Compromise Solutions for Rough Multiple Objective Decision Making Problems

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Abstract

The technique for order preference by similarity to ideal solution (TOPSIS) is introduced to develop a methodology to find compromise solutions for the Multiple Objective Decision Making (MODM) Problems with Rough intervals parameters in the objective functions (RMODM) of Mixed-type. An new algorithm is presented for the proposed TOPSIS approach for solving these types of mathematical programming problems. Also, an illustrative numerical example is solved and compared the solution of proposed algorithm with the ideal solutions.

Key words: Compromise Programming, Rough Programming, TOPSIS method, Multiple Objective Programming, Rough Intervals, and Rough Parameters.

INTRODUCTION

Compromise programming (CP) was initially proposed by Zeleny (1973) and subsequently used by many researchers. (Zeleny, M., 1973). Yu (1973) and Zeleny (1974) define the ideal solution as any solution that would simultaneously optimize each individual objective, (Yu, P.L., 1973; Zeleny, M., 1974).

Rough set theory is considered from the excellent mathematical tool for dealing with the description of vague objects. Rough set methodology has been introduced as new handling of analysis of vague concepts classificatory by Pawlak (1991). Any Vague concept is introduced by pair of precise concepts called “Lower & Upper” approximations, (Hamzhee, A., 2014; Osman, M.S., 2011; Youness, E., 2006).

Linear optimization problem is considered where some or all of its coefficients in the objective function and/or constraints are rough intervals introduced by Hamzhee et al. (2014).

Several algorithms for solving different kinds of large scale multiple objective optimization problems using TOPSIS approach are present in (Abou-El-Enien, T.H.M., 2013). TOPSIS method assumes that any DM seeks a solution which has the shortest distance from positive ideal solution (PIS), and the farthest distance from the negative ideal solution (NIS), (Lai, Y.J., 1994).

In the following sections, the formulation of RMODM problems is given in section (2). Also, transformation of RMODM problem into deterministic RMODM problems (by use of TOPSIS method and “Lower & Upper” approximations method) is introduced in section (2). An new algorithm for solving deterministic RMODM problems is proposed in section (3). For the sake of illustration, we present an example for the extended TOPSIS method and compared the solution of proposed algorithm with the ideal solutions in section (4).

Formulation of the problem:

Consider the following Linear Multiple Objective Decision Making (LMODM) Problem, (Hwang, C.L., and A.S.M. Masud, 1979; Yu, P.L., 1985; Zeleny, M., 1982), with rough parameters in the objective functions [RLMODM]:

Maximize/Minimze \( f_i(X_1, X_2, ..., X_n), ..., f_k(X_1, X_2, ..., X_n) \)

subject to

\( X \in M = \{X \in \mathbb{R}^n : DX \leq b \} \)

where

\[ f_i = \sum_{j=1}^{n} c_{ij} \left[ \left[ Q_{ij}^{hi} \right] \left[ Q_{ij}^{rl} \right] \right] Q_{ij}^{rl} x_j, \ i = 1, 2, ..., k, \]

\[ m : \text{the number of constraints}, \]

\[ n : \text{the number of variables}, \]

\[ k : \text{the number of objective functions}, \]

\( c_{ij} : \text{real constants coefficients of the objective functions,} (i = 1, 2, ..., k, j = 1, ..., n). \]

\( b: \text{non-dimensional column vector of right-hand sides of constraints} \)

\( D: \text{an (m \times n) coefficient matrix} \)

\( \mathbb{R} : \text{the set of all real numbers} \)

\( X : \text{an n-dimensional column vector of variables} \)

\( N = \{1, 2, ..., n\} \)

\( \mathbb{R}^n = \{X = (x_1, x_2, ..., x_n) \} \), \( x_i \in \mathbb{R}, i \in N \),

\[ Q^{hi}_{ij}, Q^{rl}_{ij}, Q^{hi}_{ij}, Q^{rl}_{ij} \text{are rough interval coefficients of the objective functions,} (i = 1, 2, ..., k, j = 1, ..., n). \]

