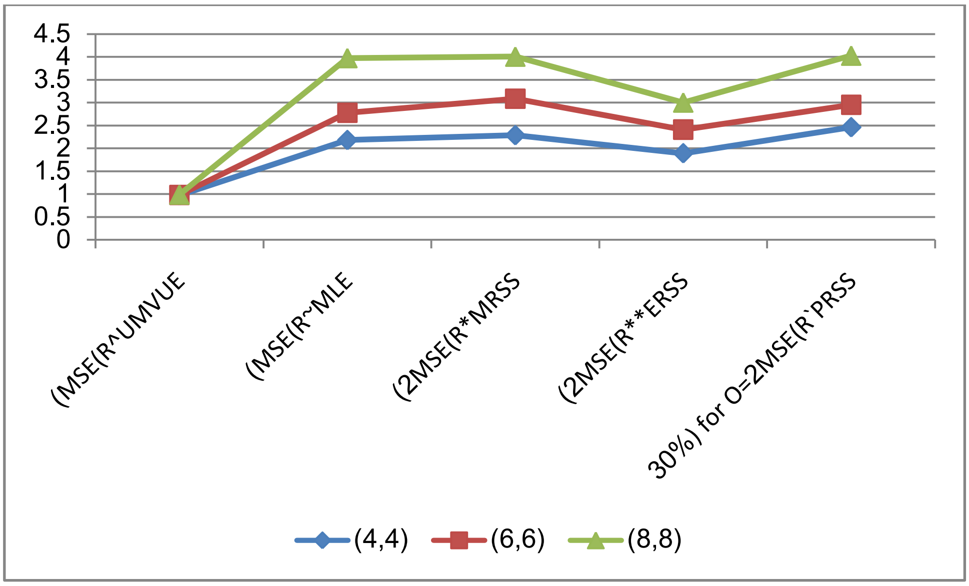

Fig. 5: The efficiencies of the estimators $\hat{R}_{UMVUE}$, $\tilde{R}_{MLE}$, $R^{*}_{MRSS\,2}$, $R^{**}_{ERSS\,2}$ and $\grave{R}_{PRSS2}$ with respect to $\hat{R}_{MLE}$ for even set size at $\rho = 1$.

Considering the efficiencies of estimators, the following conclusions can be observed:

1. The efficiencies of $\hat{R}_{UMVUE}$ with respect to $\hat{R}_{MLE}$ are greater than one in case of $\rho = 0.1$ and 6, otherwise the efficiencies of $\hat{R}_{UMVUE}$ with respect to $\hat{R}_{MLE}$ are less than one in all cases.
2. The estimators $\tilde{R}_{MLE}, R^{*}_{MRSS1}, R^{*}_{MRSS2}, R^{**}_{ERSS1}, R^{**}_{ERSS2}, \grave{R}_{PRSS1}$ and $\grave{R}_{PRSS2}$ based on RSS and its modifications are more efficient than the estimators $\hat{R}_{MLE}, \hat{R}_{UMVUE}$ based on SRS data in all cases.
3. The efficiencies of all estimators $\hat{R}_{UMVUE}, \tilde{R}_{MLE}, R^{*}_{MRSS1}, R^{*}_{MRSS2}, R^{**}_{ERSS1}, R^{**}_{ERSS2}, \grave{R}_{PRSS1}$ and $\grave{R}_{PRSS2}$ increase as n and m increase for the same value of ρ in almost all cases (see for example Figures (4-5)).
4. R^{*}_{MRSS1} and R^{*}_{MRSS2} in case of odd and even set sizes respectively are more efficient than $\tilde{R}_{MLE}, R^{**}_{ERSS1}, R^{**}_{ERSS2}$ and $\hat{R}_{UMVUE}$ respectively in all cases. However in almost all cases, $\grave{R}_{PRSS1}$ is more efficient than R^{*}_{MRSS1} in case of odd set size for $O = 0.40$ (see for example Figure (4)) and $\grave{R}_{PRSS2}$ is more efficient than R^{*}_{MRSS2} for $O = 0.30$ in case of even set size (see for example Figure (5)).

Conclusions:

In this article, the estimation of $R = \mathrm{P}(Y < X)$ when strength X and stress Y are two independent variables of BurrType XII distributionis studied. Maximum likelihood estimators of R are compared under different sampling schemes. The selected sampling schemes are SRS, RSS, MRSS, ERSS and PRSS.It is observed that, MSEs of estimators based on SRS data are greater than the corresponding MSEs based on ERSS, RSS, MRSS and PRSS data respectively.

Estimatorsof R based on MRSS have the smallest MSEs in all cases comparing with the estimators based on RSS, ERSS and SRS data respectively. However, in almost all cases, estimator of R underPRSS for $O = 0.40$ has the smallest MSEsin the case of odd set size. While,the estimator of R under PRSSfor $O = 0.30$ has the smallest MSEs in case of even set size.Also, it can be observed that, MSEs of all estimators decrease as the set size increases in all cases.

It is clear from simulation study that the efficiency of all estimators increases as the set size increases in almost all cases. The efficiencies of the estimators based on SRS data are smaller than the corresponding estimators based on RSS, MRSS, ERSS and PRSS data.

This study revealed that the estimators based on PRSS for odd set sizes when $O = 0.40$ are more efficient than the other methods of sampling procedures for estimating R. Also, the estimators based on PRSS for even set sizes when $O = 0.30$ are more efficient than the other methods of sampling procedures for estimating R. Generally, the estimators of R under PRSS with odd and even set sizes have largest efficiencies comparing with the other estimators based on MRSS, RSS and ERSS respectively.

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