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Mathematical Model of Disease Transmission with Educational Program by Media

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ABSTRACT

In this paper, we proposed a mathematical model of disease transmission with effect of educational program by media. Mathematical model is analyzed by using standard method, the equilibrium points and the stability of each equilibrium points are determined. The findings revealed that, the mathematical model of disease transmission consisting of a system of five nonlinear differential equations. We obtained two equilibrium points: disease free equilibrium and endemic equilibrium point. If the rate constant influenced by number of infectives equal to 0.01 and the rate constant corresponding to regular media coverage equal to 0.01 then the basic reproductive number $R_0 = 0.583416 < 1$, which the disease free equilibrium point is local asymptotically stable. And the disease endemic equilibrium will occur if the rate constant influenced by number of infectives equal to 0.001, and the rate constant corresponding to regular media coverage equal to 0.001. The basic reproductive number $R_0 = 1.68171 > 1$, which the endemic equilibrium point is local asymptotically stable.

INTRODUCTION

In recent years, the role of media in controlling the transmission of epidemic is well accepted. It has great influence on the individual behaviors as well as on the construction and implementation of public health intervention and control policies (Simpson, 1987). The modern communication tools like intranet/internet driven services including media enabled services, networking sites and free access to information via websites have made the information available to the human population almost in real time. These advanced technologies have strengthened the pro-active roles of the media, and nowadays media is alert everywhere and has developed the capability to capture, monitor and report even minor incidences of interest from one part of the world to another part almost in contemporary times. Infectious diseases are considered as major barrier to the social and economic development of humankind and further to the society (WHO, 2004). The main aim of epidemiological modeling is to redvce the rate of transmission and mortality caused by the diseases. There should be strong motivation and coordination between policymakers and health-care providers to accomplish the target to prepare society to fight against a pandemic and to reduce the transmission. The target population should be given appropriate information about the risk factors and about the precautionary measures to escape from the disease (Khan, 2014).

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(Mosoongnern, 2010) investigate the information management of the public health communication in response to the 2009 Influenza outbreak by the Public Health Ministry of Thailand. Furthermore it aims to analyze the various channels of communication used to target different audiences in response to the 2009 influenza outbreak by the Public Health Ministry of Thailand. (Avery and Lariscy, 2011). the Prevention of influenza type A (H1N1) through the Department of Disease Control focused on delivering newly received information to thousands of channels and to reach the public through various modes of communication, including radio and television as well as posters, leaflets and scholarly articles. (Goodwin, Haque, Neto, and Myers, 2009) the Health Promotion and communication including media education and communication in crisis. Used in a variety of disciplines, these public health communications target many issues such as SARS, anthrax disease and Max Incidents of terrorism in relation to the case of influenza type A (H1N1). The public health agency relies greatly on crisis communication management to safeguard and defend individuals in each area of the above-mentioned issues. So as avoiding travel, sourcing, purchasing mask , including . With some groups in the infection spread in the country.

(Intarasuk, 2005). To prevent and control the spreading of disease. It is necessary to know the source of infection , to gain adequate knowledge, so that measures can be taken to prevent and control disease. Media (television,posters, brochures and newspapers) is the devices of communication of Public Health Ministry of Thailand. Aims to the educate the public through the use of media, A public relations campaign has been established, and to educate and communicate to the public the risks and dangers associated with the disease, as well as an awareness that such channels of communications can in fact reduce the widespread outbreak of disease in the future. Proposed the model, is to investigate the model of disease transmission with effect of educational program by media.

2. Model Formulation:

In our model, we assume that the human is constant. We formulate the model of disease transmission by using basic ideas taken from epidemiology.The total human population *N* is divided into four compartments: namely susceptible S(t), infective I(t),recovered R(t) aware susceptible Sm(t) and M(t) be the cumulative density of the awareness programs driven. The dynamics of the disease is depicted in the flow chart shown in **Fig. 1.**

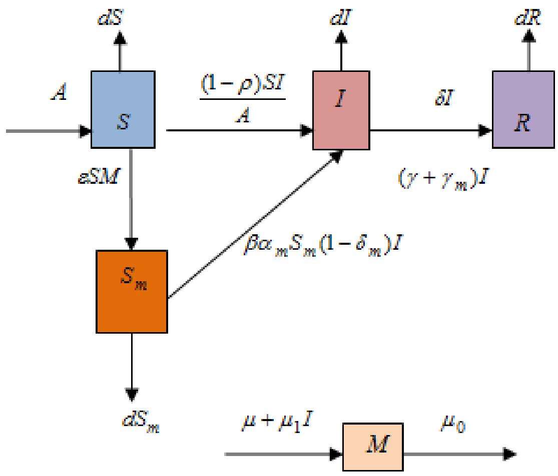

Fig. 1: Flow chart of the mathematical model of disease transmission.