Using the upper and lower approximation method, (Hamzhee, A., 2014), the multiple objective decision making problems with Rough parameters in the

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objective functions (RMODM) can be transformed to the following four deterministic LMODM problems:

**Problem (3):**

\[
\text{Maximize } 
\begin{align*}
\sum_{j=1}^{l} c_j I_1^j x_j, & \\
\text{subject to } & \\
X \in \mathbb{M}, & \\
\end{align*}
\]

**Problem (4):**

\[
\text{Maximize } 
\begin{align*}
\sum_{j=1}^{l} c_j I_2^j x_j, & \\
\text{subject to } & \\
X \in \mathbb{M}, & \\
\end{align*}
\]

**Problem (5):**

\[
\text{Maximize } 
\begin{align*}
\sum_{j=1}^{l} c_j I_3^j x_j, & \\
\text{subject to } & \\
X \in \mathbb{M}, & \\
\end{align*}
\]

**Problem (6):**

\[
\text{Maximize } 
\begin{align*}
\sum_{j=1}^{l} c_j I_4^j x_j, & \\
\text{subject to } & \\
X \in \mathbb{M}, & \\
\end{align*}
\]

**Algorithm (I):**

Use the "Lower & Upper" approximations method to transform the RLMODM Problem (1) into the four deterministic LMODM problems (3)-(6).

**Step 1:**

Find the PIS (\(F_t^{L_1}\)), PIS (\(F_t^{U_1}\)), PIS (\(F_t^{L_2}\)), PIS (\(F_t^{U_2}\)), NIS (\(F_v^{L_3}\)), NIS (\(F_v^{U_3}\)) and NIS (\(F_v^{L_4}\)) which are (Abou-El-Enien, T.H.M., 2013; Lai, Y.J., 1994):

**Step 2:**

Construct PIS and NIS payoff tables for the four deterministic LMODM problems (3)-(6).

**Step 3:**

Let \(w_i = w_i^*, \ i = 1, 2, ..., k\), where \(\sum_{i=1}^k w_i = 1\) and \(p^*, p^* \in [1, 2, ..., \infty]\).

**Step 4:**


**II:**

Construct distance functions for problem (4) as following:

**II:**

Construct distance functions for problem (5) as following:

**III:**

Construct distance functions for problem (6) as following:
IV- Construct distance functions $d_p^{PIS}$ and $d_p^{NIS}$ for problem (6) as following:

$$d_p^{PIS} = \left( \sum_{c=1}^{m} \left( \frac{e_p^{H(X,Y)}}{\varepsilon_{c}^{H(X,Y)}} \right)^p \right)^{1/p}$$

and

$$d_p^{NIS} = \left( \sum_{c=1}^{m} \left( \frac{e_p^{H(X,Y)}}{\varepsilon_{c}^{H(X,Y)}} \right)^p \right)^{1/p}$$

Step 5:
I- Construct the following bi-objective problem with two commensurable (but conflicting) objectives, (Abou-El-Enien, T.H.M., 2013; Lai, Y.J., 1994), using the distance functions $d_p^{PIS}$ and $d_p^{NIS}$:

Minimize $d_p^{PIS}(X)$
Maximize $d_p^{NIS}(X)$
subject to $X \in \mathbb{M}$

where $p = 1, 2, \ldots, \infty$.

II- Construct the following bi-objective problem with two commensurable (but conflicting) objectives using the distance functions $d_p^{PIS}$ and $d_p^{NIS}$:

Minimize $d_p^{PIS}(X)$
Maximize $d_p^{NIS}(X)$
subject to $X \in \mathbb{M}$

where $p = 1, 2, \ldots, \infty$.

III- Construct the following bi-objective problem with two commensurable (but conflicting) objectives using the distance functions $d_p^{PIS}$ and $d_p^{NIS}$:

Minimize $d_p^{PIS}(X)$
Maximize $d_p^{NIS}(X)$
subject to $X \in \mathbb{M}$

where $p = 1, 2, \ldots, \infty$.

