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Neutrosophic TOPSIS Based Game Theory for Solving MCGDM Problems

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ABSTRACT

Background: Multi-Criteria Group Decision Making (MCGDM) problems can be defined as the process of choosing the most suitable alternative among a set of alternatives. MCGDM problems can contain more than one participant, so the competition can possibly arise between the participants of any MCGDM Problem. MCGDM problems also contain complex, uncertain, incomplete, and imprecise information, so the crisp sets cannot be precious for representing this information. **Objective:** A new approach for solving the MCGDM problems which have competition between their participants under the condition of the uncertainty in data is proposed in this paper. In this approach, Single Valued Neutrosophic Sets (SVNs) is firstly used for representing the data uncertainty. Then, TOPSIS technique is extended to handle the SVN environment. After that, the game theory is used to find the optimal solution due to its superiority in dealing with the competition situations. **Results:** Finding the Nash equilibrium for the competitive MCGDM problem under neutrosophic environment (two-player non-constant sum game). **Conclusion:** The proposed technique opens the door of utilizing neutrosophic sets in conjunction with game theory principles in solving competitive MCGDM problems under uncertainty conditions.

INTRODUCTION

Multi-Criteria Group Decision Making (MCGDM) problems can be defined as the process of choosing the most suitable alternative among a set of alternatives with respect to several conflict criteria with the help of a decision-making group. In classical methods of solving MCGDM problems, such as TOPSIS Hwang and Yoon (1981), PROMETHEE Brans (1986), ELECTRE Bernard (1991), and VIKOR Opricovic (2004), decision makers used crisp numbers for evaluating criteria and alternatives. However, simplicity introduced in using crisp numbers, they are not appropriate for handling real case problems as most of real problems suffer from incomplete, imprecise and vague information which all lead to the problem of data uncertainty.

Fuzzy set Zadeh (1965) introduced an effective way to model data uncertainty by defining membership function in the range of [0,1]. The Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) is one of the most popular classical methods for solving MCGDM problems. TOPSIS has been fitted to solve MCGDM problems in fuzzy environment Chen (2000). A fuzzy TOPSIS approach to deal with the fuzzy multi-attribute decision making (MADM) problems under a group of decision makers was presented by Wu (2007). Madi (2011) applied fuzzy TOPSIS in the application of selecting investment boards on bursa Malaysia under a set of criteria. Saghafian (2005) introduced a modified fuzzy TOPSIS procedure with a new distance measure for solving MCGDM problems. However, the fuzzy set has demonstrated a good performance in handling data

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uncertainty by using membership function, they failed to handle non-membership degree and indeterminacy membership degree.

Intuitionistic fuzzy set (IFs) introduced by Atanassov (1986) is an extension of fuzzy set in order to overcome the lack of knowledge about the non-membership degree. It is characterized by a membership degree and a non-membership degree functions. TOPSIS has been extended to solve MCGDM problems with intuitionistic fuzzy data. Pramanik (2011) studied the teacher selection in intuitionistic fuzzy environment. Intuitionistic fuzzy TOPSIS has been used for the employee performance appraisal by Yinghui (2015). However, the intuitionistic fuzzy set can handle the membership degree and the non-membership degree, it can't handle problems involving indeterminate and inconsistent information.

Neutrosophic set (NS) first introduced by Smarandache (1999) in order to handle the problems with indeterminate and inconsistent information. NS is a generalization of crisp sets, fuzzy sets, intuitionistic fuzzy sets, interval-valued fuzzy sets, and interval-valued intuitionistic fuzzy sets. NS is characterized by three membership functions, Truth membership function (T), Indeterminacy membership function (I), and Falsity membership function (F). NS is difficult to apply in real problems, so the single-valued neutrosophic set was introduced by Wang (2010) to be applied to real scientific and engineering situations. Biswas (2016) extended TOPSIS method to solve multi-attribute group decision making problem under SVN environment.

MCGDM problems can include more than one participant, each of them needs to take the best decision, so competition may appear between these participants. Game theory is a powerful tool which used to handle the competition situations between two or more participants. A game is a formal description of a strategic situation. Game theory is the formal study of decision making where several players must make choices that potentially affect the interests of the other players (Turocy 2001). Fuzzy TOPSIS has been used with game theory in order to solve a MCGDM problem and applied with a numerical example (Aplak 2013). Game theory was used with a MCDM framework (SWARA) for personnel selection problem (Zolfani 2014).

