Bayesian inference for Parameter and Reliability function of Inverse Rayleigh Distribution Under Modified Squared Error Loss Function

Huda A. Rasheed and Raghda Kh. Aref

AL-Mustansiriya University College of Science, Dept. of Math.

ABSTRACT

In this study, obtained some Bayes estimators based on Modified squared error loss function as well as Maximum likelihood estimator for scale parameter and reliability function of Inverse Rayleigh distribution. In order to get better understanding of our Bayesian analysis, we consider non-informative prior for the scale parameter using Jeffreys prior information as well as informative prior density represented by Gamma distribution. Based on Monte-Carlo simulation study, the behavior of Bayes estimates of the scale parameter of inverse Rayleigh distribution have been compared depending on the mean squared errors (MSE’s), while the estimates of the reliability function have been compared depending on the integrated mean squared errors (IMSE’s). In the current study, we observed that, the performance of Bayes estimator for the scale parameter and reliability function under Modified squared error loss function with Gamma prior is better than the corresponding estimators with Jeffreys prior, for all cases.

INTRODUCTION

The inverse Rayleigh distribution is introduced by Voda (1972). He studies some properties of the MLE of the scale parameter of inverse Rayleigh distribution which is also being used in lifetime experiments. Gharraph (1993) derived five measures of location for the Inverse Rayleigh distribution. These measures are the mean, harmonic mean, geometric mean, mode, and the median. He, also, estimated the unknown parameter using different methods of estimation. A comparison of these estimates was discussed numerically in term of their bias and root mean square error. In (2010) Soliman and other researchers studied the estimation and prediction from Inverse Rayleigh distribution based on lower record values, Bayes estimators have been developed under squared error and zero one-loss functions. In (2012) Dey discussed the Bayesian estimation of the parameter and reliability function of an Inverse Rayleigh distribution using different loss function represented by Square error, LINEX loss function. In (2013) Tabassum and others studied the Bayes estimation of the parameters of the Inverse Rayleigh distribution for left censored data under Symmetric and asymmetric loss functions. In (2014) Muhammad Shuaib Khan obtained the Modified Inverse Rayleigh Distribution as a special case of Inverse Weibull, which is extension to it. In (2015) Guobing discussed Bayes estimation for Inverse Rayleigh model under different loss functions represented by squared error loss, LINEX loss and entropy loss functions. In (2015) Rasheed and others compared between some classical estimators with the Bayes estimators of one parameter Inverse Rayleigh distribution under Generalized squared error loss function. In (2016) Rasheed and Aref obtained and discussed the Bayesian approach for estimating the scale parameter of Inverse Rayleigh distribution under different loss function. Finally, in (2016) Rasheed and Aref obtain Reliability Estimation in Inverse Rayleigh Distribution using Precautionary Loss Function.
1. **One parameter Inverse Rayleigh distribution:**

The probability density function (pdf) of the Inverse Rayleigh distribution with scale parameter \( \theta \) is defined as follows: (Shawky, A.I. and M.M. Badr, 2012)

\[
f(x; \theta) = \frac{2\theta}{x^3} \exp \left( -\frac{\theta}{x^2} \right) \quad x > 0, \theta > 0
\]

(1)

The corresponding cumulative distribution function (C.D.F), is given by:

\[
F(x; \theta) = \exp \left( -\frac{\theta}{x^2} \right) \quad ; \quad x > 0, \theta > 0
\]

(2)

The Reliability function of inverse Rayleigh distribution is given by:

\[
R(x; \theta) = 1 - F(x; \theta)
\]

\[
= 1 - \exp \left( -\frac{\theta}{x^2} \right) \quad x > 0, \theta > 0
\]

(3)

The Hazard function of Inverse Rayleigh distribution is given by:

\[
h(x; \theta) = \frac{f(x; \theta)}{R(x; \theta)}
\]