IV- Construct the following bi-objective problem with two commensurable (but conflicting) objectives using the distance functions $d_p^{PIS}$ and $d_p^{NIS}$:

Minimize $d_p^{PIS}(X)$
Maximize $d_p^{NIS}(X)$
subject to $X \in \mathbb{M}$

where $p = 1, 2, \ldots, \infty$.

Step 6:
I- Construct PIS Payoff table for problem (11):
At $p = 1$, use the simplex method or the interior point method,
At $p \geq 2$, use the generalized reduced gradient method, and obtain

$$d_p^{UL} = \left( (d_p^{PIS})^*, (d_p^{NIS})^* \right), \quad d_p^{LR} = \left( (d_p^{PIS})^*, (d_p^{NIS})^* \right)$$

where

$$\left( d_p^{PIS} \right)^* = \frac{\text{Minimize}}{X \in \mathbb{M}} d_p^{PIS}(X) \quad \text{and the solution is} \ X^{PIS}_{opt}\)$$

$$\left( d_p^{NIS} \right)^* = \frac{\text{Maximize}}{X \in \mathbb{M}} d_p^{NIS}(X) \quad \text{and the solution is} \ X^{NIS}_{opt}\)$$

II- Construct PIS Payoff table for problem (12):
At $p = 1$, use the simplex method or the interior point method,
At $p \geq 2$, use the generalized reduced gradient method, and obtain

$$d_p^{UL} = \left( (d_p^{PIS})^*, (d_p^{NIS})^* \right), \quad d_p^{LR} = \left( (d_p^{PIS})^*, (d_p^{NIS})^* \right)$$

III- Construct PIS Payoff table for problem (13):
At $p = 1$, use the simplex method or the interior point method,
At $p \geq 2$, use the generalized reduced gradient method, and obtain

$$d_p^{UL} = \left( (d_p^{PIS})^*, (d_p^{NIS})^* \right), \quad d_p^{LR} = \left( (d_p^{PIS})^*, (d_p^{NIS})^* \right)$$

IV- Construct PIS Payoff table for problem (14):
At $p = 1$, use the simplex method or the interior point method,
At $p \geq 2$, use the generalized reduced gradient method, and obtain

$$d_p^{UL} = \left( (d_p^{PIS})^*, (d_p^{NIS})^* \right), \quad d_p^{LR} = \left( (d_p^{PIS})^*, (d_p^{NIS})^* \right)$$

Step 7:

Maximize $\delta^{UL}$
subject to
where \( \delta^{\text{LL}} \) is the satisfactory level for both criteria of the shortest distance from the PIS and the farthest distance from the NIS.

II. Construct the following satisfactory level model (for finite value of \( p \)) for problem (12):

Maximize \( \delta^{\text{LL}} \), subject to

\[
\left( \frac{d_{p}^{\text{L}}(X) - d_{p}^{\text{H}}(X)}{d_{p}^{\text{H}}(X)} \right) \geq \delta^{\text{LL}},
\]

\[
\left( \frac{d_{p}^{\text{H}}(X) - d_{p}^{\text{L}}(X)}{d_{p}^{\text{L}}(X)} \right) \geq \delta^{\text{LL}},
\]

\( X \in \mathbb{M}, \quad \delta^{\text{LL}} \in [0,1] \)

where \( \delta^{\text{LL}} \) is the satisfactory level for both criteria of the shortest distance from the PIS and the farthest distance from the NIS.

III. Construct the following satisfactory level model (for finite value of \( p \)) for problem (13):

Maximize \( \delta^{\text{LH}} \), subject to

\[
\left( \frac{d_{p}^{\text{L}}(X) - d_{p}^{\text{H}}(X)}{d_{p}^{\text{H}}(X)} \right) \geq \delta^{\text{LH}},
\]

\[
\left( \frac{d_{p}^{\text{H}}(X) - d_{p}^{\text{L}}(X)}{d_{p}^{\text{L}}(X)} \right) \geq \delta^{\text{LH}},
\]

\( X \in \mathbb{M}, \quad \delta^{\text{LH}} \in [0,1] \)

where \( \delta^{\text{LH}} \) is the satisfactory level for both criteria of the shortest distance from the PIS and the farthest distance from the NIS.

