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A Heuristic for Spectrum Assignment based on Maximal Independent Set in Offline Ring Optical Networks

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ABSTRACT

Background: Elastic optical networks (EONs) are deployed to serve the huge traffic demands of high-bandwidth multimedia applications in optical networks. The assignment of each lightpath in an EON to a set of contiguous spectrum slots through the network to accommodate a traffic demand is referred to as a spectrum assignment (SA) problem. Many studies have proved that offline SA is NP-hard problem meaning that no algorithm can solve it optimally in polynomial time. **Objective:** In this paper, we propose a new heuristic algorithm based on maximal independent set (MIS) to solve the offline SA problem on unidirectional ring topology. **Results:** Our results indicate that, on average, compared with the reference algorithm the spectrum consumption of the proposed method is lower than that obtained using the reference algorithm for different network sizes. An average spectrum utilization of our proposed algorithm is 15% closer to the optimal solution for ring topology less than 7 nodes. **Conclusion:** The proposed algorithm outperforms the well-known reference method Maximum Reuse Spectrum Allocation (MRSA) in term of spectrum utilization in average for networks size ranging from four to sixteen nodes.

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INTRODUCTION

Wavelength-division multiplexing (WDM) networks have emerged as viable architectural solutions to serve the high-speed transmission of traffic and induced rapid expansion of optical networks (Ramaswami, 2006). WDM networks are strictly regulated by fixed ITU-T grids and spacing policies that yield poor exploitation of the fiber's capacity. In such networks, the wavelength capacity may not be fully utilized if the demand requires a very little amount of the wavelength channel (Gerstel *et al.,* 2012, Chen *et al.,* 2012).

Technological breakthroughs in the field of Optical Orthogonal Frequency Division Multiplexing (OOFDM) enabled the development of flexible spectrum assignment modulation. As opposed to the WDM network, OOFDM imposes extra robustness on allocating different channels without interfering with each other. This development alleviated the low utilization efficiency reflected by WDM fixed ITU-T grids and spacing policies (Gerstel *et al.,* 2012).

In the optical network, a logical path between a pair of source-destination nodes that may span multiple fiber links is referred to as a lightpath. Each lightpath in an EON must be assigned a path and a set of spectrum slots through the network. This process is defined as a routing and spectrum assignment (RSA) problem. In an EON, any lightpath needing allocation must satisfy three constraints:

(1) *The spectrum contiguity constraint:* ensures the assigned spectrum slots are contiguous.

(2) *The spectrum continuity constraint*: ensures the assigned spectrum slots are the same for each demand along the selected routing path.

(3) *The distinct spectrum constraint*: if two or more lightpaths overlap on a routing path, each must be assigned distinct slots.

Fully grid-less tenability cannot be achieved by most optical components (Shen *et al.,* 2011). Thus, it is necessary to have a guard carrier (GC) to separate two neighbor channels. This ensures that adjacent lightpaths do not interfere with each other.

In unidirectional ring topology, the lightpaths are limited to only one route; hence, the RSA problem is reduced to a spectrum assignment (SA) problem. Ring topology is the most popular infrastructure for SONET architecture (Alewayn, 2004). According to a new study by (Talebi *et al.* 2014), the SA problem in a unidirectional ring is NP-hard for three or more nodes. The authors proved that when the topology is a chain, SA is equivalent to the multiprocessor task-scheduling problem and is NP-

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complete for four or more processors. In the case of a bidirectional ring network with a predetermined route, the SA is NP-complete at a size greater than four nodes (Talebi *et al*., 2015). Although wavelength assignments in WDM networks (WA) is NP-hard in undirected stars and polynomial in directed stars, SA is NP-hard for undirected stars of three links and directed stars of four links (Bermond *et al.*, 2015). A recent study by (Shirazipourazad *et al.*2013) solved the RSA problem in ring networks with an approximation ratio $4 + 2\epsilon$. A literature exploring the advantages, technologies and RSA algorithms in EONs is presented in (Talebi *et al.*, 2014).The paper survey current state of the art of works that have already been done to manage the traffic in EON, and discuss generally the various research issues in EON.

