Australian Journal of Basic and Applied Sciences, 10(11) Special 2016, Pages: 126-132

AUSTRALIAN JOURNAL OF BASIC AND APPLIED SCIENCES

ISSN:1991-8178 EISSN: 2309-8414
Journal home page: www.ajbasweb.com

Effect of Educational Campaign on the Mathematical Model of Chickenpox

Jittiporn Preecha, Surapol Naowarat and Surin Sammana

Department of Mathematics, Faculty of Science and Technology, Suratthani Rajabhat University,Surat Thani, 84100, Thailand.

Address For Correspondence:
Surapol Naowarat, Department of Mathematics, Faculty of Science and Technology, Suratthani Rajabhat University,Surat Thani, 84100, Thailand.

ARTICLE INFO

Article history:
Received 3 March 2016
Accepted 2 May 2016
published 26 May 2016

Keywords:
Mathematical model, Chickenpox, Educational campaign, Stability, Equilibrium points, Basic reproductive number.

ABSTRACT

The objectives of this study were to develop and analyze the effect of educational campaign on the dynamics of chickenpox. Mathematical model is analyzed by using standard method, the equilibrium points and the stability of the equilibrium points are determined. The research results, we obtained the mathematical model of chickenpox which represented by a system of four nonlinear differential equations. The human population is divided into four compartments, susceptible, exposed, infectious and recovered. By numerical analysis we obtained the disease free equilibrium point which the basic reproductive number is 0.642857, 0.428571 and 0.214286 with the value of educational campaign at 0.25, 0.50 and 0.75, respectively. For the endemic equilibrium point, the basic reproductive number is 5.7551, 5.32653 and 4.04081 with the value educational campaign at 0.25, 0.50 and 0.75, respectively. It can conclude that if we increase the effectiveness of educational campaign then the number of chickenpox case will decrease.

INTRODUCTION

Chickenpox is a common disease in both children and adults. Symptoms of infection Vallicella, which can enter the body by touching the blister water. Chickenpox spreads in the air through coughing or sneezing. Chickenpox most commonly causes an illness that lasts about 5-10 days (CDC,2015).

The situation for the spread of chickenpox in Thailand from January to June in 2014 are reported the number of patients with chickenpox 63,510 patients or representing a rate of 99.12 cases per hundred thousand population. Considering the trend of chickenpox in the past 10 years find the number of patients increasing.

From evaluation to information and public conduct in 2011. It is evident that the public has very little knowledge of chickenpox and the trend of increasing outbreaks of chickenpox. Due to the proposed media campaign to educate the media exposure and awareness of the human in Thailand are low. For this reason, we proposed the model of chickenpox with effect of educational campaign.

Mathematical studies enable us to understand the spread of chickenpox and the results of the study. It can also help us understand the factors to control the epidemic. The development of the model can be adjusted according to the specific characteristics of the epidemic. This study is based on Baffoe (2013) by adding a parameter that is educational campaign to control the chickenpox . We have also used the model of Edward (2014) to develop the model. Halloran et al. (1994) proposed an age-structured varicella model with vaccination, but without zoster, to study the incidence, age distribution of varicella cases, and the sensitivity to estimated parameter values. Macartney et al. (2005) proposed the model of varicella. The research results showed that the vaccine is safe and highly effective in reducing varicella-related disease with universal varicella vaccination of

To Cite This Article: Jittiporn Preecha, Surapol Naowarat and Surin Sammana., Effect of Educational Campaign on the Mathematical Model of Chickenpox. ***Aust. J. Basic & Appl. Sci.,*** *10(11): 126-132, 2016*

children in the USA over the past 10 years. Edmunds and Brisson (2002) brought to bear that infant VZV vaccination could lead to an increase in adult disease or a temporary increase in the incidence of shingles. Studies indicate more effective vaccination is at preventing varicella. Understanding the mechanisms for maintaining immunity against varicella is critical for predicting the long-term effects of vaccination. Garnett and Greenfell (1992) examined the possible influence of zoster on the transmission dynamics of varicella, but did not investigate the impact of vaccination on the incidence of zoster (their model assumes a constant background force of infection of zoster which remains unchanged through time). The objectives of this study were to develop and analyze a mathematical model of chickenpox with the effect educational campaign, to achieve a better understanding of the spread of chickenpox. It can reduce the spread of chickenpox and to guide surveillance and control chickenpox.

2. Model Formulation:

Our SEIR model was developed from the chickenpox model given in Baffoe (2013) and adding educational campaign. The human population is divided into four sub-classes, that is the susceptible human(S), exposed human(E), infected human(I), and recovered human (R). We denote the total numbers of human by N(t) and denote sub-classes of human by S(t), E(t), I(t), and R(t), respectively, so that

$S(t)+E(t)+I(t)+R(t)=N(t)$.