The objective of this paper is to introduce a proposed technique for solving MCGDM problem with competition between the participants. These problems are being represented as a game matrix using the game theory principles. SVNs with TOPSIS will be used in order to handle the data uncertainty and formulate the payoff matrix of the game.

The remainder of this paper is organized as follows: Section. 2 briefly introduces some basic preliminaries related with neutrosophic sets. In Section. 3 TOPSIS and neutrosophic TOPSIS methods are discussed. Principles of game theory are included in Section. 4. The proposed approach for solving MCGDM problem with uncertainty and competition is introduced in Section. 5. A numerical example is illustrated in Section.6. Finally, conclusions and future work are pointed out at the end of this paper.

2. Preliminaries:

In this section, a brief review of some definitions has been introduced to be used in this paper.

2.1. Neutrosophic Set:

Neutrosophic set is a part of neutrosophy, which studies the origin, nature, and scope of neutralities as well as their interactions with different ideational spectra. Smarandache (1999) gave the following definitions of a neutrosophic set:

Definition 1:

Let X be a universal space of points (objects), with a generic element in X denoted by x. A neutrosophic set $NS \subset X$ is characterized by three membership functions, a truth-membership function $T_{NS}(x)$, an indeterminacy-membership function $I_{NS}(x)$, and a falsity-membership function $F_{NS}(x)$. $T_{NS}(x)$, $I_{NS}(x)$ and $F_{NS}(x)$ are real standard or nonstandard subsets of $]^-0,1^+[$, so that

$$T_{NS} : X \to \left]^-0,1^+\right[$$
$$I_{NS} : X \to \left]^-0,1^+\right[$$
$$F_{NS} : X \to \left]^-0,1^+\right[. \quad (1)$$

There is no restriction on the sum of the three neutrosophic membership components, so

$$^-0 \le \sup T_{NS}(x) + \sup I_{NS}(x) + \text{sub } F_{NS}(x) \le 3 . \quad (2)$$

2.2. Single Valued Neutrosophic Set (SVNs):

The single valued neutrosophic set is a special case or a subclass of neutrosophic set. SVNs was introduced to overcome the difficulties of using neutrosophic set, as neutrosophic set (NS) is difficult to apply directly in real engineering and scientific applications. Some basic definitions of SVNs are given by Wang (2010) as follows:

Definition 2:

Wang (2010) Let X be a universal space of points (objects), with a generic element in X denoted by x. A single valued neutrosophic set $SVN \subset X$ is characterized by three membership functions, a truth-membership function $T_{SVN}(x)$, an indeterminacy-membership function $I_{SVN}(x)$,and a falsity-membership function $F_{SVN}(x)$, such that $T_{SVN}(x), I_{SVN}(x), F_{SVN}(x) \in [0,1]$ for all $x \in X$. Therefore, SVNs can be written in the form $SVN = \{< x, T_{SVN}(x), I_{SVN}(x), F_{SVN}(x) >: x \in X\}$.

The sum of the three-single valued neutrosophic membership functions is

$$0 \le T_{NS}(x) + I_{NS}(x) + F(x) \le 3 \text{ for all } x \in X. \quad (3)$$

Definition 3:

Wang (2010) Let A, B be two SVNs, then the summation of two SVNs A, B is defined as:

$$A \oplus B = \left\langle \left(T_A(x) + T_B(x) - T_A(x) \cdot T_B(x)\right), \left(I_A(x) \cdot I_B(x)\right), \left(F_A(x) \cdot F_B(x)\right) \right\rangle. \quad (4)$$

Definition 4:

Wang (2010) Let A, B be two SVNs, then the multiplication of two SVNs A, B is defined as:

$$A \otimes B = \left\langle \left(T_A(x) \cdot T_B(x)\right), \left(I_A(x) + I_B(x) - I_A(x) \cdot I_B(x)\right), \left(F_A(x) + F_B(x) - F_A(x) \cdot F_B(x)\right) \right\rangle. \quad (5)$$