A maximal independent set is a promising technique likely to be a viable solution in a wide range of practical problems arising in wireless and optical networks (Wang *et al.*, 2002; Wang *et al.*, 2009; Yetginer *et al.,* 2011).A fast exact algorithm based on MIS managing the connections in WDM networks is proposed in (Yetginer *et al.*, 2011). The study designed a new ILP formulation that trades off the large set of integer variables and constraints. The main idea of this study centered on decomposing the ring graph recursively to yield smaller number of maximal independent sets (MISs) computed in the decomposition phase. Therefore, the set of integer variables is quite small for even a large problem. The paper decomposed the network into 2^x partitions while the assignment consisted of allocating one lightpath to different MISs. The experimental results indicated that decomposing the network reduced several orders of magnitude in running time (Yetginer *et al.*, 2011).

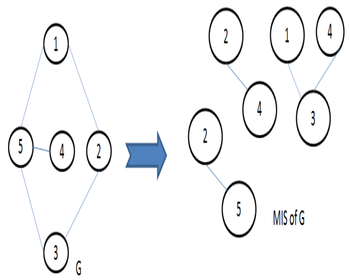

Fig. 1: The MISs of graph G.

Although MIS has applications in a number of different areas, it is not yet adopted to solve the offline SA problems. (Talebi *et al.* 2015) argued the importance of providing a new algorithm to allocate traffic demands into a unidirectional ring network up to sixteen nodes (the maximum size of a SONET/SDH ring). Regarding current literature, there is no other known work related to employing MIS to solve the offline SA problem. In this paper, we study the offline version of the SA problem to minimize the total bandwidth requirement for a set of connection requests. The contributions include:

(1) Employing a maximal independent set to solve offline SA. To our best knowledge, this would be the first instance of employing MIS for solving offline SA.

(2) The proposed algorithm outperforms the reference method maximum reuse spectrum allocation (MRSA) (Wang *et al.,* 2011) in terms of spectrum utilization for networks ranging from four to sixteen nodes.

Background:

Graph Coloring Problem:

The graph coloring problem (GCP), a well-known NP-hard problem, is one of the most typical problems in computer science. A graph coloring problem presents the issue of finding the minimum number of colors needed to paint each node such that adjacent nodes are assigned different colors (Méndez-Díaz*et al.*, 2008).The wavelength assignment problem (WA) can be considered as the graph coloring problem by transforming each connection into a node where two nodes are connected if they share a common link. Thus, the constrain that each wavelength used by only one connection is equivalent to the graph coloring constrain (Li *et al.*, 2000).Unlike the GCP, SA introduces contiguity constraints making it more complicated to directly transform GCP into SA.

Maximal Independent Set (MIS):

An independent set (IS) is a subset of graph **G** with no edge between the vertices i.e., the subgraph induced by these vertices is edgeless. An independent set is a maximal independent set if no other vertex can be added to the set (Busygin *et al.*, 2002).Figure 1 illustrates the concept of MIS. MIS can also be used to solve the graph coloring problem since all vertices in one MIS are assigned the same color.

Applying MIS to solve offline SA:

The optimal assignment of spectrums is an NP-hard problem. We must rely on a heuristic algorithm to obtain an approximate solution for this issue within a reasonable timeframe. The proposed algorithm inherits the advantages offered by the MIS model. The main advantage of MIS stems from the observation that the slots assigned to one MIS will satisfy the three aforementioned constraints (the spectrum contiguity constraint, the spectrum continuity constraint, and distinct spectrum constraint). In the algorithm's standard form, it consists of an arbitrary number of independent sets comprised of dis-joint lightpaths; one lightpath may be a vertex in many sets. For any two MISs (m,$\overline{m}$), they share at least one common edge in both sets. To achieve the goal of minimizing the index of the highest slot used in any link, we propose a new algorithm MIS-SA to minimize the required number of slots on each fiber link.

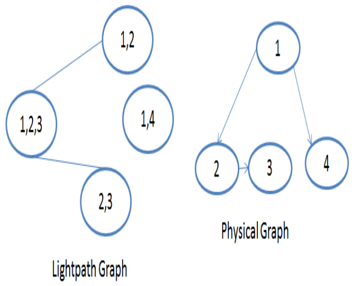

Fig. 2: The lightpath graph & the physical graph.