The transmission dynamics associated with these sub-classes are illustrated in **Fig.1**

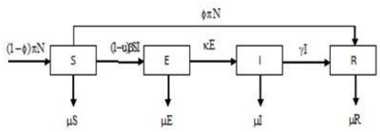

Fig. 1: Flow chart of chickenpox model.

The dynamics of flow chart are described by the following ordinary differential equations:

$$\frac{dS}{dt} = (1-\phi)\pi N - (1-u)\beta SI - \mu S \quad (1)$$

$$\frac{dE}{dt} = (1-u)\beta SI - (\kappa+\mu)E \quad (2)$$

$$\frac{dI}{dt} = \kappa E - (\gamma+\mu)I \quad (3)$$

$$\frac{dR}{dt} = \phi\pi N + \gamma I - \mu R \quad (4)$$

Where

S	is the susceptible human
E	is the exposed human
I	is the infected human
R	is the recovered human
π	is the birth rate of human
μ	is the death rate of human
β	is the exposure rate of infection
κ	is the incubation of chickenpox
γ	is the convalescence of patients with chickenpox
ϕ	is the vaccination rate
u	is the effect of educational campaign
N	is the total number of human

3. *Analysis of the Model:*

Equilibrium Points:

By using the standard method for analyzing our model, this system has two equilibrium points; disease free and endemic equilibrium points. We obtained these by setting the right hand side of equation (1)-(4) to zero. Doing this, we obtained.

Disease Free Equilibrium Point (E_0):

In the absence of the disease in the community, that is $E = I = 0$, we obtained,

$$E_0(S,E,I,R) = E_0\left(\frac{(1-\phi)\pi N}{\mu},0,0,\frac{\phi\pi N}{\mu}\right)$$

Endemic Equilibrium Point (E_1):

In case the disease is presented in the community, $I \neq 0$, we obtained, $E_1(S^*,E^*,I^*,R^*)$

$$S^* = \frac{(1-\phi)\pi N}{(1-u)\beta I^* + \mu},$$

$$E^* = \frac{(1-u)\beta(1-\phi)\pi N I^*}{(\kappa+\mu)((1-u)\beta I^* + \mu)},$$

$$I^* = \frac{\kappa(1-\mu)\beta(1-\phi)\pi N - (\gamma+\mu)(\kappa+\mu)\mu}{(\gamma+\mu)(\kappa-\mu)(1-u)\beta},$$

$$R^* = \frac{\phi\pi N + \gamma I^*}{\mu}.$$

Local Asymptotically Stability:

The local stability of each equilibrium point is determined from the Jacobian matrix of the system of equations (1)-(4) by considering the signs of real parts of all eigenvalues. The eigenvalues (λ) are the solutions of the characteristic equation $\det(J - \lambda I) = 0$. Where J is the Jacobian matrix at equilibrium point, I is the identity matrix dimension 4×4. Equilibrium point is locally stability, when all eigenvalues are negative real part.

The Jacobian matrix at E_0, we get

$$J_0 = \begin{bmatrix} -\mu & 0 & \frac{-\beta(1-u)(1-\phi)\pi N}{\mu} & 0 \\ 0 & -\kappa-\mu & \frac{\beta(1-u)(1-\phi)\pi N}{\mu} & 0 \\ 0 & \kappa & -\gamma-\mu & 0 \\ 0 & 0 & \gamma & -\mu \end{bmatrix}$$

The eigenvalues of the J_0 are obtained by solving $\det(J_0 - \lambda I) = 0$. From Eqn. (1)-(4), we obtain the characteristic equation,

$$(-\mu-\lambda)(-\mu-\lambda)\left(\lambda^2 + A\lambda + B\right) = 0$$

Where

$$A = \kappa + 2\mu + \gamma,$$

$$B = (\gamma+\kappa)\mu + \kappa\gamma + \mu^2 - \frac{\kappa(1-u)\beta(1-\phi)\pi N}{\mu}.$$

From the characteristic equation, we see that two eigenvalues are $\lambda_{1,2} = -\mu < 0$. The other two are the solutions of the characteristic equation. The roots of this equation will be negative if two coefficients satisfied with the Routh-Hurwitz criteria (Allen,2006).

1) $A > 0$, 2) $B > 0$.