The mathematical model of disease transmission with effect of educational program by media is described by the following ordinary differential equations:

$$\frac{dS}{dt} = A - \frac{(1-\rho)SI}{A} - \varepsilon SM - dS \quad (1)$$

$$\frac{dI}{dt} = \frac{(1-\rho)SI}{A} + \beta\alpha_m S_m (1-\delta_m) I - (\delta + \gamma + \gamma_m + d) I \quad (2)$$

$$\frac{dR}{dt} = (\delta + \gamma + \gamma_m) I - dR \quad (3)$$

$$\frac{dS_m}{dt} = \varepsilon SM - \beta\alpha_m S_m (1-\delta_m) I - dS_m \quad (4)$$

$$\frac{dM}{dt} = \mu + \mu_1 I - \mu_0 M \quad (5)$$

with $N = S + I + R + S_m$

where

S is the susceptible individuals,
I is the infective individuals ,
R is the recovered individuals ,
S_m is the susceptible of the awareness programs individuals ,
M is the cumulative density of the awareness programs driven,
A is the recruitment rate of human,
ε is the dissemination rate of awareness among susceptibles due to media awareness programs,
γ_m is the driven by media awareness programs,
β is the transmission rate,
α_m is the aware susceptible interacts with infective ,
d is the natural death rate,
δ is the recovery rate,
μ is the constant corresponding to regular media coverage rate,
μ_1 is the constant influenced by number of infectives rate,
μ_0 is the natural decay rate constant of media awareness programs,
δ_m is the fraction of infectives are interacting with susceptible,
ρ is the fraction of individuals that are going to be infectious,
γ is the natural recovery rate .

3. *Analysis of the Model:*

Equilibrium Points:

The system has two equilibrium points; a disease free equilibrium point and an endemic equilibrium point. We obtained these by setting the right hand sides of equations. (1) - (5) to zero. Doing this, we obtained

Disease Free Equilibrium Point: (E_0):

In the absence of the disease**, i.e.,** I = 0, R = 0 , equation (1), (4) and (5) becomes

$$\frac{dS}{dt} = A - \frac{(1-\rho)SI}{A} - \varepsilon SM - dS \ ,$$

$$\frac{dS_m}{dt} = \varepsilon SM - \beta\alpha_m S_m (1-\delta_m) I - dS_m \quad \text{and}$$

$$\frac{dM}{dt} = \mu + \mu_1 I - \mu_0 M$$

The solution of this equation is $S = \dfrac{\mu_0 A}{\mu\varepsilon + \mu_0 d}$, $S_m = \dfrac{\varepsilon A\mu}{(\mu\varepsilon + \mu_0 d)d}$ and $M = \dfrac{\mu}{\mu_0}$.

The disease free state is $E_0 = (\dfrac{\mu_0 A}{\mu\varepsilon + \mu_0 d}, 0, 0, \dfrac{\varepsilon A\mu}{(\mu\varepsilon + \mu_0 d)d}, \dfrac{\mu}{\mu_0})$

Endemic Equilibrium Point: (E_1):

In this case, the disease is presented, by setting

$I^* \neq 0, R^* \neq 0$. This gives

$$S^* = \frac{\mu_0 A^2}{(1-\rho)\mu_0 I^* + (\mu + \mu_1 I^*)\varepsilon A + \mu_0 dA},$$

$$R^* = \frac{(\delta + \gamma + \gamma_m)I^*}{d},$$

$$M^* = \frac{\mu + \mu_1 I^*}{\mu_0},$$

$$S_m{}^* = \frac{\varepsilon A^2(\mu + \mu_1 I^*)}{[(1-\rho)\mu_0 I^* + (\mu + \mu_1 I^*)\varepsilon A + \mu_0 dA][\beta\alpha_m(1-\delta_m)I^* + d]},$$

