A Novel Analysis and Control of Matrix Converter using Mathematical Model under Distorted Input Voltage Conditions

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ABSTRACT

Matrix converter (MC) is a three phase forced commutated cyclo-converter. It converts three phase fixed frequency AC input into three phase Variable frequency AC output. It consists of nine bi-directional switches that connect each output phase to each input phase. This paper analyses the closed loop control of matrix converter under distorted input voltage conditions. A mathematical model is used to represent the power circuit and the switching techniques. The MC is controlled using the PI controller based compensation method. The control signals for power switches are generated using Venturini method. As a result a very simple mathematical model has been developed for the closed loop operation. The simulation results have shown to prove the reduction in harmonics under closed loop operation.

INTRODUCTION

Matrix converter principle was introduced in 1976. The development of this converter starts after the papers proposed by Venturini and Alesina in 1980 and 1989 (Bhimbra, P.S., 2003). The Matrix converter is an array of bidirectional switches as the main power elements, which interconnects the power supply directly to the load, without using any dc - link or large energy storage elements. The Matrix converter has many advantages over traditional topologies (Rashid, M.H., 2000; Zuckerberger, A., 1997; Ned Mohan, M.). The most important characteristics of Matrix converter are(a) Simple and compact power circuit,(b) sinusoidal input and output current, (c) Regeneration capability (d) operation with unity displacement factor ,(d)Generation of load voltage with arbitrary amplitude and frequency and (e) High reliability and long life. These highly attractive characteristics are the reasons for the tremendous interest in this topology. A complete mathematical model for the matrix converter is found in (Rodriguez, J.). In this present work, a PI based compensation scheme has been introduced in this mathematical model and a closed loop circuit is developed and the analysis was carried out using MATLAB/SIMULINK. The advantages of mathematical model are very low computation time, simple, flexible and can be accommodated with any type of load and low memory requirement. Simulation results are presented to prove the effectiveness of the proposed compensation technique.

Analysis of Matrix Converter:

In general matrix converter is a single stage converter that consists of m x n bidirectional power switches, to connect an m-phase voltage source to an n-phase load. Theoretically, the number of the input phases, m must be
at least here, and the number of output phases, n can be chosen from one to infinity \([6, 7, 8]\). Fig.1 shows the basic Matrix converter topology (Zuckerberger, A., 1997), from a practical point of view. The Matrix converters of 3 x 3 switches are shown in Fig.2. Their respective three phase input voltages vector and output voltage vector are given by eq.1 and eq.2 respectively (Zuckerberger, A., 1997; Wheeler, P.W., 2002)

\[
\begin{align*}
\mathbf{V}_a &= \begin{bmatrix}
V_a \cos(\omega_i t) \\
V_a \cos\left(\omega i + \frac{2\pi}{3}\right) \\
V_a \cos\left(\omega i + \frac{4\pi}{3}\right)
\end{bmatrix}
\end{align*}
\]

(1)

\[
\begin{align*}
\mathbf{V}_o &= \begin{bmatrix}
V_o \cos(\omega_o t) \\
V_o \cos\left(\omega o + \frac{2\pi}{3}\right) \\
V_o \cos\left(\omega o + \frac{4\pi}{3}\right)
\end{bmatrix}
\end{align*}
\]

(2)

Where \(\omega_i\) and \(\omega_o\) are the input and output frequencies of the Matrix converter respectively. The relationship between the output and the input voltages is given as follows,

\[V_o(t) = M(t) V_i(t)\] (3)

In the same way the input current is given as follows,

\[i_i(t) = M(t)^T i_o(t)\] (4)

Where \(M_{ij} = \int_{t_{ij}}^{t_{ij+1}}\) is the time during \(S_{ij}\) is ON

\(T_s\) - sampling period

The power filter located at the input of the converter has two main purposes. First, it filters the high-frequency components of the matrix converter input currents, generating almost sinusoidal source currents. Also it avoids the generation of over voltage produced by the fast commutation of currents due to the presence of the short circuit reactance of any real power supply.

Each switch can connect or disconnect input phase to output phase with a proper combination of the conduction.

Where, \(\omega_o = \omega_i, \omega_o\) and \(q\) is the voltage transfer ratio.