Definition 5:

Euclidean distance is used to measure the separation distance between any two SVNs. Let A, B be two SVNs for $x_i \in X(i = 1, 2, ..., n)$, then the Euclidean distance can be defined as:

$$D_{Euclidean} = \sqrt{\frac{1}{3n} \sum_{i=1}^{n} \left\langle \left(T_A(x) - T_B(x)\right)^2 + \left(I_A(x) - I_B(x)\right)^2 + \left(F_A(x) - F_B(x)\right)^2 \right\rangle}. \quad (6)$$

3. TOPSIS Method:

TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) was first introduced by Hwang and Yoon (1981) to be used in solving Multi-Criteria Decision Making (MCDM) problems. It finds the best alternative based on the concept that the most suitable solution is the one that has the shortest distance from the Positive Ideal Solution (PIS), and the longest distance from the Negative Ideal Solution (NIS). PIS and NIS are calculated according to the Euclidean distance measure. At the last step of TOPSIS, closeness coefficients of each alternative are calculated to determine the ranking order of all alternatives.

3.1 Fuzzy TOPSIS:

Crisp numbers are inadequate to represent the real-case problems so fuzzy data was applied. Fuzzy TOPSIS is a classical method for solving MCGDM problems. It is applied to the problems that have data uncertainty. Fuzzy TOPSIS is based on fuzzy data which means that it can only handle the problems that can be represented by the truth membership degree. It fails to represent the problems that contain the false membership degree as well as the indeterminate membership degree, so neutrosophic TOPSIS started to be applied. The steps of fuzzy TOPSIS summarized by Aplak (2013).

(1) Determination of the weights of each criterion.
(2) Construction of the aggregated fuzzy decision matrix based on decision makers' assessments.
(3) Normalization of the fuzzy decision matrix.
(4) Construction of the weighted normalized fuzzy decision matrix.
(5) Calculation of the fuzzy positive-ideal solution (FPIS) and the fuzzy negative-ideal solution (FNIS).
(6) Calculation of the distance measure of each alternative from FPIS and FNIS.
(7) Calculation of the closeness coefficients.
(8) Ranking the alternatives.

3.2. Single Valued Neutrosophic TOPSIS:

The concept of TOPSIS was extended to be used in solving MCDM problems with the neutrosophic environment as all the information is in the form of SVNs. Single valued neutrosophic TOPSIS is applied instead of fuzzy TOPSIS as it can be used to solve the problems which contain truth, false, and indeterminate information. It depends on finding the best alternative which is nearest to the single-valued neutrosophic positive ideal solution (SVN-PIS) and farthest from the single-valued neutrosophic negative ideal solution (SVN-NIS). The steps of Neutrosophic TOPSIS as summarized by Biswas (2016) are included in the proposed technique in section.5.

4. Game Theory:

Game theory is developed to study decision making in conflict and sometimes cooperation situations, in which each player must make decisions that affect the interests of other players Turocy (2001). Game theory is a useful tool for representing the competition situations of MCDM problems. Any finite game must have a set of players, a set of strategies for each player, and the payoff of each strategy. The solution of the game is a set of strategies, one for each player, which is called the Nash equilibrium such that no player has the incentive to change his or her strategy given what the other players are doing. Best response method is used to find the Nash Equilibrium, but if there is no equilibrium, the players must play using mixed strategies with probabilities. A Nash equilibrium of a strategic game is a profile of strategies $\left(s_1^*,...,s_2^*\right)$, where $s_i^* \in S_i$ (S_i is the strategy set of player *i*), such that for each player *i*, $\forall s_i \in S_i$, $u_i(s_i^*, s_{-i}^*) \geq u_i(s_i, s_{-i}^*)$ Nash (1951).

The proposed technique opens the door of utilizing neutrosophic sets in conjunction with game theory principles in solving competitive MCGDM problems under uncertainty conditions. Instead of ranking the alternatives using the neutrosophic TOPSIS, a payoff matrix is formulated to solve the problem. After calculating the closeness coefficients of each alternative from the SVN-PIS and the SVN-NIS, they are used to formulate the payoff matrix. After that, the best response method is used to find the optimal solution of the problem.