Proposed Algorithm:

The first step is to construct the undirected lightpath graph $\bar{G}$=(V, E) with V corresponding to the set of all lightpaths. There is an edge between two vertices in $\bar{G}$ if they share at least one common link. Figure 2 illustrates the relation between the physical graph and the lightpath graph, in which the physical graph G is converted into the lightpath graph$\bar{G}$. The main idea of the proposed algorithm is to select the paths that fall in only one MIS. The total number of used slots is computed by summing up the number of slots allocated per MIS plus the GC slots that separate two MISs. The algorithm MIS-SA is stated in Algorithms 1 and we utilize the notation from Table 1 for convenience.

Given the need to satisfy the all non-zero demands, the first step of the algorithm is to remove zero demands from the traffic demand matrix and from all MISs constructed by these zero demands (Procedure 1). Then we search for a lightpath l that is part of only one MISm. According to the definition of MIS, a lightpath $l \not\subset \overline{m} \rightarrow l$ shares at least one link with $\overline{m}$. Thus, the only way to allocate the lightpath l is by assigning a sufficient number of slots to MIS m to support traffic demand l.

After a lightpath l is assigned to a particular MIS m, removing the lightpath from the set of MIS will simplify the problem without affecting the solution. Repeatedly we look for a lightpath l, where l size-request is less than or equal to the number of slots assigned to a MIS m, where $l \subset m$ (Procedure 4).

Whenever a vertex (lightpath) has been assigned and removed from the set of MISs, the set of MISs must be updated. In this way, some of the MISs will be transformed into an IS (Procedure 3). An example is shown in Figure 3(A)—the assigned lightpath (2-3-1) from MIS b is shown as shaded black lines. Removing this lightpath will result in converting MIS b to an independent set as shown in Figure 3(B).

Once the above processes cannot be executed, and a lightpath still needs to be allocated, the procedure skips to a deadlock phase. The assignment in this phase may lead to insufficient utilization of spectrums but must be executed to break this deadlock (Procedure 5).

Deadlock Phase:

Whenever l falls in two MISs $m1, m2$ where ($m1.v$ or $m2.v > 0$). If the demand required by l less than the sum number of slots assigned to $m1, m2$ orderthe MISs to be contiguous. Two constraints have to be considered to make two MISs contiguous:

- Each MIS is contiguous by less than three MISs.
- Two MISs (the first and the last) are contiguous to only one MIS.

Otherwise, we check for any lightpath $\subset m$, where m.v >0, and change the value of m to be equal to l size-request. Else, we pick a random lightpath l and a random MIS m where $l \subset m$ and assign the MIS m the number of slots equal to l size-request.

Table 1: Notations For MIS-SA.

L(l)	lightpath l request-size.
LU(l)	one of MIS that don't satisfy lightpath l
m.v	number of slots was assigned to MIS m
countNZ	number of MISs that having a non-zero value
sumMIS	sum of numbers of slots assigned to all MISs

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Algorithm 1: Maximal Independent Set For Spectrum Assignment (MIS-SA)

Input:
1. A list of MISs M, each MIS m having a value of 0 represents the number of slots assigned to it.
2. A list of MISs C, each MIS c having a value of 0 represents the numberof MIS that is contiguous with c MIS.
3. A list of lightpaths L, each lightpath l having a value d represents the required demand in terms of number of slots.
4. A list of lightpaths LU each lightpath lu initializing with an empty list representing the MISs that didn't satisfy lu demand.
5. A list of lightpath LP each lightpath lp having a value p represents the number of MISs that lp belongs to them.
6. A list of lightpaths LM, each lightpath Lmhaving a list of MISs represents the MISs that L is part of them.
7. GC: An integer value represents the size of a guard-carrier in terms of the number of slots.