$$I^* = \frac{-Y + \sqrt{Y^2 - 4XY}}{2X}.$$

Where

$$X = (\delta + \gamma + \gamma_m + d)(1-\delta_m)\beta\alpha_m[(1-\rho)\mu_0 + \mu_1\varepsilon A],$$

$$Y = -(1-\rho)\mu_0 A\beta\alpha_m(1-\delta_m) - \varepsilon A^2\mu_1\beta\alpha_m(1-\delta_m) + (\delta + \gamma + \gamma_m + d)$$
$$[(1-\delta_m)\beta\alpha_m\mu\varepsilon A + (1-\delta_m)\beta\alpha_m\mu_0 dA + (1-\rho)\mu_0 d + \mu_1\varepsilon Ad],$$

$$Z = -(1-\rho)\mu_0 Ad - \varepsilon A^2\mu\beta\alpha_m(1-\delta_m) + Ad(\delta + \gamma + \gamma_m + d)(\mu\varepsilon + \mu d).$$

Basic Reproductive Number:

The basic reproductive number is obtained by the next generation matrix. In the notation of Van den Driessche and Watmough (2002), we start with

$$\frac{dx}{dt} = F(x) - V(x) \qquad (6)$$

where $F(x)$ is the matrix of new infectious and $V(x)$ is the matrix of the transfers between the compartments in the infective equations. We obtained

$$F(x) = \begin{bmatrix} 0 \\ \frac{(1-\rho)SI}{A} + \beta\alpha_m S_m(1-\delta_m)I \\ 0 \\ 0 \\ 0 \end{bmatrix}, \text{ and}$$

$$V(x) = \begin{bmatrix} \frac{A-(1-\rho)SI}{A} + \varepsilon SM + dS - A \\ \delta I + (\gamma + \gamma_m)I + dI \\ dR - \delta I - (\gamma + \gamma_m)I \\ \beta\alpha_m S_m(1-\delta_m)I + dS_m - \varepsilon SM \\ \mu_0 M - \mu - \mu_1 I \end{bmatrix}.$$

where $F = \left[\frac{\partial F_i(E_0)}{\partial X_i}\right]$ and $V = \left[\frac{\partial V_i(E_0)}{\partial X_i}\right]$.

for all $i, j = 1, 2, 3, 4, 5$. This are the Jacobian matrix of $F(x)$ and $V(x)$ at E_0. The basic reproductive number, R_0, is the threshold for indicating the degree of epidemiology of the disease. It can be determined by noting that

$$R_0 = \rho(FV^{-1})$$

For our model, the Jacobian matrices are

$$F=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \dfrac{(1-\rho)\mu_0 d+\beta\alpha_m\varepsilon A\mu(1-\delta_m)}{(\mu\varepsilon+\mu_0 d)d} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

and

$$V=\begin{bmatrix} \dfrac{\varepsilon\mu}{\mu_0}+d & \dfrac{(1-\rho)\mu_0}{\mu\varepsilon+\mu_0 d} & 0 & 0 & \dfrac{\varepsilon A\mu_0}{\mu\varepsilon+\mu_0 d} \\ 0 & \delta+\gamma+\gamma_m+d & 0 & 0 & 0 \\ 0 & -\delta-\gamma-\gamma_m & d & 0 & 0 \\ -\dfrac{\varepsilon\mu}{\mu_0} & \dfrac{\beta\alpha_m\varepsilon A\mu(1-\delta_m)}{(\mu\varepsilon+\mu_0 d)d} & 0 & \beta\alpha_m(1-\delta_m)+d & -\dfrac{\varepsilon A\mu_0}{\mu\varepsilon+\mu_0 d} \\ 0 & -\mu_1 & 0 & 0 & \mu_0 \end{bmatrix}$$

This leads to

$$FV^{-1}=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \dfrac{(1-\rho)\mu_0 d+\beta\alpha_m\varepsilon A\mu(1-\delta_m)}{(\delta+\gamma+\gamma_m+d)(\mu\varepsilon+\mu_0 d)d} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Thus,

$$\rho(FV^{-1}-\lambda I)=\frac{(1-\rho)\mu_0 d+\beta\alpha_m\varepsilon A\mu(1-\delta_m)}{(\delta+\gamma+\gamma_m+d)(\mu\varepsilon+\mu_0 d)d}$$

$$R_0=\frac{(1-\rho)\mu_0 d+\beta\alpha_m\varepsilon A\mu(1-\delta_m)}{(\delta+\gamma+\gamma_m+d)(\mu\varepsilon+\mu_0 d)d} \tag{7}$$