IV. Construct the following satisfactory level model (for finite value of \( p \)) for problem (14):

Maximize \( \delta^{\text{HH}} \), subject to

\[
\left( \frac{d_{p}^{\text{L}}(X) - d_{p}^{\text{H}}(X)}{d_{p}^{\text{H}}(X)} \right) \geq \delta^{\text{HH}},
\]

\[
\left( \frac{d_{p}^{\text{H}}(X) - d_{p}^{\text{L}}(X)}{d_{p}^{\text{L}}(X)} \right) \geq \delta^{\text{HH}},
\]

\( X \in \mathbb{M}, \quad \delta^{\text{HH}} \in [0,1] \)

where \( \delta^{\text{HH}} \) is the satisfactory level for both criteria of the shortest distance from the PIS and the farthest distance from the NIS.

Step 8:

If the solution of problems (15)-(18) yield optimal solutions \( \delta^{\text{LH}}, \delta^{\text{LH}}, \delta^{\text{HH}}, \delta^{\text{HH}} \) and \( \delta^{\text{HH}} \), then \( X^{\text{LH}}, X^{\text{LH}}, X^{\text{LH}}, X^{\text{LH}} \) and \( X^{\text{LH}} \) are non-dominated solutions of problems (3)-(6) and compromise solutions of problem (1), then go to step (10). Otherwise go to step (9).

Step 9:

Go to step (3).

Step 10:

Stop.

Illustrative Numerical Example for the Proposed Algorithm (I):

The proposed algorithm (I) is used to solve the following RLMODM problem:

Maximize \( f_1(X) = 7([2,3],[1,5])x_1 + 4([2,5],[2,7]) x_2 \)

Minimize \( f_2(X) = 5([2,3],[1,4])x_1 + 11([3,4],[2,9]) x_2 \)

subject to

\( X \in \mathbb{M} = \{3x_1 + 5x_2 \leq 35, 2x_1 - x_2 \leq 20, 5x_2 \leq 16, x_1 \geq 1, x_1, x_2 \geq 0\} \)

Solution:

\( p^{\text{LL}} \):

Maximize \( F^{\text{LL}}_1 = 14x_1 + 8x_2 \)

Minimize \( F^{\text{LL}}_2 = 10x_1 + 33x_2 \)

subject to

\( X \in \mathbb{M} \)

\( p^{\text{LH}} \):

Maximize \( F^{\text{LH}}_1 = 21x_1 + 20x_2 \)

Minimize \( F^{\text{LH}}_2 = 15x_1 + 44x_2 \)

subject to

\( X \in \mathbb{M} \)

\( p^{\text{HH}} \):

Maximize \( F^{\text{HH}}_1 = 7x_1 + 8x_2 \)

Minimize \( F^{\text{HH}}_2 = 5x_1 + 22x_2 \)

subject to

\( X \in \mathbb{M} \)

\( p^{\text{HH}} \):

Maximize \( F^{\text{HH}}_1 = 35x_1 + 28x_2 \)

Minimize \( F^{\text{HH}}_2 = 20x_1 + 99x_2 \)

subject to

\( X \in \mathbb{M} \)

Obtain PIS and NIS payoff tables for problem \( p^{\text{LL}} \).
Table 1: PIS payoff table for problem $P^L_i$:

<table>
<thead>
<tr>
<th>$F_i^L(x)$</th>
<th>$F_j^L(x)$</th>
<th>$x_1$</th>
<th>$x_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>151.53855</td>
<td>129.2308164</td>
<td>10.38462</td>
<td>0.7692308</td>
</tr>
</tbody>
</table>

PIS: $F^L = (151.5385, 10)$

Table 2: NIS payoff table for problem $P^L_i$:

<table>
<thead>
<tr>
<th>$F_i^L(x)$</th>
<th>$F_j^L(x)$</th>
<th>$x_1$</th>
<th>$x_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>10</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

NIS: $F^L = (14, 168.9333)$

Next, construct equation and obtain the following equations:

$$d_{P^L_i}^o = \left[ \frac{F_i^L(x) - F_{i^L}(x)}{151.5385 - 14} \right] + \left[ \frac{F_j^L(x) - F_{j^L}(x)}{129.2308164 - 10} \right]^{1/p}$$

$$d_{N^L_i}^o = \left[ \frac{F_i^L(x) - F_{i^L}(x)}{14} \right] + \left[ \frac{F_j^L(x) - F_{j^L}(x)}{10} \right]^{1/p}$$

Thus, problem (11) is obtained. In order to get numerical solutions, assume that $w_{1^L} = w_{2^L} = 0.5$ and $p = 2$.

Table 3: PIS payoff table of problem (11) when $p=2$.

<table>
<thead>
<tr>
<th>$d_{P^L_i}^{o^{11}}$</th>
<th>$d_{N^L_i}^{o^{11}}$</th>
<th>$F_i^{11}(x)$</th>
<th>$F_j^{11}(x)$</th>
<th>$x_1$</th>
<th>$x_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>75.21841</td>
<td>0.1249037754</td>
<td>151.5385264</td>
<td>129.2305616</td>
<td>10.38462</td>
<td>0.7692308</td>
</tr>
</tbody>
</table>

NIS: $F^L = (14, 168.9333)$

Now, it is easy to compute step (10):

Maximize $d_{P^L_i}^{o^{11}}$

Subject to $x_1, x_2 \geq 35, 2x_1 - x_2 \leq 20, 5x_2 \leq 16, x_1, x_2 \geq 1, x_1, x_2 \geq 0$

The maximum "satisfactory level" ($\delta^{11} = 0.3960653$) is achieved for the solution $x_1^{11} = 1, x_2^{11} = 1.930737$.

Obtain PIS and NIS payoff tables for problem $P^H_i$:

Table 4: PIS payoff table for problem $P^H_i$:

<table>
<thead>
<tr>
<th>$F_i^{H}(x)$</th>
<th>$F_j^{H}(x)$</th>
<th>$x_1$</th>
<th>$x_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>233.4615</td>
<td>189.6154552</td>
<td>10.38462</td>
<td>0.7692308</td>
</tr>
</tbody>
</table>

PIS: $F^H = (233.4615, 15)$

Table 5: NIS payoff table for problem $P^H_i$:

<table>
<thead>
<tr>
<th>$F_i^{H}(x)$</th>
<th>$F_j^{H}(x)$</th>
<th>$x_1$</th>
<th>$x_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>21</td>
<td>15</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

NIS: $F^H = (15, 235.8)$

Next, construct equation and obtain the following equations:

$$d_{P^H_i}^o = \left[ \frac{F_i^{H}(x) - F_{i^H}(x)}{233.4615 - 21} \right] + \left[ \frac{F_j^{H}(x) - F_{j^H}(x)}{189.6154552 - 15} \right]^{1/p}$$

$$d_{N^H_i}^o = \left[ \frac{F_i^{H}(x) - F_{i^H}(x)}{233.4615 - 21} \right] + \left[ \frac{F_j^{H}(x) - F_{j^H}(x)}{189.6154552 - 15} \right]^{1/p}$$

Thus, problem (12) is obtained. In order to get numerical solutions, assume that $w_{1^H} = w_{2^H} = 0.5$ and $p = 2$.

Table 6: PIS payoff table of problem (12) when $p=2$.

