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Complex of Lascoux in the Case of Partition (7,6,3)

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ABSTRACT

in this paper, the complex of Lascoux in the case of partition (7,6,3) has been studie by using diagrams, divided power of the place polarization $\partial_{21}^{(k)}$, capelli identities and the idea of mapping cone.

INTRODUCTION

Let R be a commutative ring with 1, F be free R-module and D_t be the divided power of degree t Akin, Buchsbaum and Weyman (1982). Consider the map

$\partial_{21}^{(k)} : D_{p+k} F \otimes D_{q-k} F \otimes D_r F \to D_P F \otimes D_q F \otimes D_r F,$

this map is a place polarization from place one to place two where place one and two are denoted by $D_{p+k} F$ and $D_{q-k} F$ respectively, and the map

$\partial_{32}^{(k)} : D_p F \otimes D_{q+k} F \otimes D_{r-k} F \to D_P F \otimes D_q F \otimes D_r F$

is the place polari zation from place two to place three where place two and three are denoted by $D_{q+k} F$ and $D_{r-k} F$ respectively, Buchsbaum and Rota (2001) and Buchsbum and Rota (1994).

The complex of characteristic zero is studied in Hassan (2006), Hassan (2012) and Buchsboum (1986).in the partition (2,2,2), (3,3,3) and (4,4,3) using this modified and the letter place methods. In this paper we study the complex of Lascoux in the case of partition (7,6,3) as a diagram by using the idea of the mapping cone Rotman (1979), and the map $\partial_{ij}^{(k)}$ which means the k^{th} divided power of the place polarization ∂ij where j must be less than i with it's capelli identities Buchsbaum and Rota (2001). specificly, in this work we used only the following identies

$$\partial_{21}^{(a)} \partial_{32}^{(b)} = \sum_{\propto \geq 0} (-1)^{\propto} \partial_{32}^{(b-\propto)} \partial_{21}^{(a-\propto)} \partial_{31}^{(\propto)} \qquad (1.1)$$

$$\partial_{32}^{(b)} \partial_{21}^{(a)} = \sum_{\propto \geq 0} \partial_{21}^{(a-\propto)} \partial_{32}^{(b-\propto)} \partial_{31}^{(\propto)} \qquad (1.2)$$

$$\partial_{32}^{(1)} o\, \partial_{31}^{(1)} = \partial_{31}^{(1)} o\, \partial_{32}^{(1)} \qquad (1.3)$$

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$$\partial_{21}^{(1)} o\, \partial_{31}^{(1)} = \partial_{31} o\, \partial_{21}^{(1)} \tag{1.4}$$

Where ∂_{ij} is the place polarization form place j to place i,

II. The Terms Of Lascoux Complex In The Case Of Partition (7, 6, 3):

The terms of the Lascaux complex are obtained from the determinantal expansion of the Jocobi-Trudi matrix of the partition. The positions of the terms of the complex are determined by the length of the permutation to which they correspond Akin, Buchsbaum and Weyman (1982) and Buchsboum (1986). Now in the case of the partition $\lambda = (7,6,3)$, we have the following matrix:

$$\begin{bmatrix} D_7 F & D_5 F & D_1 F \\ D_8 F & D_6 F & D_2 F \\ D_9 F & D_7 F & D_3 F \end{bmatrix}$$

Then the Lascoux complex has the correspondence between it's terms as follows:

$D_7 F \otimes D_6 F \otimes D_3 F \leftrightarrow identity$

$D_8 F \otimes D_5 F \otimes D_3 F \leftrightarrow (12)$

$D_7 F \otimes D_7 F \otimes D_2 F \leftrightarrow (23)$

$D_9 F \otimes D_5 F \otimes D_2 F \leftrightarrow (123)$

$D_8 F \otimes D_7 F \otimes D_1 F \leftrightarrow (132)$

$D_9 F \otimes D_6 F \otimes D_1 F \leftrightarrow (13)$

So, the complex of Lascoux in the case of the partition $\lambda = (7,6,3)$ has the form:

$$D_9 F \otimes D_6 F \otimes D_1 F \to \begin{array}{c} D_8 F \otimes D_7 F \otimes D_1 F \\ \oplus \\ D_9 F \otimes D_5 F \otimes D_2 F \end{array} \to \begin{array}{c} D_7 F \otimes D_7 F \otimes D_2 F \\ \oplus \\ D_8 F \otimes D_5 F \otimes D_3 F \end{array} \to D_7 F \otimes D_6 F \otimes D_3 F$$