5. Proposed approach for solving MCGDM problem with uncertainty and competition:

The proposed approach is formed to solve the MCGDM problem with uncertainty information, which has two participants and a competition between them and represent it as a two player non-constant sum game. This approach uses single valued neutrosophic sets to represent data uncertainty. Neutrosophic TOPSIS is applied for obtaining neutrosophic values to the payoff matrix. Finally, the game theory principles are applied to find the best combination of strategies by finding the Nash equilibrium using the best response method.
The main steps of the proposed approach are discussed in the following subsections.

(1) Determination of the Criteria Used in The Evaluation of the Problem:

Firstly, decision makers analyze the situation in order to determine the suitable criteria that can be used in order to evaluate the problem.

(2) Determination of Strategies of Each Player:

In this step, after determining the used criteria, decision making group intends to determine the strategies used by each player.

(3) Determination of Linguistic Scales Used for Evaluation of Criteria and Strategies:

Three linguistic scales expressed in SVNs are formed. The first scale is used for the rating of the decision makers. The second scale is used for the criteria evaluation. The third scale is used for the evaluation of strategies of each player.

(4) Solving TOPSIS for Formulating the Payoff Matrix Instead of Ranking the Alternatives:

let $K = \{K_1, K_2, \ldots, K_P\}$ be a group of decision makers
$A = \{A_1, A_2, \ldots, A_m\}$ be a set of alternatives
$C = \{C_1, C_2, \ldots, C_n\}$ be a set of criteria
$D = \{d_{ij}\}, \quad i = 1, 2, \ldots, m \quad , \quad j = 1, 2, \ldots, n$ be the performance ratings of alternatives with respect to criteria

The following 8 steps summarize the solving of TOPSIS for formulating the payoff matrix.
Determination of The Weights of the Decision Makers:

Each one in the group of decision makers has his own decisions and preferences, so the importance of decision makers' opinions may not be equal. The importance of each decision maker is defined as a linguistic term and expressed by SVNs. The weight of K_{th} decision maker is:

$$\lambda_K = \frac{1-\sqrt{\frac{1}{3}\left[(1-T_K)^2 + I_K^2 + F_K^2\right]}}{\sum_{K=1}^{P}\left(1-\sqrt{\frac{1}{3}\left[(1-T_K)^2 + I_K^2 + F_K^2\right]}\right)} \quad (7)$$

where, $\langle T_k, I_k, F_k \rangle$ a neutrosophic number defined for the rating of Kth decision makers.

Determination of The Weights of the Criteria:

There is a different weight for each criterion used in the decision process due to its importance to the decision makers. Criteria weight w_j can be calculated by formula:

$$w_j = \left\langle 1-\prod_{K=1}^{P}(1-T_j)^{\lambda_K}, \prod_{K=1}^{P}(I_j)^{\lambda_K}, \prod_{K=1}^{P}(F_j)^{\lambda_K} \right\rangle \quad (8)$$

where, $w_j = \langle T_j, I_j, F_j \rangle$ for $j = 1, 2, \cdots, n$ is a SVN number represent the criteria

Construction of The Aggregated Single-Valued Neutrosophic (SVN) Decision Matrix Based on Decision Makers' Assessments:

The assessments of the decision-making group are gathered to construct the neutrosophic decision matrix by using the single valued neutrosophic weighted averaging (SVNWA) operator as follows:

$$d_{ij} = \left\langle 1-\prod_{K=1}^{P}(1-T_{ij})^{\lambda_K}, \prod_{K=1}^{P}(I_{ij})^{\lambda_K}, \prod_{K=1}^{P}(F_{ij})^{\lambda_K} \right\rangle \quad (9)$$