Each switch is characterized with a commutation function defined as in eq.6,
where $i = \{A,B,C\}$ on input side $j = \{a,b,c\}$ on load side

In order to provide the safe operation of the converter, when operating with bidirectional switches, two rules have to be followed. Initially, the input terminals should not be short circuited, since Matrix converter is fed by voltage source. Also the output phase must never be opened because the loads are typically inductive in nature which leads to over voltage. These conditions can be stated as in eq.7.

$$\sum_{i \in \{A,B,C\}} S_{ij}(t) = 1 \text{ if } j = \{a,b,c\}$$

These rules are used to reduce the possible 512 switching combinations to only 27 different switching states

**Venturini Switching Algorithm:**

Venturini modulation is a scalar control method. In this type of modulation technique voltage transfer ratio is limited to 50% under normal operation. The common mode addition technique is used to achieve a maximum voltage ratio of 87%. This algorithm can be simpler and because of its unity displacement factor it is suited for real time implementation. After injecting the harmonics, output voltage is given in eq.8[9,6,3]

$$[V_{j}(t)] = qV_{m} \begin{bmatrix} \cos(\omega_{m}t) - \frac{1}{6}\cos(3\omega_{m}t) + \frac{1}{2\sqrt{3}}\cos(3\beta_{m}t) \\ \cos(\omega_{m}t) + \frac{2\beta_{m}}{3\pi} - \frac{1}{6}\cos(3\omega_{m}t) + \frac{1}{2\sqrt{3}}\cos(3\beta_{m}t) \\ \cos(\omega_{m}t) + \frac{\beta_{m}}{3\pi} - \frac{1}{6}\cos(3\omega_{m}t) + \frac{1}{2\sqrt{3}}\cos(3\beta_{m}t) \end{bmatrix}$$

Where, $q$ is the voltage gain or voltage transfer ratio.

**Mathematical Model:**

The duty cycle of the matrix converter can be calculated from the required voltage transfer ratio($q$), input, output and switching frequencies. The duty cycle calculations for 0.5 and 0.866 are given in eq.9 and 10. Duty cycles for transfer ratio of 0.5 are (HulusiKaraca, RamazanAkkaya, 2011; HulusiKaraca, RamazanAkkaya, 2009; Rodriguez, J.,)

$$M_{Aa} = \frac{1}{3}(1 + 2q\cos(\omega_{in}t))$$
$$M_{Ba} = \frac{1}{3}(1 + 2q\cos(\omega_{in}t - \frac{2\pi}{3}))$$
$$M_{Ca} = \frac{1}{3}(1 + 2q\cos(\omega_{in}t - \frac{4\pi}{3}))$$
$$M_{Ab} = \frac{1}{3}(1 + 2q\cos(\omega_{in}t - \frac{4\pi}{3}))$$
$$M_{Bb} = \frac{1}{3}(1 + 2q\cos(\omega_{in}t))$$
$$M_{Cb} = \frac{1}{3}(1 + 2q\cos(\omega_{in}t - \frac{2\pi}{3}))$$
$$M_{Ac} = \frac{1}{3}(1 + 2q\cos(\omega_{in}t))$$
$$M_{Bc} = \frac{1}{3}(1 + 2q\cos(\omega_{in}t - \frac{4\pi}{3}))$$
$$M_{Cc} = \frac{1}{3}(1 + 2q\cos(\omega_{in}t))$$

$$T_{ij} = \frac{T_{f}}{3} \begin{bmatrix} 1 + \frac{2qV_{j}^{*}}{3V_{m}^{*}} + \frac{2q}{3\beta_{m}} \sin(q\beta_{j} - \frac{2\pi}{3}) \sin(3q\beta_{j}) \\ \frac{2qV_{j}^{*}}{3V_{m}^{*}} + \frac{2q}{3\beta_{m}} \sin(q\beta_{j} - \frac{2\pi}{3}) \sin(3q\beta_{j}) \\ \frac{2qV_{j}^{*}}{3V_{m}^{*}} + \frac{2q}{3\beta_{m}} \sin(q\beta_{j} - \frac{2\pi}{3}) \sin(3q\beta_{j}) \end{bmatrix}$$

Where $\beta = 0, \frac{2\pi}{3}, \frac{4\pi}{3}$ ………… (10)

$$q_{m}^{*} \text{ is the maximum voltage ratio(0.866), } q \text{ is the required voltage ratio, } V_{in} \text{ is the input voltage and } V_{j} \text{ is given in eq.11. Using this equation duty cycle waveforms can be developed.}$$

$$P_{j}(t) = qV_{m}^{*} \left[ \cos(\omega_{in}t + \phi) - \frac{1}{6}\cos(3\omega_{in}t) + \frac{1}{2\sqrt{3}}\cos(3\phi_{in}t) \right]$$

Where $\phi = 0, \frac{2\pi}{3}, \frac{4\pi}{3}$ ………… (11)

Mathematical model for “a” phase is shown in Fig. 3, and the load is modeled using transfer function shown in eq. 12 (Rodriguez, J., 2000).

$$\frac{I_{out}(t)}{V_{j}(t)} = \frac{1}{3\beta_{m}}$$

**PI Compensation Scheme:**

PI controller is used to reduce harmonic content and also to insure stable control of the load currents. The PI compensation scheme is shown in Fig.4.