III. The Complex of Lascoux As A Diagram:

Consider the following diagram :

$$\begin{array}{ccccc} D_9 F \otimes D_6 F \otimes D_1 F & \xrightarrow{f_1} & D_8 F \otimes D_7 F \otimes D_1 F & \xrightarrow{f_2} & D_7 F \otimes D_7 F \otimes D_2 F \\ s_1 \downarrow & \mathrm{M} & s_2 \downarrow & \mathrm{N} & s_3 \downarrow \\ D_9 F \otimes D_5 F \otimes D_2 F & \xrightarrow{g_1} & D_8 F \otimes D_5 F \otimes D_3 F & \xrightarrow{g_2} & D_7 F \otimes D_6 F \otimes D_3 F \end{array}$$

So, if we define

$f_1 : D_9 F \otimes D_6 F \otimes D_1 F \to D_8 F \otimes D_7 F \otimes D_1 F$ as $f_1(v) = \partial_{21}(v)$; where $v \in D_9 F \otimes D_6 F \otimes D_1 F$

$s_1 : D_9 F \otimes D_6 F \otimes D_1 F \to D_9 F \otimes D_5 F \otimes D_2 F$ as $s_1(v) = \partial_{32}(v)$; where $v \in D_9 F \otimes D_6 F \otimes D_1 F$

and

$s_2 : D_8 F \otimes D_7 F \otimes D_1 F \to D_8 F \otimes D_5 F \otimes D_3 F$ as $s_2(v) = \partial_{32}^{(2)}(v)$; where $v \in D_8 F \otimes D_7 F \otimes D_1 F$.

Now, we have to define the map

$g_1 : D_9 F \otimes D_5 F \otimes D_2 F \to D_8 F \otimes D_5 F \otimes D_3 F$

which makes the diagram M commutative i.e.

$g_1 \circ s_1 = s_2 \circ f_1$

which implies that

$g_1 \circ \partial_{32} = \partial_{32}^{(2)} \circ \partial_{21}$

Now if we use Capelli identities (1.2) we get :

$$\begin{aligned} \partial_{32}^{(2)} \circ \partial_{21} &= \partial_{21}^{(1)} \circ \partial_{32}^{(2)} + \partial_{31}^{(1)} \circ \partial_{32}^{(1)} \\ &= (\partial_{21}^{(1)} \circ \tfrac{1}{2} \partial_{32}^{(1)} + \partial_{31}^{(1)}) \circ \partial_{32}^{(1)} \end{aligned}$$

Thus, $g_1 = \frac{1}{2} \partial_{21}^{(1)} \circ \partial_{32}^{(1)} + \partial_{31}^{(1)}$.

On the other hand, if we define

$g_2 : D_8 F \otimes D_5 F \otimes D_3 F \to D_7 F \otimes D_6 F \otimes D_3 F$

as $q_2(v) = \partial_{21}(v)$; where $v \in D_8 F \otimes D_5 F \otimes D_3 F$

and $s_3 : D_7 F \otimes D_7 F \otimes D_2 F \to D_7 F \otimes D_6 F \otimes D_3 F$ as $s_3(v) = \partial_{32}(v)$; where $v \in D_7 F \otimes D_7 F \otimes D_2 F$.

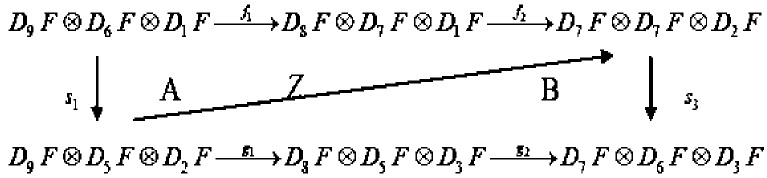
Now we need to define f_2 to make the diagram N commute:

$f_2 : D_8 F \otimes D_7 F \otimes D_1 F \to D_7 F \otimes D_7 F \otimes D_2 F$ Such that $s_3 \circ f_2 = g_2 \circ s_2$ i.e. $\partial_{32} \circ f_2 = \partial_{21} \circ \partial_{32}^{(2)}$
again by using Capelli identities (1.1) we get

$$\partial_{21}^{(1)} \circ \partial_{32}^{(2)} = \partial_{32}^{(2)} \circ \partial_{21}^{(1)} - \partial_{32}^{(1)} \circ \partial_{31}^{(1)}$$
$$= \partial_{32} \circ (\tfrac{1}{2} \partial_{32}^{(1)} - \partial_{21}^{(1)} \circ \partial_{31}^{(1)})$$

then $f_2 = \frac{1}{2} \partial_{32}^{(1)} \circ \partial_{21}^{(1)} - \partial_{31}^{(1)}$.

Now consider the following diagram:



Define $Z : D_9 F \otimes D_5 F \otimes D_2 F \to D_7 F \otimes D_7 F \otimes D_2 F$ by $Z(v) = \partial_{21}^{(2)}(v)$; where $v \in D_9 F \otimes D_5 F \otimes D_2 F$

Proposition 3. 1.:

The diagram A is commutative.