After substitution (1) and (3) into \( h(x; \theta) \), yields:

\[
h(x; \theta) = \frac{2\theta^2}{1 - \exp \left( -\frac{\theta}{x^2} \right)} = \frac{2\theta}{x^3} \left( \exp \left( \frac{\theta}{x^2} \right) - 1 \right)
\]

(4)

2. **Estimation of parameter and the Reliability function**

In this section, we estimate the scale parameter \( \theta \) and the Reliability function \( R(t) \) using Maximum Likelihood Estimator in addition of some Bayesian estimators, as follows

1. **Maximum likelihood estimator:**

   The Maximum likelihood estimator of the scale parameter \( \theta \) and the reliability function \( R(t) \) of inverse Rayleigh distribution have been derived as one of the classical estimators.

   The maximum likelihood estimator \( \hat{\theta}_{ML} \) of the parameter \( \theta \) that maximizes the likelihood function is defined as:

   \[
   L(x_1, x_2, ..., x_n; \theta) = 2^n \theta^n \prod_{i=1}^{n} \frac{1}{x_i^3} \exp \left( -\theta \sum_{i=1}^{n} \frac{1}{x_i^2} \right)
   \]

   (5)

   Taking the partial derivatives for the natural log-likelihood function, with respect to \( \theta \) and then, equating to zero we have:

   \[
   \frac{\partial \ln L(x; \theta)}{\partial \theta} = \frac{n}{\theta} - \sum_{i=1}^{n} \frac{1}{x_i^2} = 0
   \]

   Hence, the MLE of \( \theta \) denoted by \( \hat{\theta}_{ML} \) is:

   \[
   \hat{\theta}_{ML} = \frac{n}{\sum_{i=1}^{n} \frac{1}{x_i^2}} = \frac{n}{T}
   \]

   (6)

   Where \( T = \sum_{i=1}^{n} \frac{1}{x_i^2} \)

   Since the Maximum likelihood estimator is invariant and one to one mapping (Singh, S.K., et al., 2011), hence the Maximum likelihood estimator of reliability function will be:

   \[
   \hat{R}_{ML}(t) = 1 - \exp \left( -\frac{\hat{\theta}_{ML}}{t^2} \right)
   \]

   (7)

2. **Bayes estimators:**

   We provide Bayesian estimation method including informative and non-informative priors, under Modified squared error loss function to estimate scale parameter and reliability function of Inverse Rayleigh distribution.

2.1. **Jeffreys prior information:**

   Assume that \( \theta \) has a non-informative prior density defined as using Jeffreys prior information \( g_1(\theta) \) which is given by (Rasheed, H. A. and Khalifa, Z. N. 2016)

   \[
   g_1 \propto \sqrt{I(\theta)}
   \]

   Where \( I(\theta) \) represented Fisher information, defined as, follows:

   \[
   I(\theta) = -nE \left[ \frac{\partial^2 \ln f(x; \theta)}{\partial \theta^2} \right]
   \]
Therefore,
\[ g(\theta) = b \sqrt{n} \exp \left( \frac{3}{2} \ln f(x; \theta) \right), \quad b \text{ is a constant} \]  

(8)

Now, taking the second partial derivative of \( \log f(x; \theta) \) with respect to \( \theta \), gives
\[ \frac{\partial^2 \ln f(x_i, \theta)}{\partial \theta^2} = -\frac{1}{\theta^2} \]

Hence,
\[ E \left( \frac{\partial^2 \ln f(x_i; \theta)}{\partial \theta^2} \right) = -\frac{1}{\theta^2} \]

After substitution into (8), we get
\[ g_1(\theta) = \frac{b}{\theta} \sqrt{n} \quad , \quad \theta > 0 \]  

(9)