Output

Mapping each lightpath to one MIS or more MIS where each MIS is assigned a set of slots.

```
Begin ()
    removeZeroDemand()
    allocateNonZeroDemands()
    the number of slots = sumMIS + ((countNZ − 1) ∗ GC)
```

Procedure 1 **Find and remove any zero demand**

```
Procedure removeZeroDemand ()
while there exists lightpath l where L(l) == 0 do
    L.remove(l)
    Remove (l)   // remove l from all MISs
    FindIS(LM(l))
```

Procedure 2 Allocate NonZero Demands

```
Procedure allocateNonZeroDemands ()
while there lightpath l where L(l) != 0 do
    if there are lightpath l fall on only one MIS m
    m.v =L(l)
        Remove (l)   // remove l from all MISs
        FindIS(LM(l))
        LookingForLightpaths(m)
    else
    BreakingDeadlock()
```

Procedure 3:Find and remove any independent set

```
Procedure FindIS(listMIS)
    for each MIS m in listMIS
        If m⊆ (m̄) in M
    remove(m) //remove m from the set of MISs
```

Procedure 4: Looking for lightpaths

```
Procedure LookingForLightpaths(m)
    for all the remaining lightpath r in MIS m
    if L(r) ≤ m.v then
            Remove (l)   // remove l from all MISs
            FindIS(LM(r))
    else
        LU.add(r, m)
```

Procedure 5 Breaking the deadlock

```
Procedure BreakingDeadlock ()
if there l satisfied by the sum of two MIS m1, m2
    if (C(m1) ,and C(M2) < 2 )
        C(m1)++
        C(m2)++
        LU.remove(l)
        Remove (l)   // remove l from all MISs
        FindIS(LM(l))
    else if there are a lightpath l where LU(l) ! = ∅
        m.v =L(l)
        LU.remove(l)
        FindIS(LM(l))
        Remove (l)   // remove l from all MISs
        LookingForLightpaths(LU(l))
    else choose l randomly that falls to random MISm
        m.v =L(l)
        FindIS(LM(l))
        Remove (l)   // remove l from all MISs
        LookingForLightpaths (m)
```

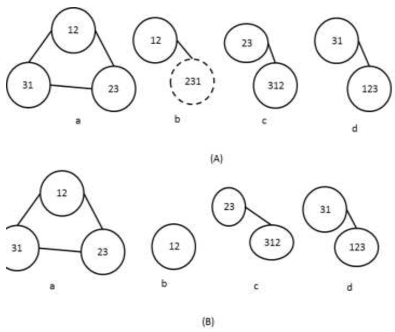

Fig. 3: MIS b before and after removing lightpath 231.

Experimental results and discussion:

In the following two subsections, we discuss two sets of experiments we conducted to investigate the performance of the MIS-SA. Results were obtained using a machine with an Intel GHz i7-processor, a Windows 7 operating system, 3.7GHz speed, and 32GB RAM. The ILP model was implemented using the ILOG CPLEX 12 while heuristic algorithms were programmed and implemented using python programming language. Unidirectional ring networks were considered to investigate the performance of the proposed algorithm.

MIS-SA vs. MRSA and ILP:

In this section, we study the performance of the MIS-SA by comparing it with a well-known heuristic algorithm, Maximum Reuse Spectrum Assignment (MRSA), and the ILP approach (Wang *et al.,* 2011). The random traffic data rates matrix is generated according to uniform distribution for each source-destination pair within the range of [0, 6] slots while the guard-band between two connections is set to be two slots. The comparison of the average spectrum utilization between the optimal and two heuristic algorithms is an important metric for the quality of any heuristic algorithm. In Figure 4, we analyzed the spectrum consumption in the two heuristic approaches. The x-axis in the model represents the number of nodes in the ring network while the y-axis is the ratio between the optimal model and the heuristic method in terms of spectrum consumption. The two ratios are computed as follows:

$$Ratio - MIS - SA = \frac{Max\ slot\ index\ \ (MIS - SA)}{Max\ slot\ index\ (Optimal)}$$

$$Ratio - MRSA\ = \frac{Max\ slot\ index\ \ (MRSA)}{Max\ slot\ index\ (Optimal)}$$

The gap between the two models increases when the network size increases from four to six in total spectrum consumption (recall that the SA problem is NP-hard for a network-size ≥ 3). The results show that our proposed algorithm achieves an approximation ratio ≅ 1.15 for number of nodes n < 7.