Local Asymptotically Stability:

The local stability of an equilibrium point is determined from the Jacobian matrix of the ordinary differential equation (1) – (5) evaluated at E_0 . The Jacobian matrix at E_0 is

$$J_0=\begin{bmatrix} \dfrac{-\varepsilon\mu}{\mu_0}-d & \dfrac{-(1-p)\mu_0}{\mu\varepsilon+\mu_0 d} & 0 & 0 & \dfrac{-\varepsilon\mu_0 A}{\mu\varepsilon+\mu_0 d} \\ 0 & \dfrac{(1-p)\mu_0}{\mu\varepsilon+\mu_0 d}+\dfrac{\beta\alpha_m(1-\delta_m)\varepsilon A\mu}{(\mu\varepsilon+\mu_0 d)d}-\delta-\gamma-\gamma_m-d & 0 & 0 & 0 \\ 0 & \delta-\gamma-\gamma_m & -d & 0 & 0 \\ \dfrac{\varepsilon\mu}{\mu_0} & \dfrac{-\beta\alpha_m(1-\delta_m)\varepsilon A\mu}{(\mu\varepsilon+\mu_0 d)d} & 0 & -d & \dfrac{\mu_0 A}{\mu\varepsilon+\mu_0 d} \\ 0 & \mu_1 & 0 & 0 & -\mu_0 \end{bmatrix}$$

The eigenvalues of the J_0 are obtained by solving $det(J_0-\lambda I)=0$. From this, we obtain the characteristic equation,

$$A_1\lambda^5+A_2\lambda^4+A_3\lambda^3+A_4\lambda^2+A_5\lambda+Y_6=0$$

Where

$A_1=-(2d+1)$,

$A_2=2dA+A-2d\mu_0-\mu_0-B,$

$A_3=2d\mu_0 A+\mu_0 A+AB-\mu_0 B-d^2-2dB,$

$A_4=\mu_0 AB+d^2A+2dAB-d^2\mu_0-2d\mu_0 B-d^2B,$

$A_5=d^2\mu_0 A+2d\mu_0 AB+d^2AB-d^2\mu_0 B$,

$A_6=d^2\mu AB,$

$$A=\frac{(1-\rho)\mu_0}{\mu\varepsilon+\mu_0 d}+\frac{\beta\alpha_m(1-\delta_m)\varepsilon A\mu}{(\mu\varepsilon+\mu_0 d)d}-(\delta+\gamma+\gamma_m+d),\quad B=\left(\frac{\varepsilon\mu}{\mu_0}+d\right).$$

So

$\lambda_1 = -\mu_0$, $\lambda_2 = -d$, $\lambda_3 = -d$, $\lambda_4 = -\left(\frac{\varepsilon\mu}{\mu_0} + d\right)$,

$$\lambda_5 = \frac{(1-\rho)\mu_0}{\mu\varepsilon + \mu_0 d} + \frac{\beta\alpha_m(1-\delta_m)\varepsilon A\mu}{(\mu\varphi + \mu_0 d)d} - (\delta + \gamma + \gamma_m + d).$$

From the characteristic equation, we see that five eigenvalues are $\lambda_{1,2,3,4} < 0$ and $\lambda_5 < 0$, if $\frac{(1-\rho)\mu_0}{\mu\lambda + \mu_0 d} + \frac{\beta\alpha_m(1-\delta_m)\lambda A\mu}{(\mu\lambda + \mu_0 d)d} < (\delta + \gamma + \gamma_m + d)$.

The roots of this equation will be negative if the coefficients satisfied with the Routh-Hurwitz criteria (Allen,2006).