The posterior density function is defined as:
\[ h(\theta|x_1, ..., x_n) = \frac{g(\theta)L(\theta; x_1, ..., x_n)}{\int_0^\infty g(\theta)L(\theta; x_1, ..., x_n) d\theta} \]

\[ h_1(\theta|x_1, ..., x_n) = \frac{b}{\theta} \sqrt{n} 2^n \theta^n \prod_{i=1}^n \frac{1}{x_i^2} \exp \left( -\theta \sum_{i=1}^n \frac{1}{x_i^2} \right) \]

\[ \int_0^\infty \frac{b}{\theta} \sqrt{n} 2^n \theta^n \prod_{i=1}^n \frac{1}{x_i^2} \exp \left( -\theta \sum_{i=1}^n \frac{1}{x_i^2} \right) d\theta = \Gamma_n \]

Hence, the posterior density function of \( \theta \) with Jeffreys prior can be written as
\[ h_1(\theta|x_1, ..., x_n) = \frac{T^{n-1} e^{-\theta T}}{\Gamma_n} \]  

(10)

It is clear, \( h_1(\theta|x_1, ..., x_n) \) is recognized as the density of the Gamma distribution, i.e., \( \theta|x \sim \text{Gamma}(n, T) \), with
\[ E(\theta) = \frac{n}{T}, \quad \text{Var}(\theta) = \frac{n}{T^2} \]

2.2. Gamma prior distribution:

Assuming that \( \theta \) has informative prior as Gamma prior which takes the following form
\[ g_2(\theta) = \frac{\beta^\alpha \theta^{\alpha-1} e^{-\theta \beta}}{\Gamma(\alpha)} \quad ; \quad \theta > 0 \quad , \quad \alpha > 0 \quad , \quad \beta > 0 \]  

(11)

Where, \( \beta, \alpha \) are the shape and the scale parameters respectively.

From Bayesian theorem the posterior density function of \( \theta \) denoted by \( h_2(\theta|x) \) can be obtained as
\[ h_2(\theta|x) = \frac{g_2(\theta)L(\theta; x_1 x_2 \ldots \ldots x_n)}{\int_0^\infty g_2(\theta)L(\theta; x_1 x_2 \ldots \ldots x_n) d\theta} \]

Now, combining (5) and (11), gives
\[ h_2(\theta|x) = \frac{e^{-\theta T + \beta} \phi^{a+n} e^{-\theta \phi}}{\phi^{a+n} e^{-\theta \phi} \int_0^\infty e^{-\theta T + \beta} \phi^{a+n} e^{-\theta \phi} d\theta} \]

So, the posterior density function of \( \theta \) with Gamma prior is:
\[ h_2(\theta|x) = \frac{e^{-\theta T + \beta} \phi^{a+n} e^{-\theta \phi}}{\phi^{a+n} e^{-\theta \phi} \Gamma(a + n)}, \quad \theta > 0 \]  

(12)

Where, \( P = T + \beta \)

Notice that: \( \theta|x \sim \text{Gamma}(a + n, P) \), with \( E(\theta) = \frac{a+n}{P}, \quad \text{Var}(\theta) = \frac{a+n}{P^2} \)

2.3. Bayes estimator for \( \theta \) under Modified squared error loss function:

The modified squared error loss function can be defined as follows (Al-Baldawi, T. H., 2013)
L(\theta, 0) = \theta^2(\theta - 0)^2

The Risk function under the Modified squared error loss function which is denoted by \( R_{MS}(\hat{\theta}, \theta) \) is

\[
R_{MS}(\hat{\theta}, \theta) = E[L(\hat{\theta}, \theta)]
\]

\[
R_{MS}(\hat{\theta}, \theta) = \int_0^\infty \theta^2(\hat{\theta} - \theta)^2 h(\theta|x) d\theta
\]

(13)

Taking the partial derivative for \( R_{MS}(\hat{\theta}, \theta) \) with respect to \( \hat{\theta} \) and setting it equal to zero, gives the Bayes estimator relative to Modified square error loss function which is denoted by \( \hat{\theta}_{MS} \) as

\[
\hat{\theta}_{MS} = \frac{E(\theta^{r+1}|x)}{E(\theta^r|x)}
\]