MIS-SA vs. MRSA:

In this section, we present the performance of MIS-SA for larger networks (> 6 nodes). MIS-SA was evaluated compared to the heuristic algorithm (MRSA) outlined in the study by (Wang *et al.*2011). We adopted two values of guard-carrier one and two slots to test guard-carrier effect on the MIS-SA performance. The traffic matrix was generated by three distributions:

1. Uniform: traffic demands may take any of the discrete values in the interval [0, 6].
2. Discrete high: traffic demands may take any of the discrete values in the interval [2, 8] with probabilities [0.05 0.05 0.1 0.1 0.15 0.15 0.4], respectively.
3. Discrete low: traffic demands may take any of the discrete values in the interval [2, 8] with probabilities [0.4 0.15 0.15 0.1 0.1 0.05 0.05], respectively.
4. In Figure 5 we plot the ratio as a function of the network-size where GC = 1 between the two methods. The ratio is computed as follows:
5. $Ratio = \frac{\max slot\ index\ (MRSA)}{\max slot\ index\ (MIS-SA)}$

Each data-point in the figure represents the average of twenty random instances of traffic matrices. We observe that the spectrum consumption of the MIS-SA, on average, is lower than that obtained by MRSA for all network sizes and for each of the three traffic distributions. However, in each case there are a few instances (<7) where our proposed method required more slots than MRSA. In these cases, the short paths required more slots than the long paths.

For different network sizes and specific distributions, the ratios between two approaches are very low for large network sizes (>=10 nodes). For these larger networks, MIS-SA executes the deadlock procedure many times (especially the last case in the deadlock procedure). Consequently, the average spectrum consumpation ratio of MIS-SA is close to the MRSA.

To investigate the impact of guard-band size, similar experiments (as mentioned above) changed guard-carrier size from one to two slots. As shown in Figure 6, the difference between the results of GC=1 and GC=2 is negligible for all cases in terms of the performance of the two methods. The average ratio performance of these two algorithms is not sensitive to the guard-carrier size because changing the guard-carrier size does not affect the steps of both algorithms.

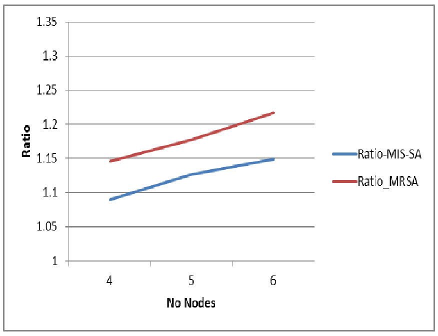

Fig. 4: SA spectrum consumption ratio vs. N.

Conclusion Remarks:

Elastic optical networks based on OOFDM technology have been deployed to utilize optical resources more efficiently. The SA problem mentioned throughout this article is extensively studied and various heuristic scheme solutions have been proposed. Since SA is NP-hard, we designed a new heuristic algorithm to gain a better solution. Our algorithm was compared to a well-known MRSA spectrum assignment algorithm used for different sizes of networks. Simulation results show the proposed method performs better compared to MRSA by lowering the highest spectrum index in all fibers.

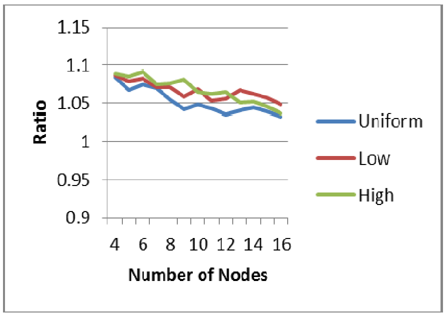

Fig. 5: Spectrum consumption ratio vs. N and Guard-band =1.

Future Work:

Our intent is to improve the performance of the proposed algorithm. We will add and modify the steps and policies in the deadlock procedure to enhance the performance of allocating the lightpaths. We will also implement the proposed method on different topology, bidirectional rings, and unidirectional and bidirectional mesh networks. In these topologies, the number of MISs is very large and greatly affects the performance of the method. We will employ the proposed method by implementing several techniques (e.g., k-shortest path) using part of the MISs without impairing overall performance**.**

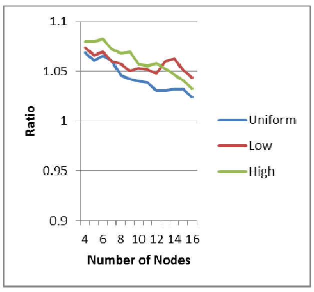

Fig. 6: Spectrum consumption vs. N and Guard-band =2.