The output currents and the reference currents are used to calculate the error value for PI controller. The output obtained from the PI controller is the value of voltage transfer ratio or voltage gain. The output currents can be measured by using the eq. 13. If the input voltage of the matrix converter is sinusoidal and balanced, $I_{in}$ is constant. Then the output current is also sinusoidal.
\[ I_d = \sqrt{\frac{2}{3}i^2_{m}(t) + i^2_{s}(t) + i^2_{m}(t)} \]

**Fig. 3:** Mathematical model of the MC for phase a.

**Fig. 4:** PI compensation scheme for the MC.

**Simulation Result Analysis:**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Source voltage, ( V_1 )</td>
<td>311V</td>
</tr>
<tr>
<td>Source frequency, ( f_1 )</td>
<td>50Hz</td>
</tr>
<tr>
<td>Output frequency, ( f_0 )</td>
<td>50Hz to 60Hz</td>
</tr>
<tr>
<td>Load resistance, ( R )</td>
<td>10Ω</td>
</tr>
<tr>
<td>Load inductance, ( L )</td>
<td>20mH</td>
</tr>
<tr>
<td>Switching frequency, ( f_s )</td>
<td>10KHz</td>
</tr>
</tbody>
</table>

The performance analysis of the matrix converter under normal and distorted input voltage conditions for 50 Hz and 60Hz has been presented. Simulation has been carried out in MATLAB/SIMULINK using the design specifications given in Table I (6, 5). The results are discussed as under. The input voltages under normal and distorted conditions are given in Fig. 5 and Fig. 6.

**Fig. 5:** Input voltage under normal operation.

The variation of voltage transfer ratio for uncompensated system is shown in Fig. 7.
Voltage transfer ratio is constant for uncompensated system and variable for compensated system. Pulse waveform can be generated from the obtained voltage transfer ratio by using the equation (8) and (9). Pulse waveform is illustrated in Fig. 8 and 9.

The output current, output voltage and THD analysis waveform for output frequency = 50Hz under normal operation is shown in Fig. 10, Fig.11 and Fig. 12.

![Input voltage under distorted input voltage condition.](image6)

Fig. 6: Input voltage under distorted input voltage condition.

![Voltage transfer ratio for compensated system.](image7)

Fig. 7: Voltage transfer ratio for compensated system.

![Pulse waveform for uncompensated system.](image8)

Fig. 8: Pulse waveform for uncompensated system.

![Pulse waveform for compensated system.](image9)

Fig. 9: Pulse waveform for compensated system.
Fig. 10: (a) Output voltage for uncompensated system.

Fig. 10: (b) Output voltage for compensated system.

Figure 10 Output voltage waveform.

Fig. 11: (a) Output current for uncompensated system.

Fig. 11: (b) Output current for compensated system.

Fig. 11 Output current waveform
Figure 12 THD analysis waveform.

The output current, output voltage and THD analysis of the waveform for output frequency of 60Hz after injecting the third harmonics is shown in Fig.13, Fig.14 and Fig.15.
Fig. 12: (a) THD analysis for uncompensated system.

Fig. 13 Output voltage waveform.
Fig. 14 Output current waveform
Fig. 15 THD analysis waveform.

Under normal operation, the Total harmonic distortion for the uncompensated and compensated system is 4.05% and 1.49%. After injecting the harmonics, the Total harmonic distortion for the uncompensated and compensated systems are 32.32% and 20.38%. As it is shown, if the input voltages of the MC are distorted or unbalanced, the low order harmonics occur on the output current of the MC. But, under the same conditions, the proposed PI controller based compensation method has effectively decreased these harmonics.

Fig. 12 (b) THD analysis for compensated system.

Fig. 13: (a) Output voltage for uncompensated system.

Fig. 13: (b) Output voltage for compensated system.
Conclusions:
In this paper, the modeling of Matrix converter and the switching technique has been carried out for the closed loop operation. The switching techniques have been analyzed using Venturini modulation. In Venturini modulation switching pulse can be obtained by comparing voltage transfer ratio (q) and saw tooth carrier waveform of switching frequency 10 kHz. Simulation has been done for this mathematical model with and without injecting third order harmonics for open loop and closed loop systems. The above model was implemented using MATLAB/SIMULINK. The THD analysis has been done and compared for uncompensated and compensated systems. It is found that the closed loop PI response is better than the open loop response and the THD reduces considerably while using PI controller.
REFERENCES