(14)

(i) With Jeffreys prior information:

According to the posterior density function (10), the Bayes estimator for \( \theta \) under Modified squared error loss function can be derived as follows:

\[
E(\theta^m|x) = \int_0^\infty \theta^m h_1(\theta|x) d\theta
\]

\[
E(\theta^m|x) = f_0 e^{-\theta^{m+1} e^{-\theta} \gamma_1} \gamma_0 d\theta
\]

\[
E(\theta^m|x) = \frac{\gamma_1 \theta^{m+1} e^{-\theta} \gamma_1}{\gamma_0} d\theta
\]

\[
E(\theta^m|x) = \frac{\gamma_1 \theta^{m+1} e^{-\theta} \gamma_1}{\gamma_0} d\theta
\]

(15)

We get the Bayes estimator for the scale parameter of inverse Rayleigh distribution under Modified square loss function with Jeffreys prior denoted by \( \hat{\theta}_{MS1}, \hat{\theta}_{MS2} \) with \( r = 1, 3 \) respectively, are:

\[
\hat{\theta}_{MS1} = \frac{E(\theta^{1+1}|x)}{E(\theta^1|x)} = \frac{n + 1}{T}
\]

(16)

\[
\hat{\theta}_{MS2} = \frac{E(\theta^{3+1}|x)}{E(\theta^3|x)} = \frac{n + 3}{T}
\]

(17)

(ii) With Gamma prior information:

According to the posterior density function (12), the Bayes estimator of \( \theta \) of Inverse Rayleigh distribution under Modified squared error loss function we substitute two values of \( r, r = 1, 3 \) respectively into (14) as follows:

\[
E(\theta^m|x) = \int_0^\infty \theta^m h_2(\theta|x) d\theta
\]

\[
E(\theta^m|x) = \frac{\gamma_1 \theta^{m+1} e^{-\theta} \gamma_1}{\gamma_0} d\theta
\]

\[
E(\theta^m|x) = \frac{\gamma_1 \theta^{m+1} e^{-\theta} \gamma_1}{\gamma_0} d\theta
\]

\[
E(\theta^m|x) = \frac{\gamma_1 \theta^{m+1} e^{-\theta} \gamma_1}{\gamma_0} d\theta
\]

(18)

the Bayes estimator for the scale parameter of inverse Rayleigh distribution under Modified square loss function with Gamma prior denoted by \( \hat{\theta}_{MSG1}, \hat{\theta}_{MSG2} \) with \( r = 1, 3 \) respectively, are:

\[
\hat{\theta}_{MSG1} = \frac{E(\theta^{1+1}|x)}{E(\theta^1|x)} = \frac{\alpha + n + 1}{P}
\]

(19)

\[
\hat{\theta}_{MSG2} = \frac{E(\theta^{3+1}|x)}{E(\theta^3|x)} = \frac{\alpha + n + 3}{P}
\]

(20)

2.4 Bayes estimator for R(t) Under Modified squared error loss function:

We can find the Bayes estimator for the reliability function R(t) by using the probability density function for \( \theta \). According to (14), the Bayes estimator for R(t) under Modified squared error loss function, will be:
\[ \hat{R}(t)_{MS} = \frac{E((R(t))^{r+1} | t)}{E((R(t))^{r} | t)} \]  

(i) Bayes estimator for \( R(t) \) based on Jeffreys prior information:

To derive the Bayes estimator for the \( R(t) \) under Modified squared error loss function (MSLF) with Jeffreys prior denoted by \( \hat{R}(t)_{MSJ1} \), we substitute two values of \( r \), \( r = 1, 3 \) respectively into (21) which required to obtain \( E(R(t)|t), E((R(t))^2|t), E((R(t))^3|t) \) and \( E((R(t))^4|t) \) as follows:

\[ E(R(t)|t) = \int_0^\infty R(t)h_1(\theta|t) \, d\theta \tag{22} \]