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APPENDIX A:
An Example of Solving SA using MIS-SA:

Consider a ring network with 4 nodes. It composes of 11 MISs (Fig. 7). Assuming the random traffic demand matrix is:

Destinations / Sources	1	2	3	4
1	0	3	0	5
2	4	0	6	2
3	5	4	0	3
4	3	0	3	0

The first step is to remove zero lightpaths (1-3 and 4-2) from all MISs (see Figure 8). Next, update the set of MISs. From the figure, one can see that MISs E, F, H, I are transformed into an IS that was removed from the overall MIS set (Figure 9). In the MIS-SA algorithm, the priority is to satisfy lightpaths that fall in only one MIS1-4, 2-1, 3-2, 4-3, 3-1, and 2-4and assign a number slot over each MIS to be equal to these lightpaths' size (Figure 10). For other lightpaths that fall in more than one MIS, we have to check if they are satisfied by their already assigned MIS. For example, the value of MIS D is enough to satisfy the lightpath 4-1. In contrast, no MIS values can satisfy the lightpath 2-3. At the end of this step, the lightpaths 1-2, 3-4, and 4-1 utilize some or all of the slots provided by MISs G, C, and D, respectively (Figure 11). To simplify the problem, the already assigned lightpaths have to be removed across all MISs (Figure 3.10). Since the lightpath 2-3 has not been allocated we have to repeat the above processes and remove the MIS K from the overall set. We now reach the deadlock phase because the remaining lightpath falls in more than one MIS we must look to see if any two MISs satisfy the remaining lightpath. Since the summation value of two MISs B and G are greater than the request size of the remaining lightpath, we order these MISs to be contiguous (Figure 12). Now every lightpath is located and the maximum number of slots to satisfy all lightpaths equals the total value of the MISs A, B, C, D, G, and J (4 + 4 + 3 + 5 + 5 + 2 = 23 slots).

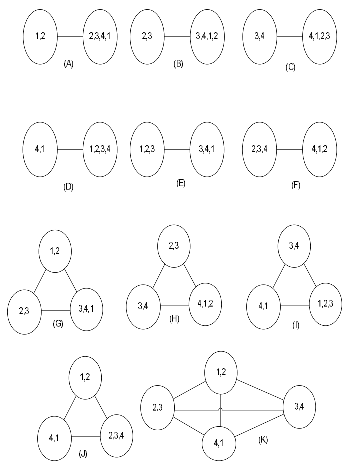

Fig. 7: The Set of all MISs.

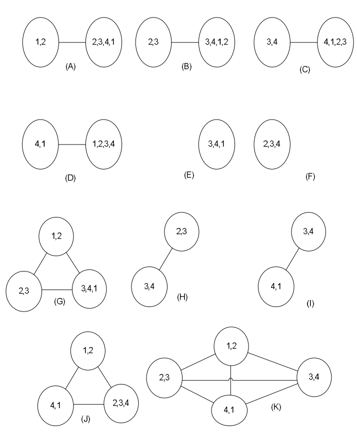

Fig. 8: The set of MISs after removing zero lightpaths.

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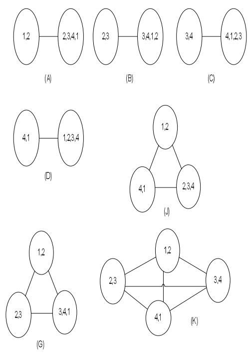

Fig. 9: The set of MISs after removing E, F, H and I MISs.

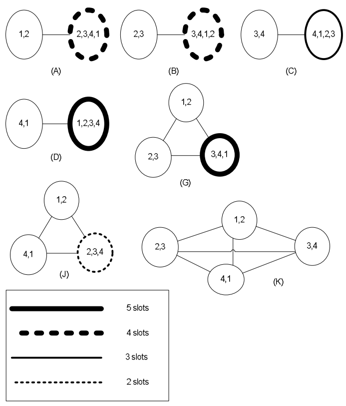

Fig. 10: Allocating the lightpaths that fall on one MIS.

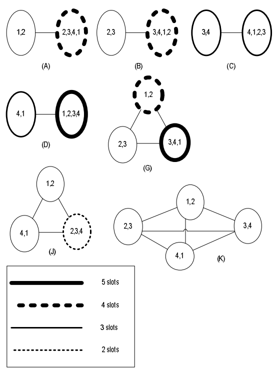

Fig. 11: Allocating the lightpaths (3-4, 4-1, 1-2).