The single valued neutrosophic decision matrix D can be expressed as:

$$D = (d_{ij})_{m\times n} = \begin{array}{c} \\ A_1 \\ A_2 \\ \cdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \begin{pmatrix} \langle T_{11}, I_{11}, F_{11} \rangle & \langle T_{12}, I_{12}, F_{12} \rangle & \cdots & \langle T_{1n}, I_{1n}, F_{1n} \rangle \\ \langle T_{21}, I_{21}, F_{21} \rangle & \langle T_{22}, I_{22}, F_{22} \rangle & \cdots & \langle T_{2n}, I_{2n}, F_{2n} \rangle \\ \cdots & \cdots & \cdots & \cdots \\ \langle T_{m1}, I_{m1}, F_{m1} \rangle & \langle T_{m2}, I_{m2}, F_{m2} \rangle & \cdots & \langle T_{mn}, I_{mn}, F_{mn} \rangle \end{pmatrix} \end{array}$$

where, $d_{ij} = \langle T_{ij}, I_{ij}, F_{ij} \rangle$ is the aggregated element of neutrosophic decision matrix D for $i = 1, 2, \cdots, m$ and $j = 1, 2, \cdots, n$, T_{ij}, I_{ij}, F_{ij} represent the degree of truth, indeterminacy, and falsity membership values.

Normalization of The SVN Decision Matrix:

The normalized decision matrix d_{ij}^N is constructed as:

$$d_{ij}^N = \frac{d_{ij} - d_j^-}{d_j^+ - d_j^-} \text{ for benefit criteria (larger the better)} \quad (10)$$

$$d_{ij}^N = \frac{d_j^- - d_{ij}}{d_j^- - d_j^+} \text{ for cost criteria (smaller the better)} \quad (11)$$

where $d_j^+ = \max_i(d_{ij})$ and $d_j^- = \min_i(d_{ij})$

Construction of The Weighted Normalized SVN Decision Matrix:

The weighted normalized decision matrix d_{ij}^* is constructed by the multiplication of the criteria weight with the normalized decision matrix.

$$d_{ij}^* = w_j \otimes d_{ij}^N = \langle d_{ij}^{wj} \rangle_{m\times n} = \langle T_{ij}^{wj}, I_{ij}^{wj}, F_{ij}^{wj} \rangle_{m\times n}$$

$$d_{ij}^* = \begin{array}{c} \\ A_1 \\ A_2 \\ \cdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \begin{pmatrix} \langle T_{11}^{w1}, I_{11}^{w1}, F_{11}^{w1} \rangle & \langle T_{12}^{w2}, T_{11}^{w2}, T_{11}^{w2} \rangle & \cdots & \langle T_{1n}^{wn}, I_{1n}^{wn}, F_{1n}^{wn} \rangle \\ \langle T_{21}^{w1}, I_{21}^{w1}, F_{21}^{w1} \rangle & \langle T_{22}^{w2}, T_{22}^{w2}, T_{22}^{w2} \rangle & \cdots & \langle T_{2n}^{wn}, I_{2n}^{wn}, F_{2n}^{wn} \rangle \\ \cdots & \cdots & \cdots & \cdots \\ \langle T_{m1}^{w1}, I_{m1}^{w1}, F_{m1}^{w1} \rangle & \langle T_{m2}^{w2}, T_{m2}^{w2}, T_{m2}^{w2} \rangle & \cdots & \langle T_{mn}^{wn}, I_{mn}^{wn}, F_{mn}^{wn} \rangle \end{pmatrix} \end{array} \quad (12)$$

Calculation of The Single Valued Neutrosophic Positive-Ideal Solution (SVN-PIS) and The Single Valued Neutrosophic Negative-Ideal Solution (SVN-NIS):

Criteria used in decision making process is divided into two categories. The first category G_1 is the collection of benefit criteria, and the second category G_2 is the collection of cost criteria. SVN-PIS (Q_N^+) and SVN-NIS (Q_N^-) can be defined as:

$$Q_N^{+} = \left[d_1^{w+}, d_2^{w+}, \ldots, d_n^{w+} \right] \quad (13)$$
where $d_j^{w+} = \left\langle T_j^{w+}, I_j^{w+}, F_j^{w+} \right\rangle$ for $j = 1, 2, \ldots, n$

$$T_j^{w+} = \left\{ \left(\max_i \left\{ T_{ij}^{wj} \right\} | j \in G_1 \right), \left(\min_i \left\{ T_{ij}^{wj} \right\} | j \in G_2 \right) \right\} \quad (14)$$