Since \( R(t) = 1 - \exp \left(-\frac{\theta}{t^2} \right) \),

\[ E(R(t)|t) = \int_0^\infty \left(1 - \exp \left(-\frac{\theta}{t^2} \right) \right) \frac{\theta^n e^{-\theta T}}{\Gamma(n)} \, d\theta \]

\[ E(R(t)|t) = 1 - \left( \frac{t^2}{T^2 + 1} \right)^n \tag{23} \]

By the same way, we can find \( E((R(t))^2|t), E((R(t))^3|t) \) and \( E((R(t))^4|t) \) so,

\[ E((R(t))^2|t) = 1 - 2 \left( \frac{T t^2}{T t^2 + 1} \right)^n + \left( \frac{T t^2}{T t^2 + 2} \right)^n \tag{24} \]

\[ E((R(t))^3|t) = 2 - 3 \left( \frac{T t^2}{T t^2 + 1} \right)^n + \left( \frac{T t^2}{T t^2 + 2} \right)^n \tag{25} \]

\[ E((R(t))^4|t) = 2 - 4 \left( \frac{T t^2}{T t^2 + 1} \right)^n + 2 \left( \frac{T t^2}{T t^2 + 2} \right)^n \tag{26} \]

Hence, from (24), (23) we get the Bayes estimator for the \( R(t) \) of inverse Rayleigh distribution under Modified squared error loss function with Jeffreys prior with \( r = 1 \), which is denoted by \( \hat{R}(t)_{MSJ1} \) as follows

\[ \hat{R}(t)_{MSJ1} = \frac{1 - 2 \left( \frac{T t^2}{T t^2 + 1} \right)^n + \left( \frac{T t^2}{T t^2 + 2} \right)^n}{1 - \left( \frac{T t^2}{T t^2 + 1} \right)^n} \tag{27} \]

Now, from (26) and (25) we obtain the Bayes estimator for the \( R(t) \) of inverse Rayleigh distribution under Modified squared error loss function with Jeffreys prior with \( r = 3 \), which is denoted by \( \hat{R}(t)_{MSJ2} \)

\[ \hat{R}(t)_{MSJ2} = \frac{2 - 4 \left( \frac{T t^2}{T t^2 + 1} \right)^n + 2 \left( \frac{T t^2}{T t^2 + 2} \right)^n}{2 - 3 \left( \frac{T t^2}{T t^2 + 1} \right)^n + \left( \frac{T t^2}{T t^2 + 2} \right)^n} \tag{28} \]

(ii) Bayes estimator for \( R(t) \) based on Gamma prior information:

To derive the Bayes estimator for the \( R(t) \) under Modified squared error loss function (MSLF) with Gamma prior, that is denoted by \( \hat{R}(t)_{MSG} \), we’ll derive \( E(R(t)|t), E((R(t))^2|t), E((R(t))^3|t) \) and \( E((R(t))^4|t) \) as follows:

\[ E(R(t)|t) = \int_0^\infty R(t)h_2(\theta|t) \, d\theta \tag{29} \]

since \( R(t) = 1 - \exp \left(-\frac{\theta}{t^2} \right) \),

\[ E(R(t)|t) = \int_0^\infty \left(1 - \exp \left(-\frac{\theta}{t^2} \right) \right) \frac{\theta^n e^{-\theta p}}{\Gamma(\alpha+n)} \, d\theta \]

\[ E(R(t)|t) = 1 - \left( \frac{P t^2}{P t^2 + 1} \right)^{\alpha+n} \tag{30} \]