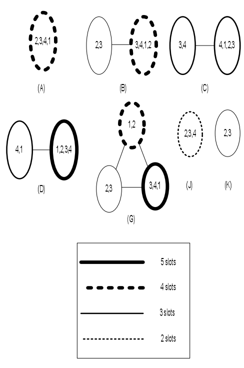

Fig. 12: The set of MISs after removing lightpaths (1-2, 3-4, and 4-1).

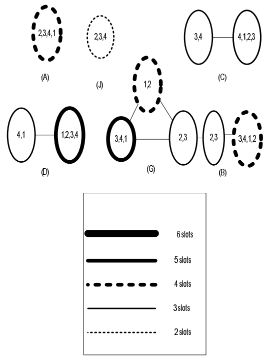

Figure 13: Making MISs G and B contiguous

REFERENCES

Alwayn V., 2004. "Optical Network Design and Implementatio", Cisco Press.

Bermond, J.C. and F.Z. Moataz, 2015. "On Spectrum Assignment in Elastic Optical Tree-Networks," Inria Sophia Antipolis; Université Nice Sophia Antipolis.

Busygin, S., S. Butenko and P.M. Pardalos, 2002. "A heuristic for the maximum independent set problem based on optimization of a quadratic over a sphere," Journal of Combinatorial Optimization, 6(3): 287–297.

Gerstel, O., M. Jinno, A. Lord and S.B. Yoo, 2012. "Elastic optical networking: A new dawn for the optical layer?" IEEE Communications Magazine, 50(2): s12–s20.

Méndez-Diaz Iand P. Zabala, 2008. "A cutting plane algorithm for graph coloring," Discrete Applied Mathematics, 156: 159–179.

Li, G. and R. Simha, 2000. "The partition coloring problem and its application to wavelength routing and assignment," in Proceedings of the First Workshop on Optical Networks.

Ramaswami, R., 2006. "Optical networking technologies: what worked and what didn't, IEEE Communications Magazine", 44(9): 132–139.

Shen, G. and M. Zukerman, 2012. "Spectrum-efficient and agile CO-OFDM optical transport networks: architecture, design, and operation," Commun. Mag. IEEE, 50(5): 82–89.

Shen, G. and Q. Yang, 2011. "From coarse grid to mini-grid to gridless: How much can gridless help contentionless?," in Optical Fiber Communication Conference, pp: 1-3.

Shirazipourazad, S., C. Zhou, Z. Derakhshandeh, Sen, 2013. "On routing and spectrum allocation in spectrum-sliced optical networks," in INFOCOM, 385–389, Italy.

Talebi, S., F. Alam, I. Katib and G.N. Rouskas, 2015. "Spectrum assignment in rings with shortest path routing: Complexity and approximation algorithms," in Computing, Networking and Communications, pp: 642 – 647, CA, USA.

Talebi, S., F. Alam, I. Katib, M. Khamis, R. Salama and G.N. Rouskas, 2014. "Spectrum management techniques for elastic optical networks: A survey," Optical Switching and Networking, 13: 34–48.

Talebi, S., E. Bampis, G. Lucarelli, I. Katib and G.N. Rouskas, 2014. "Spectrum assignment in optical networks: A multiprocessor scheduling perspective," Optical Communications and Networking, 6(8): 754–763.

Wan, P.J., K.M. Alzoubi and O. Friede, 2002. "Distributed construction of connected dominating set in wireless ad hoc networks," in INFOCOM, 3: 1597–1604.

Wang Y, W. Wang and M. Li, 2006. "Efficient distributed low-cost backbone formation for wireless networks," Parallel and Distributed Systems, 17(7): 681–693.

Wang, Y., X. Cao and Y. Pan, 2011. "A study of the routing and spectrum allocation in spectrum-sliced elastic optical path networks," in INFOCOM, pp: 1503–1511. China.

Yetginer, E., Z. Liu and G.N. Rouskas, 2011. "Fast exact ILP decompositions for ring RWA," Optical Communications and Networking, 3(7): 577–586.