$$I_j^{w+} = \left\{ \left(\min_i \left\{ I_{ij}^{wj} \right\} | j \in G_1 \right), \left(\max_i \left\{ I_{ij}^{wj} \right\} | j \in G_2 \right) \right\} \quad (15)$$

$$F_j^{w+} = \left\{ \left(\min_i \left\{ F_{ij}^{wj} \right\} | j \in G_1 \right), \left(\max_i \left\{ F_{ij}^{wj} \right\} | j \in G_2 \right) \right\} \quad (16)$$

$$Q_N^{-} = \left[d_1^{w-}, d_2^{w-}, \ldots, d_n^{w-} \right] \quad (17)$$
where $d_j^{w-} = \left\langle T_j^{w-}, I_j^{w-}, F_j^{w-} \right\rangle$ for $j = 1, 2, \ldots, n$

$$T_j^{w-} = \left\{ \left(\min_i \left\{ T_{ij}^{wj} \right\} | j \in G_1 \right), \left(\max_i \left\{ T_{ij}^{wj} \right\} | j \in G_2 \right) \right\} \quad (18)$$

$$I_j^{w-} = \left\{ \left(\max_i \left\{ I_{ij}^{wj} \right\} | j \in G_1 \right), \left(\min_i \left\{ I_{ij}^{wj} \right\} | j \in G_2 \right) \right\} \quad (19)$$

$$F_j^{w-} = \left\{ \left(\max_i \left\{ F_{ij}^{wj} \right\} | j \in G_1 \right), \left(\min_i \left\{ F_{ij}^{wj} \right\} | j \in G_2 \right) \right\} \quad (20)$$

Calculation of The Distance Measure of Each Alternative from SVN-PIS and SVN-NIS:

Distance between any two alternatives is measured by using the Euclidean distance measure. The distance measure of each alternative from the SVN-PIS $\left(S_i^{+}\right)$ is measured by:

$$S_i^{+} = \sqrt{\frac{1}{3n} \sum_{j=1}^{n} \left(\left(T_{ij} - T_j^{w+} \right)^2 + \left(I_{ij} - I_j^{w+} \right)^2 + \left(F_{ij} - F_j^{w+} \right)^2 \right)} \quad (21)$$

The distance measure of each alternative from the SVN-PIS $\left(S_i^{-}\right)$ is measured by:

$$S_i^{-} = \sqrt{\frac{1}{3n} \sum_{j=1}^{n} \left(\left(T_{ij} - T_j^{w-} \right)^2 + \left(I_{ij} - I_j^{w-} \right)^2 + \left(F_{ij} - F_j^{w-} \right)^2 \right)} \quad (22)$$

Calculation of the closeness coefficient $\left(CC_i\right)$:

Finally, the closeness coefficient of each alternative is calculated to determine the ranking order of each alternative in descending manner.

$$CC_i = \frac{S_i^{-}}{S_i^{+} + S_i^{-}} \text{ where, } 0 \le CC_i \le 1 \quad (23)$$

(5) Construction of The Payoff Matrix:

The closeness coefficients extracted from the neutrosophic TOPSIS in the previous step are used in order to formulate the payoff matrix.

(6) Finding the Solution of the Game (Nash Equilibrium):

As a last step, the best response method is used to find the Nash equilibrium of the game which represents the optimal solution of the problem.

6. Numerical Example:

Suppose a group of six decision makers $\left(DM_1, DM_2, DM_3, DM_4, DM_5, DM_6\right)$ intended to solve a competition problem between two participants (players). Player A has three different strategies $\left(AS_1, AS_2, AS_3\right)$ to choose from, while player B also has three different strategies $\left(BS_1, BS_2, BS_3\right)$Three criteria $\left(C_1, C_2, C_3\right)$ have been suggested by the decision makers to be used for the evaluation process. According to the proposed approach, this problem is solved by the following steps:

Step 1: Determination of the weights of the decision makers.

The importance of a decision maker's opinion in the decision-making group is not equal to others, so this importance is expressed by linguistic terms shown in Table 1. The corresponding single valued neutrosophic numbers (SVNNs) to these linguistic terms are shown in Table 2. The weights of the decision makers are determined using Eq. (7). Thus, the weights of the six decision makers are (0.2734 0.2476 0.1665 0.1059 0.0401 0.1665).