Proof:

To prove A is commutative, we need to prove $f_2 \circ f_1 = z \circ s_1$

$$f_2 \circ f_1 = \left(\tfrac{1}{2} \partial_{32}^{(1)} \circ \partial_{21}^{(1)} - \partial_{31}^{(1)}\right) \circ \partial_{21}^{(1)}$$
$$= \partial_{32}^{(1)} \circ \partial_{21}^{(2)} - \partial_{31}^{(1)} \circ \partial_{21}^{(1)}$$
$$= \partial_{21}^{(2)} \circ \partial_{32}^{(1)} + \partial_{21}^{(1)} \circ \partial_{31}^{(1)} - \partial_{31}^{(1)} \circ \partial_{21}^{(1)}$$
$$= \partial_{21}^{(2)} \circ \partial_{32}^{(1)}$$
$$= z \circ s_1 .$$

□

ProPosition 3. 2.:

The diagram B is commutative

Proof:

To prove B is commutative, we need to prove $g_2 \circ g_1 = s_3 \circ z$

$$g_2 \circ g_1 = \partial_{21}^{(1)} \circ \left(\tfrac{1}{2} \partial_{21}^{(1)} \circ \partial_{32}^{(1)} + \partial_{31}^{(1)}\right)$$
$$= \partial_{21}^{(2)} \circ \partial_{32}^{(1)} + \partial_{21}^{(1)} \circ \partial_{31}^{(1)}$$
$$= \partial_{32}^{(1)} \circ \partial_{21}^{(2)} - \partial_{21}^{(1)} \circ \partial_{31}^{(1)} + \partial_{21}^{(1)} \circ \partial_{31}^{(1)}$$
$$= \partial_{32}^{(1)} \circ \partial_{21}^{(2)}$$
$$= s_3 \circ z .$$

Finally, we can define the maps σ_1, σ_2 and σ_3 where:

$$\sigma_3 : D_9 F \otimes D_6 F \otimes D_1 F \longrightarrow \begin{matrix} D_8 F \otimes D_7 F \otimes D_1 F \\ \oplus \\ D_9 F \otimes D_5 F \otimes D_2 F \end{matrix}$$

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$$\sigma_2 : \begin{array}{c} D_8 F \otimes D_7 F \otimes D_1 F \\ \oplus \\ D_9 F \otimes D_5 F \otimes D_2 F \end{array} \longrightarrow \begin{array}{c} D_7 F \otimes D_7 F \otimes D_2 F \\ \oplus \\ D_8 F \otimes D_5 F \otimes D_3 F \end{array}$$

and

$$\sigma_1 : \begin{array}{c} D_7 F \otimes D_7 F \otimes D_2 F \\ \oplus \\ D_8 F \otimes D_5 F \otimes D_3 F \end{array} \longrightarrow D_7 F \otimes D_6 F \otimes D_3 F$$

by

- $\sigma_3(x) = (f_1(x), s_1(x)); \quad \forall x \in D_9 F \otimes D_6 F \otimes D_1 F$
- $\sigma_2((x_1, x_2)) = (f_2(x_1) - z(x_2),\ s_1(x_2) - s_2(x_1)); \quad \forall (x_1, x_2) \in \begin{array}{c} D_8 F \otimes D_7 F \otimes D_1 F \\ \oplus \\ D_9 F \otimes D_5 F \otimes D_2 F \end{array}$
- $\sigma_1((x_1, x_2)) = (s_3(x_1) + g_2(x_2));$

$$\forall (x_1, x_2) \in \begin{array}{c} D_7 F \otimes D_7 F \otimes D_2 F \\ \oplus \\ D_8 F \otimes D_5 F \otimes D_3 F \end{array}$$

Proposition 3. 3:

$$0 \to D_9 F \otimes D_6 F \otimes D_1 F \xrightarrow{\sigma_3} \begin{array}{c} D_8 F \otimes D_7 F \otimes D_1 F \\ \oplus \\ D_9 F \otimes D_5 F \otimes D_2 F \end{array} \xrightarrow{\sigma_2}$$

$$\xrightarrow{\sigma_2} \begin{array}{c} D_7 F \otimes D_7 F \otimes D_2 F \\ \oplus \\ D_8 F \otimes D_5 F \otimes D_3 F \end{array} \xrightarrow{\sigma_1} D_7 F \otimes D_6 F \otimes D_3 F$$

is complex.

Proof:

From the definition we know that $\partial_{32}^{(1)}$ and $\partial_{21}^{(1)}$ are injective (see Buchsbaum and Rota (2001)), then we get σ_3 is injective.