Disease Endemic Equilibrium Point: (E_1) :

To determine the stability of the endemic equilibrium point. We examine the eigenvalues of Jacobian matrix at E_1 , which is

$$J_1 = \begin{bmatrix} -(1-\rho)I^* - \varepsilon M^* - d & \frac{-(1-\rho)S^*}{A} & 0 & 0 & -\varepsilon S^* \\ \frac{-(1-\rho)I^*}{A} & -(1-\rho)S^* + \beta\alpha_m S_m^*(1-\delta_m) - \delta - \gamma - \gamma_m - d & 0 & \beta\alpha_m(1-\delta_m)I^* & 0 \\ 0 & \delta + \gamma + \gamma_m & -d & 0 & 0 \\ \varepsilon M^* & -\beta\alpha_m S_m^*(1-\delta_m) & 0 & -\beta\alpha_m(1-\delta_m)I^* & \varepsilon S^* \\ 0 & \mu_1 & 0 & 0 & -\mu_0 \end{bmatrix}$$

Where are given by equations (6) . The characteristic equation of Jacobian matrix at E1 given by equations (1) – (5), becomes,

$$\lambda^4 + B_1\lambda^3 + B_2\lambda^2 + B_3\lambda + B_4 = 0$$

Where

$B_1 = \mu_0 - L - C - G,$

$B_2 = CG - KJ - (C+G)\mu_0 + (C+G)L - DQ,$

$B_3 = KJ(C - \mu_0) - KN\mu_1 - FKQ + CG\mu_0 - CGL$
$+ (C+G)L\mu_0 + DN\mu_1 - DQ\mu_0 + DLQ,$

$B_4 = CJK\mu_0 + FKN\mu_1 + CKN\mu_1 - FKQ\mu_0 - CGL\mu_0$
$- L\mu_0 - DLN\mu_1 + DLQ\mu_0,$

$C = -(1-\rho)I^* - \varepsilon M^* - d, D = \frac{-(1-\rho)I^*}{A},$

$F = \varepsilon M^*, G = -(1-\rho)S^* + \beta\alpha_m S_m^*(1-\delta_m) - \delta - \gamma - \gamma_m - d,$

$H = \delta + \gamma + \gamma_m, J = -\beta\alpha_m S_m^*(1-\delta_m),$

$K = \beta\alpha_m(1-\delta_m)I^*, L = -\beta\alpha_m(1-\delta_m)I^*,$

$N = \varepsilon S^*, Q = \frac{-(1-\rho)S^*}{A}.$

We obtained one eigenvalue $\lambda_1 = -d < 0$, and the other eigenvalves four the form the solutions of $\lambda^4 + Y_1\lambda^3 + Y_2\lambda^2 + Y_3\lambda + Y_4 = 0$. will have negative real part if they satisfy the Routh - Hurwitz criteria (Allen,2006), that is. $n = 4 : a_1 > o, a_3 > 0, a_4 > 0, a_1 a_2 a_3 > a_3^2 + a_1^2 a_4$.

4 Numerical Results:

The value of parameters used in the numerical simulation are given in Table 1.

Table 1: Parameter values used in numerical simulations at disease free state.

Parameter	Values	Unit
A	100	person
ε	0.01	day^{-1}
γ_m	0.01	day^{-1}
β	0.000005	-
α_m	0.02	day^{-1}
δ	0.003	day^{-1}
μ	0.01	day^{-1}
μ_1	0.01	day^{-1}
μ_0	0.000003	day^{-1}
δ_m	0.2	day^{-1}
ρ	0.4	-
γ	0.002	day^{-1}
d	0.01666	day^{-1}

Stability of the disease free state: Using the values of parameters listed in Table 1. We obtain the eigenvalues and basic reproductive number to be:

$\lambda_1 = -0.000003$, $\lambda_2 = -1.6666$, $\lambda_3 = -0.16666$, $\lambda_4 = -33.34999$, $\lambda_5 = -0.013189$

and $R_0 = 0.583416 < 1$.

Since all of the eigenvalues are negative and the basic reproductive number is less than one, then the disease free state E_0 is local asymptotically stable as show in **Fig. 2**