Table 1: Importance of decision makers expressed with SVNs

	DM1	DM2	DM3	DM4	DM5	DM6
Linguistic term	VI	I	M	UI	VUI	M

Table 2: Linguistic Terms used for rating decision makers

Linguistic terms	SVNs
Very Important (VI)	(0.90,0.10,0.10)
Important (I)	(0.80,0.20,0.15)
Medium (M)	(0.50,0.40,0.45)
Unimportant (UI)	(0.35,0.60,0.70)
Very Unimportant (VUI)	(0.10,0.80,0.90)

Step 2: Determination of the weights of the criteria.
Table 3 shows the linguistic variables used for the evaluation of criteria importance expressed in SVNs. The importance of each criterion for every decision maker is shown Table 4. In order to calculate the combined weight of each criterion, Eq. (8) is used. The calculated criteria weights are shown in Table 5.

Table 3: Linguistic terms for rating criteria

Linguistic terms	SVNs
No Importance (N)	(0.20,0.75,0.80)
Very Low (VL)	(0.35,0.65,0.60)
Low (L)	(0.50,0.50,0.45)
Medium (M)	(0.65,0.35,0.30)
High (H)	(0.80,0.20,0.15)
Very High (VH)	(0.90.0.10,0.05)
Extreme Importance (E)	(1.00,0.00,0.00)

Table 4: Importance of criteria expressed in SVNs

	DM1	DM2	DM3	DM4	DM5	DM6
C1	E	VH	E	H	VH	H
C2	H	M	M	M	M	H
C3	L	M	L	VL	VL	L

Table 5: The weights of criteria

Criteria	Weight
C1	(1,0,0)
C2	(0.7264,0.2736,0.2212)
C3	(0.5244,0.4756,0.4245)

Step 3: Construction of the aggregated single-valued neutrosophic (SVN) decision matrix based on decision makers' assessments.
As we mentioned above, player A has three strategies (AS_1, AS_2, AS_3), while player B has three different strategies (BS_1, BS_2, BS_3). Strategies of each player are compared with the strategies of the other player reciprocally. The linguistic terms used in the evaluation of strategies are shown in Table 6.

Table 6: Linguistic Terms for Strategy evaluation expressed in SVNs

Linguistic terms	SVNs
Very Poor (VP)	(0.20,0.75,0.80)
Poor (P)	(0.35,0.65,0.60)
Medium Poor (MP)	(0.50,0.50,0.45)
Medium (M)	(0.65,0.35,0.30)
Medium Good (MG)	(0.80,0.20,0.15)
Good (G)	(0.90.0.10,0.05)
Very Good (VG)	(1.00,0.00,0.00)

Table 7 represents the opinions of all decision makers for the evaluation of player-A strategies with respect to the first strategy of player-B, (BS_1) according to the used criteria. To calculate the decision matrix shown in Table 8, Eq. (9) is used.

Step 4: Normalization of the SVN decision matrix
Using Eq. (10) and Eq. (11), the normalized decision matrix is constructed as shown in Table 9.

Table 7: AS's evaluation according to criteria with linguistic variables (BS1 case)

		DM1	DM2	DM3	DM4	DM5	DM6
C1	AS1	MP	VP	VP	VP	VP	MP
	AS2	P	VP	P	P	P	MP
	AS3	VP	P	MP	VP	VP	P
C2	AS1	VG	VG	MG	G	G	VG
	AS2	VG	MG	G	MG	MG	VG
	AS3	MP	VP	P	MP	MP	MP
C3	AS1	G	G	G	M	M	M
	AS2	VP	VP	VP	P	MP	MP
	AS3	MG	G	G	MG	VG	VG

Table 8: Single valued neutrosophic decision matrix (BS1 case)

	C1	C2	C3
AS1	(0.3494,0.6275,0.6211)	(1,0,0)	(0.8521,0.1479,0.0875)
AS2	(0.3450,0.6447,0.6142)	(1,0,0)	(0.2898,0.6794,0.6890)
AS3	(0.3212,0.6607,0.6453)	(0.4132,0.5775,0.5444)	(1,0,0)

Table 9: Normalized neutrosophic decision matrix

	C1	C2	C3
AS1	(0.0832,0.9022,0.8834)	(1,0,0)	(.8521,.1479,0.0875)
AS2	(0.0701,0.9527,0.8629)	(1,0,0)	(0.2898,0.6794,0.6890)
AS3	(0,1,0.9546)	(0.4132,0.5775,0.5444)	(1,0,0)

Step 5: Construction of the weighted normalized SVN decision matrix

The weighted normalized decision matrix is constructed using the values of Table 5 with the values in Table 9 by applying Eq. (12). The weighted normalized decision matrix is shown in Table 10.