Now

$$\begin{aligned} \sigma_2 \circ \sigma_3(x) &= \sigma_2(f_1(x), s_1(x)) \\ &= \sigma_2(\partial_{21}(x), \partial_{32}(x)) \\ &= (f_2(\partial_{21}(x) - z(\partial_{32}(x)), g_1(\partial_{32}(x)) - s_2(\partial_{21}(x)))\cdot \end{aligned}$$

Now

$$\begin{aligned} f_2(\partial_{21}(x)) - z(\partial_{32}(x)) &= (\tfrac{1}{2}\partial_{32}^{(1)} \circ \partial_{21}^{(1)} - \partial_{31}^{(1)}) \circ \partial_{21}(x) - \partial_{21}^{(2)} \circ \partial_{32}(x) \\ &= (\partial_{32}^{(1)} \circ \partial_{21}^{(2)} - \partial_{31}^{(1)} \circ \partial_{21} - \partial_{21}^{(2)} \circ \partial_{32})(x) \\ &= (\partial_{32}^{(1)} \circ \partial_{21}^{(2)} - \partial_{21}^{(1)} \circ \partial_{31}^{(1)} - \partial_{32}^{(1)} \circ \partial_{21}^{(2)} + \partial_{21} \circ \partial_{31})(x) \\ &= 0\cdot \end{aligned}$$

$$\begin{aligned} s_1(\partial_{32}(x)) - g_2(\partial_{21}(x)) &= (\partial_{21}^{(1)} \circ \tfrac{1}{2}\partial_{32}^{(1)} + \partial_{31}^{(1)}) \circ \partial_{31}(x) - \partial_{32}^{(2)} \circ \partial_{21}(x) \\ &= (\partial_{21}^{(1)} \circ \partial_{32}^{(2)} + \partial_{32}^{(1)} \circ \partial_{31} - \partial_{32}^{(2)} \circ \partial_{21})(x) \\ &= (\partial_{32}^{(2)} \circ \partial_{21}^{(1)} - \partial_{32} \circ \partial_{31} + \partial_{32}^{(1)} \circ \partial_{31} - \partial_{32}^{(2)} \circ \partial_{21})(x) \\ &= 0\cdot \end{aligned}$$

so we get $(\sigma_2 \circ \sigma_3)(x) = 0\cdot$

and

$$(\sigma_1 \circ \sigma_2)(x_1, x_2) = \sigma_1(f_2(x_1) - z(x_2)),\ g_2(x_2) - s_2(x_1)$$
$$= \sigma_1((\tfrac{1}{2}\partial_{32}^{(1)} \circ \partial_{21}^{(1)} - \partial_{31}^{(1)})(x_1) - \partial_{21}^{(2)}(x_2)\ ,\ (\tfrac{1}{2}\partial_{21}^{(1)} \circ \partial_{32}^{(1)} + \partial_{31}^{(1)})(x_2) - \partial_{32}^{(2)}(x_1))$$
$$= \partial_{32}^{(1)}(\tfrac{1}{2}\partial_{32}^{(1)} \circ \partial_{21}^{(1)} - \partial_{31}^{(1)})(x_1) - \partial_{32}^{(1)} \circ \partial_{21}^{(2)}(x_2) \qquad \mathit{from}\ (1.1)\mathit{and}\ (1.2)\ \mathit{then\ we\ get}$$
$$+ \partial_{21}^{(1)}(\partial_{21}^{(1)} \circ \tfrac{1}{2}\partial_{32}^{(1)} + \partial_{31}^{(1)})(x_2) - \partial_{21}^{(1)} \circ \partial_{32}^{(2)}(x_1)$$
$$= (\partial_{32}^{(2)} \circ \partial_{21}^{(1)} - \partial_{32}^{(1)} \circ \partial_{31}^{(1)} - \partial_{21}^{(1)} \circ \partial_{32}^{(2)})(x_1)$$
$$+ (\partial_{21}^{(2)} \circ \partial_{32}^{(1)} + \partial_{21}^{(1)} \circ \partial_{31}^{(1)} - \partial_{32}^{(1)} \circ \partial_{21}^{(2)})(x_2)\cdot$$
$$(\sigma_1, \sigma_2)(x_1, x_2) = (\partial_{21}^{(1)} \circ \partial_{32}^{(2)} + \partial_{32}^{(1)} \circ \partial_{31}^{(1)} - \partial_{32}^{(1)} \circ \partial_{31}^{(1)} - \partial_{21}^{(1)} \circ \partial_{32}^{(2)})(x_1)$$
$$+ (\partial_{32}^{(1)} \circ \partial_{21}^{(2)} - \partial_{21}^{(1)} \circ \partial_{31}^{(1)} + \partial_{21}^{(1)} \circ \partial_{31}^{(1)} - \partial_{32}^{(1)} \circ \partial_{21}^{(2)})(x_2)$$
$$= 0\cdot$$

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