By same way we can find \( E((R(t))^2|t), E((R(t))^3|t) \) and \( E((R(t))^4|t) \). Hence,

\[ E((R(t))^2|t) = 1 - 2 \left( \frac{P t^2}{P t^2 + 1} \right)^{\alpha+n} + \left( \frac{P t^2}{P t^2 + 2} \right)^{\alpha+n} \tag{31} \]
\[ E((R(t))^{n}) = 2 - 3 \left( \frac{Pt^2}{P^2 + 1} \right)^{\alpha+n} + \left( \frac{Pt^2}{P^2 + 2} \right)^{\alpha+n} \]  
\[ E((R(t))^{2}) = 2 - 4 \left( \frac{Pt^2}{P^2 + 1} \right)^{\alpha+n} + 2 \left( \frac{Pt^2}{P^2 + 2} \right)^{\alpha+n} \]  
(32)  
(33)

From (31), (30) we can get the Bayes estimator for the \( R(t) \) using Modified squared error loss function based on Gamma prior with \( r = 1 \), which is denoted by \( \hat{R}(t)_{MSG1} \) as follows

\[ \hat{R}(t)_{MSG1} = \frac{1 - 2 \left( \frac{Pt^2}{P^2 + 1} \right)^{\alpha+n} + \left( \frac{Pt^2}{P^2 + 2} \right)^{\alpha+n}}{1 - \left( \frac{Pt^2}{P^2 + 1} \right)^{\alpha+n}} \]  
(34)

Now, from (33), (32) we get the Bayes estimator for the \( R(t) \) under Modified squared error loss function based on Gamma prior with \( r = 3 \), that is denoted by \( \hat{R}(t)_{MSG2} \)

\[ \hat{R}(t)_{MSG2} = \frac{2 - 4 \left( \frac{Pt^2}{P^2 + 1} \right)^{\alpha+n} + 2 \left( \frac{Pt^2}{P^2 + 2} \right)^{\alpha+n}}{2 - 3 \left( \frac{Pt^2}{P^2 + 1} \right)^{\alpha+n} + \left( \frac{Pt^2}{P^2 + 2} \right)^{\alpha+n}} \]  
(35)

4. Simulation Study:
In our simulation study, the process have been repeated 5000 times (\( L=5000 \)). We generated samples of sizes \( n = 10, 25, 50, \) and 100 from Inverse Rayleigh distribution with \( \theta = 0.5, 1.5 \) and 3. The values of the parameters Gamma prior are chosen to be \( \beta = 1,2,3, \) \( \alpha = 0.3,0.8 \).

The expected values and mean squared errors (MSEs) for all estimates of the parameter \( \theta \) are obtained, where:

\[ MSE(\theta) = \frac{\sum_{i=1}^{L}(\hat{\theta}_i - \theta)^2}{L} ; \quad i = 1, 2, 3, ..., L \]

and integral mean squares error (IMSE) for all estimates of the reliability function of Inverse Rayleigh distribution which is defined as distance between the estimate value of the reliability function and actual value of reliability function that is given as follows:

\[ IMSE(\hat{R}(t)) = \frac{1}{L} \sum_{t=1}^{L} \left( \frac{1}{n_t} \sum_{i=1}^{n_t} (\hat{R}_i(t_j) - R(t_j))^2 \right) \]

The results were summarized and tabulated in the following tables for each estimator and for all sample sizes as follows:

**Table 1:** Expected Values and MSE's of the Different Estimators for the Inverse Rayleigh Distribution when \( \theta = 0.5 \) Under Modified Squared Error Loss Function

| \( \alpha \) | Estimators | Criteria | MLE | Jeffreys prior | Gamma prior | r = 1 | r = 3 | r = 1 | r = 3 | r = 1 | r = 3 | r = 1 | r = 3 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 10 | EXP | 0.55330 | 0.60863 | 0.17292 | 0.58207 | 0.60782 | 0.52841 | 0.55179 | 0.65509 | 0.71084 | 0.62193 | 0.64532 |
| 25 | EXP | 0.52179 | 0.56266 | 0.34440 | 0.53394 | 0.54550 | 0.51529 | 0.52509 | 0.57561 | 0.58574 | 0.59547 | 0.56127 |
| 50 | EXP | 0.50302 | 0.54002 | 0.54048 | 0.55791 | 0.55209 | 0.50672 | 0.51256 | 0.53317 | 0.54221 | 0.52341 | 0.53353 |
| 100 | EXP | 0.50623 | 0.50278 | 0.00320 | 0.00279 | 0.00278 | 0.00278 | 0.00278 | 0.00278 | 0.00278 | 0.00278 | 0.00278 |