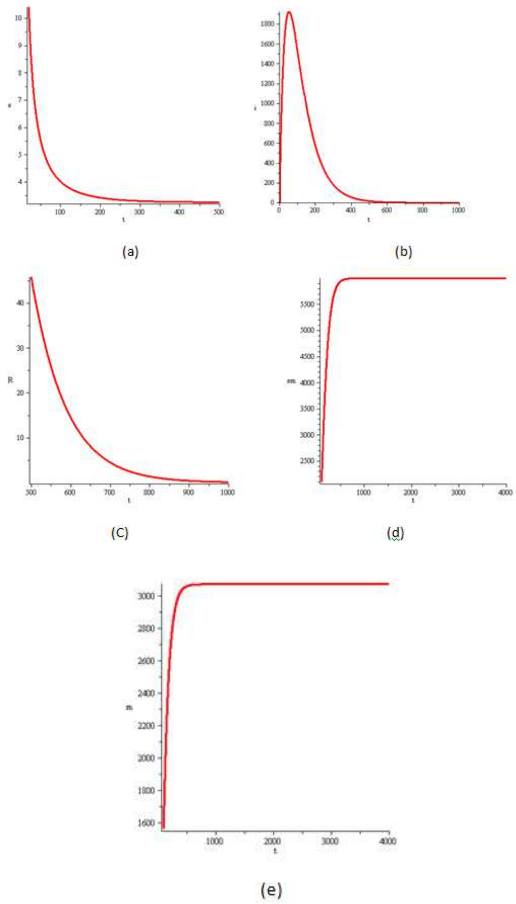

Fig. 2: The time series of (a) susceptible, (b) infective,(c) recovered, (d) aware susceptible and (e) the cumulative density of the awareness programs driven. As shown, all the state variables approach to the disease free state. $E_0 = (2.9985013, 0, 0, 5999.40245, 3333.3333)$

Stability of the endemic state: Using the values of parameter listed in Table 1. except the value of μ and μ_1 we set to be equal 0.001.

$E_1 = (169.866, 0.703455, 42.3508, 5789.48, 56.7818)$

The eigenvalues and basic reproductive number become:

$\lambda_1 = -102.939$, $\lambda_2 = -1.00659$, $\lambda_3 = -0.01666$, $\lambda_4 = -0.0000299443$,

$\lambda_5 = -4.24191 \times 10^{-8}$ and $R_0 = 1.68171 > 1$.

Since all of the eigenvalues are negative and the basic reproductive number is greater than one, the equilibrium point E_1 shown is local asymptotically stable. **Fig. 3.**

Discussion and Conclusion:

In this paper, we proposed a mathematical model of disease transmission with effect of educational program by media. From Fig. 2, we can see that if μ and μ_1 equal 0.01 the basic reproductive number $R_0 = 0.583416$, which less than one. In this case the disease will not occur. But when we change the value of μ and μ_1 equal 0.001. The basic reproductive number $R_0 = 168171$, which is greater than one. In this case the disease will persist in the as the study of Kaur, Ghosh, Bhatia (2014). We conclude that if the effectiveness of the communication educational program by media decrease then the number of infected human increase. But when the effectiveness of the educational program by media increase , the number of infected human decrease. Numerical results is satisfied with the analytic results and we applied these educational program by media as the control measure for the disease.

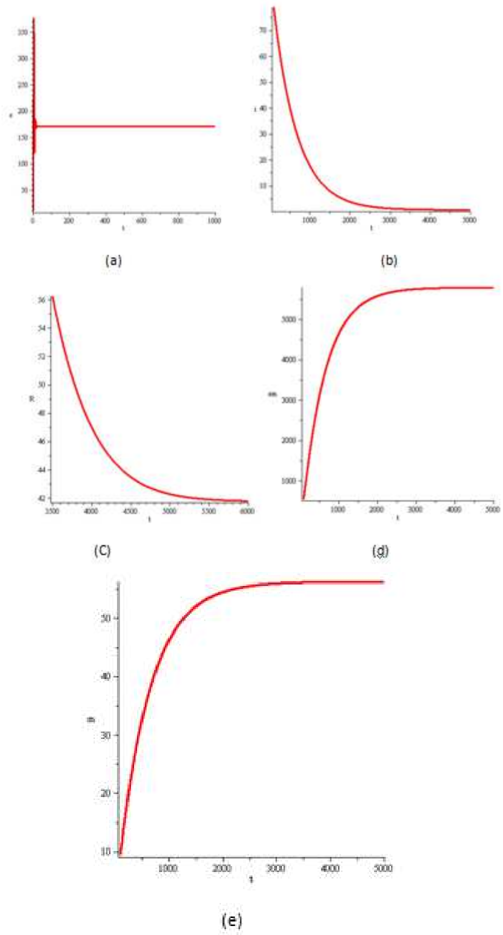

Fig. 3: The time series of (a) susceptible, (b) infective,(c) recovered, (d) aware susceptible and (e) the cumulative density of the awareness programs driven. Only the values of μ and μ_1, we set to be equal 0.001. All the state variables approach to endemic state

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