Disease Endemic Equilibrium Point:

To determine the stability of the endemic equilibrium point. We examine the eigenvalues of Jacobian matrix at E_1, which is

$$J_1 = \begin{bmatrix} -(1-u)\beta I^* - \mu & 0 & -(1-u)\beta S^* & 0 \\ (1-u)\beta I^* & -\kappa-\mu & (1-u)\beta S^* & 0 \\ 0 & \kappa & -\gamma-\mu & 0 \\ 0 & 0 & \gamma & -\mu \end{bmatrix}$$

The eigenvalues of the J_1 are obtained by solving $\det(J_1 - \lambda I) = 0$. We obtain the characteristic equation.

$$(-\mu-\lambda)(\lambda^3 + y_1\lambda^2 + y_2\lambda + y_3) = 0$$

Where

$y_1 = F + G + H,$
$y_2 = FG + FH + GH - \kappa L.$
$y_3 = FGH + \kappa LP - \kappa LF.$

With

$F = (1-u)\beta I^* + \mu,$
$G = \kappa + \mu,$
$H = \gamma + \mu,$
$L = (1-u)\beta\left(\frac{(1-\phi)\pi\mathbf{N}}{(1-u)\beta I^* + \mu}\right),$
$P = (1-u)\beta I^*.$

The three eigenvalues of $\lambda^3 + y_1\lambda^2 + y_2\lambda + y_3 = 0$ will have negative real part if they satisfy the Routh - Hurwitz criteria (Allen,2006), that is $y_1 y_2 > y_3$.

Basic Reproductive Number (R_0 ***):***

We obtained a basic reproductive number by using the next generation method. Rewriting the equations (1)-(4) in matrix form

$$\frac{dX}{dt} = F(X) - V(X) \qquad (5)$$

Where $F(X)$ is the non-negative matrix of new infection terms and $V(X)$ is the non – singular matrix of remaining transfer terms.

Letting

$$X = \begin{bmatrix} S \\ E \\ I \\ R \end{bmatrix},$$

$$F(x) = \begin{bmatrix} 0 \\ (1-u)\beta SI \\ 0 \\ 0 \end{bmatrix},$$

$$V(x) = \begin{bmatrix} (1-u)\beta SI - (1-\phi)\pi N + \mu S \\ \kappa E + \mu E \\ \gamma I - \kappa E + \mu I \\ \mu R - \gamma I - \phi\pi N \end{bmatrix}$$

For our model, the Jacobian matrix becomes

$$F(E_0) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\beta(1-u)(1-\phi)\pi N}{\mu} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}_{E_0}$$

and

$$V(E_0) = \begin{bmatrix} \mu & 0 & \frac{(1-u)\beta(1-\phi)\pi N}{\mu} & 0 \\ 0 & \kappa+\mu & 0 & 0 \\ 0 & -\kappa & \gamma+\mu & 0 \\ 0 & 0 & -\gamma & \mu \end{bmatrix}_{E_0}$$

The inverse of is V

$$V^{-1} = \begin{bmatrix} \frac{\mu(\kappa+\mu)(\gamma+\mu)}{\mu^2(\kappa+\mu)(\gamma+\mu)} & \frac{\kappa\beta(1-u)(1-\phi)\pi N}{\mu^2(\kappa+\mu)(\gamma+\mu)} & \frac{-(\kappa+\mu)\beta(1-u)(1-\phi)\pi N}{\mu^2(\kappa+\mu)(\gamma+\mu)} & 0 \\ 0 & \frac{\mu^2(\gamma+\mu)}{\mu^2(\kappa+\mu)(\gamma+\mu)} & 0 & 0 \\ 0 & \frac{\mu^2\kappa}{\mu^2(\kappa+\mu)(\gamma+\mu)} & \frac{\mu^2(\kappa+\mu)}{\mu^2(\kappa+\mu)(\gamma+\mu)} & 0 \\ 0 & \frac{\mu\gamma\kappa}{\mu^2(\kappa+\mu)(\gamma+\mu)} & \frac{\mu\gamma(\kappa+\mu)}{\mu^2(\kappa+\mu)(\gamma+\mu)} & \frac{\mu(\kappa+\mu)(\gamma+\mu)}{\mu^2(\kappa+\mu)(\gamma+\mu)} \end{bmatrix}$$ This leads to

$$FV^{-1} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{\kappa\beta(1-u)(1-\phi)\pi N}{\mu(\kappa+\mu)(\gamma+\mu)} & \frac{\beta(1-u)(1-\phi)\pi N}{\mu(\gamma+\mu)} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$ Thus,

$$\rho\left(FV^{-1}\right) = \max\left\{0, \frac{\kappa\beta(1-u)(1-\phi)\pi \mathbf{N}}{\mu(\kappa+\mu)(\gamma+\mu)}\right\}$$

We get

$$R_0 = \frac{\kappa\beta(1-u)(1-\phi)\pi N}{\mu(\kappa+\mu)(\gamma+\mu)}. \qquad (6)$$

4 Numerical simulation:

In this section, we considered the dynamic of SEIR model with educational campaign at disease free and endemic state. The parameter values leading to disease free state are show in **Table 1.**

Table 1: The values of parameters for disease free state.