**Table 2:** Expected Values and MSE's of the Different Estimators for the Inverse Rayleigh Distribution when \( \theta = 1.5 \) Under Modified Squared Error Loss Function

| \( \alpha \) | Estimators | Criteria | MLE | Jeffreys prior | Gamma prior | r = 1 | r = 3 | r = 1 | r = 3 | r = 1 | r = 3 | r = 1 | r = 3 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 10 | EXP | 1.65991 | 1.82529 | 2.17584 | 1.53852 | 1.60660 | 1.22992 | 1.27494 | 1.87891 | 1.87891 | 1.45071 | 1.45071 |
| 25 | EXP | 1.56536 | 1.62797 | 1.75320 | 1.57271 | 1.56264 | 1.37818 | 1.40438 | 1.64335 | 1.67238 | 1.48298 | 1.50918 |
| 50 | EXP | 1.53681 | 1.61921 | 1.62343 | 1.50368 | 1.52444 | 1.43598 | 1.44899 | 1.57270 | 1.58363 | 1.40197 | 1.50986 |
| 100 | EXP | 1.51607 | 1.53213 | 1.56156 | 1.50807 | 1.51551 | 1.46834 | 1.47559 | 1.53764 | 1.49733 | 1.54329 | 1.50458 |
| 100 | MLE | 0.02380 | 0.02500 | 0.02800 | 0.02256 | 0.02299 | 0.02212 | 0.02290 | 0.02483 | 0.02102 | 0.02676 | 0.02232 |
Discussion:

1. The results of the simulation study for estimating the scale parameter ($\theta$) of Inverse Rayleigh distribution show that:
   - From table (1), when the $\theta=0.5$, the performance of Bayes estimator under Modified squared error loss function with ($\beta=3$, $\alpha=0.3$ and r=1) is the best estimator comparing to the other estimators for all sample size.
   - From table (2), when the $\theta=1.5$, it is a clear that, the performance of Bayes estimator under Modified squared error loss function with ($\beta=3$, $\alpha=0.3$ and r=2) is the best estimator comparing to the other estimators for all sample size except the sample (100).
   - From table (3), we observed that, the performance of Bayes estimator under Modified squared error loss function with ($\beta=1.2$, $\alpha=0.8$ and r=1) is the best estimator comparing to the other estimators for all sample size.
   - The results of the simulation study for estimating the reliability function $R(t)$ of Inverse Rayleigh distribution show that:
   - From table (4), notice that, the performance of Bayes estimator under Modified squared error loss function with ($\beta=3$, $\alpha=0.3$ and r=1) is the best estimator comparing to the other estimators for all sample size.
   - From table (5), we observed that, the performance of Bayes estimator under Modified squared error loss function with ($\beta=3$, $\alpha=0.3$ and r=1) is the best estimator comparing to the other estimator for all sample size except the sample (10).
   - From table (6), it is a clear that, the performance of Bayes estimator under Modified squared error loss function (MSELF) with ($\beta=1.2$, $\alpha=0.8$ and r=1) is the best estimator comparing to the other estimator for all sample size except the sample (10).

In general, we conclude that, in situation involving estimation of scale parameter($\theta$) and reliability function $R(t)$ of Inverse Rayleigh distribution under Modified squared error loss function using Gamma prior for all samples sizes.

REFERENCES