Table 10: Weighted normalized decision matrix

	C1	C2	C3
AS1	(0.0832,0.9022,0.8834)	(0.7264,0.2736,0.2212)	(0.4468,0.5532,0.4749)
AS2	(0.0701,0.9537,0.8629)	(0.7264,0.2736,0.2212)	(0.1520,0.8319,0.8210)
AS3	(0,1,0.9546)	(0.3001,0.6931,0.6451)	(0.5244,0.4756,0.4245)

Step 6: Calculation of the single valued neutrosophic positive-ideal solution (SVN-PIS) and the single valued neutrosophic negative-ideal solution (SVN-NIS)

SVN-PIS and SVN-NIS are calculated from the weighted normalized decision matrix based on the criteria type (benefit or cost) by using Eq. (13-20). SVN-PIS and SVN-NIS are calculated as in Table 11.

Table 11: SVN-PIS & SVN-NIS

Criteria	SVN-PIS	SVN-NIS
C1	(0.0832,0.9022,0.8629)	(0,1,0.9546)
C2	(0.7264,0.2736,0.2212)	(0.3001,0.6931,0.6451)
C3	(0.5244,0.4756,0.4245)	(0.1520,0.8319,0.8210)

Step 7: Calculation of the distance measure of each alternative from SVN-PIS and SVN-NIS.

Distance measures of each alternative from SVN-PIS and SVN-NIS are calculated using Eq. (21) and Eq. (22). They appear as in Table 12.

Table 12: Distance measures from SVN-PIS &SVN-NIS

Strategy	Distance SVN-PIS	Distance SVN-PIS
AS1	0.0707	0.5302
AS2	0.3766	0.4293
AS3	0.4329	0.3754

Step 8: Calculation of the closeness coefficient (CC_i)

The final step in the TOPSIS algorithm is to calculate the closeness coefficients according to Eq. (23). The results shown in Table 13 will be used to construct the payoff matrix.

Table 13: Closeness Coefficients

Strategy	CC
AS1	0.8824
AS2	0.5327
AS3	0.4644

Step 9: Construction of the payoff matrix
The payoff matrix is constructed using the values of closeness coefficients as in Table 14.

Table 14: Payoff matrix for the game

	BS1	BS2	BS3
AS1	(0.8824,0.6194)	(1.0000,0.5459)	(0.6434,0.4385)
AS2	(0.5327,0.5768)	(0.5726,0.5618)	(0.3350,0.3849)
AS3	(0.4644,0.6104)	(0.0218,0.9425)	(0.5824,0.0034)

Step 10: Finding the solution of the game
As the last step, the Nash equilibrium is found by using the best response method. The optimal strategies are $(AS1, BS1)$ with the payoff values (0.8824,0.6194).

Conclusions:

This paper introduced a proposed technique for solving the multi-criteria group decision making (MCGDM) problems under uncertainty environment. These problems have a competition between their participants. In the evaluation process, the importance of decision makers, criteria weights, and strategy evaluation was provided as linguistic variables expressed in single valued neutrosophic sets (SVNs). TOPSIS method based on the neutrosophic environment was used to solve the MCGDM problem, but instead of ranking the alternatives, closeness coefficients used to construct the payoff matrix. Finally, game theory principles were applied to solve the game matrix in order to find the Nash equilibrium, which represents the optimal strategies for each player. This was applied to a two-player non-constant game matrix.

This paper demonstrated the possibility of using game theory principles with TOPSIS to solve the MCGDM problems under neutrosophic environment (two-player non-constant sum game).

Applying the proposed approach in the n-player non-constant sum game matrices will be the future work of this paper.

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