Parameters	Description	Values	Unit
π	Birth rate of human	0.40	$year^{-1}$
μ	Death rate of human	O.40	$year^{-1}$
β	Exposure rate of infection	0.0001	$year^{-1}$
κ	Incubation of chickenpox	0.30	$year^{-1}$
γ	Convalescence of patients with chickenpox	0.60	$year^{-1}$
ϕ	Vaccination rate	0.80	$year^{-1}$
u	Effect of educational campaign	0.25, 0.5, 0.75	-
N	Total number of human	100,000	person

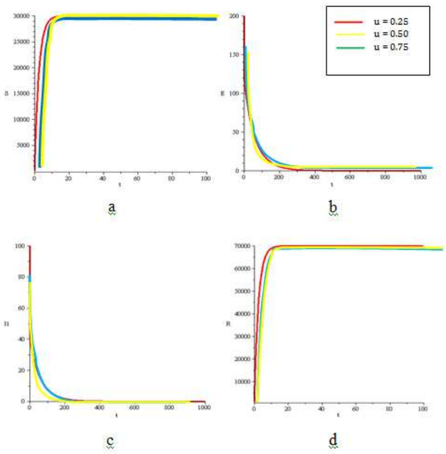

Fig. 2: (a) Time series of susceptible human (S), (b) exposed human (E), (c) infected human (I) and (d) recovered human (R). All the state variables approach the disease free state.

The values of eigenvalues and basic reproductive number as varies the values of u as shown in Table 2.

Table. 2: The values of basic reproductive number and the eigenvalues.

u = 0.25	u = 0.50	u = 0.75
R_0 = 0.642857 λ_1= -1.63899 λ_2= -0.40 λ_3= -0.40 λ_4= -0.0610133	R_0 = 0.428571 λ_1= -1.5 λ_2= -0.40 λ_3= -0.40 λ_4= -0.20	R_0 = 0.24128 λ_1= -1.3217 λ_2= -0.40 λ_3= -0.40 λ_4= -0.378301

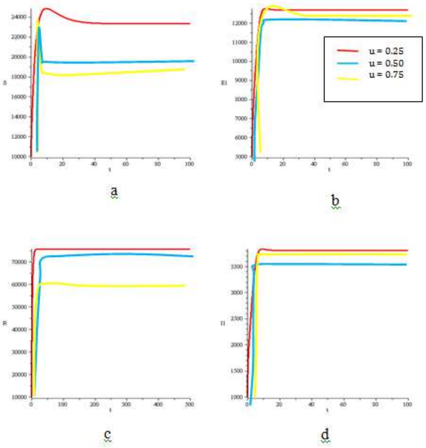

Fig. 3: (a) Time series of susceptible human (S), (b) exposed human (E), (c) infected human (I), and (d) recovered human (R). The values of all parameters used in Table.1 except $\beta = 0.0004$ and $\phi = 0.4$. All the state variables approach their endemic state.

The values of eigenvalues and basic reproductive number as varies the values of u as shown in Table. 3.

Table 3: The values of basic reproductive number and the eigenvalues.

u=0.25	u=0.5	u=0.75
R_0 = 5.7551 λ_1= -0.40 λ_2= -1.98851 λ_3= -0.503364- 0.301262i λ_4= -0.503364+ 0.301262i	R_0 = 5.32653 λ_1= -0.40 λ_2= -1.97291 λ_3= -0.477833- 0.297929i λ_4= -0.477833+ 0.297929i	R_0 = 4.04081 λ_1= -0.40 λ_2= -1.94633 λ_3= -0.391122- 0.270352i λ_4= -0.391122+ 0.270352i

5 Discussion:

In this study, the simulation results we can see that from Fig. 3, when u=0.25, u=0.50 and u=0.75 we obtain R_0 = 5.7551, 5.32653 and 4.04081, respectively. It mean that the chickenpox will occur in the community. But when the value of u=0.25, u=0.50 and u=0.75 we obtain R_0 decrease, this mean that the chickenpox will died out (Rittisak and Naowarat, 2015) as shown in Fig. 2.

Conclusion:

In this paper, we proposed the mathematical model of chickenpox with effect of educational campaign. Mathematical model consist of a system of four nonlinear differential equations. We found that there were two equilibrium points; disease free and endemic equilibrium points. The qualitative results are depended on a basic reproductive numbers. It concluded that if the effectiveness of education campaign increase, the number of infected human will be decrease.

ACKNOWEDGMENTS:

The authors would like to thank department of mathematics, faculty of science and technology, Suratthani Rajabhat University for equipment support and the anonymous reviewers for their kind helpful comments.

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