<table>
<thead>
<tr>
<th>$d_{P^H_i}^{o^{12}}$</th>
<th>$d_{N^H_i}^{o^{12}}$</th>
<th>$F_i^{12}(x)$</th>
<th>$F_j^{12}(x)$</th>
<th>$x_1$</th>
<th>$x_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>116.1814</td>
<td>0.5108208411</td>
<td>233.461636</td>
<td>189.6154552</td>
<td>10.38462</td>
<td>0.7692308</td>
</tr>
</tbody>
</table>

NIS: $F^H = (21, 235.8)$

116
Now, it is easy to compute step (10):

Maximize $\delta^{IL}$

Subject to

$3x_1 + 5x_2 \leq 35, 2x_1 - x_2 \leq 20, 5x_2 \leq 16, x_1 \geq 1, x_1, x_2, x_3 \geq 0$

$$
\begin{align*}
&\frac{d^{IL}(X) - 38.87445}{38.87445 - 38.87445} \geq \delta^{IL}, \\
&\frac{0.5008807 - 0.5008807}{0.5008807 - 0.5008807} \geq \delta^{IL}, \\
&\delta^{IL} \in [0, 1].
\end{align*}
$$

The maximum "satisfactory level" ($\delta^{IL} = 1$) is achieved for the solution $x_1^{IL} = 1.234568, x_2^{IL} = 1.234568$.

- Obtain PIS and NIS payoff tables for problem $p^{IH}$.

**Table 7:** PIS payoff table for problem $p^{IH}$.

<table>
<thead>
<tr>
<th>$F_1^{IH}(X)$</th>
<th>$F_2^{IH}(X)$</th>
<th>$x_1$</th>
<th>$x_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>78.84615</td>
<td>86.8460776</td>
<td>10.38462</td>
<td>0.7692308</td>
</tr>
</tbody>
</table>

PIS: $f^{IH} = (78.84615, 5)$

**Table 8:** NIS payoff table for problem $p^{IH}$.

<table>
<thead>
<tr>
<th>$F_1^{IH}(X)$</th>
<th>$F_2^{IH}(X)$</th>
<th>$x_1$</th>
<th>$x_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>96.9331</td>
<td>102.0667</td>
<td>6.3333</td>
<td>3.2</td>
</tr>
</tbody>
</table>

NIS: $f^{IH} = (7, 102.0667)$

- Next, construct equation and obtain the following equations:

$$
\begin{align*}
d^{IL}_{p^{IH}}^p & = \left[ w_1 \left( \frac{78.84615 - F_1^{IH}(X)}{78.84615 - 7} \right)^p + w_2 \left( \frac{F_2^{IH}(X) - 5}{102.0667 - 5} \right)^p \right]^{1/p}, \\
d^{IL}_{p^{IH}}^p & = \left[ w_1 \left( \frac{78.84615 - 7}{78.84615 - 7} \right)^p + w_2 \left( \frac{102.0667 - F_2^{IH}(X)}{102.0667 - 5} \right)^p \right]^{1/p}.
\end{align*}
$$

Thus, problem (13) is obtained. In order to get numerical solutions, assume that $w_1^p = w_2^p = 0.5$ and $p = 2$.

**Table 9:** PIS payoff table of problem (13) when $p = 2$.

<table>
<thead>
<tr>
<th>Min. $d^{IL}_{p^{IH}}^p$</th>
<th>Max. $d^{IL}_{p^{IH}}^p$</th>
<th>$F_1^{IH}(X)$</th>
<th>$F_2^{IH}(X)$</th>
<th>$x_1$</th>
<th>$x_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.32887737472</td>
<td>38.87445</td>
<td>78.84615</td>
<td>68.8460776</td>
<td>10.38462</td>
<td>0.7692308</td>
</tr>
</tbody>
</table>

Now, it is easy to compute step (10):

Maximize $\delta^{IL}$

Subject to

$3x_1 + 5x_2 \leq 35, 2x_1 - x_2 \leq 20, 5x_2 \leq 16, x_1 \geq 1, x_1, x_2, x_3 \geq 0$

$$
\begin{align*}
&\frac{d^{IL}(X) - 38.87445}{38.87445 - 38.87445} \geq \delta^{IL}, \\
&\frac{0.5008807 - 0.5008807}{0.5008807 - 0.5008807} \geq \delta^{IL}, \\
&\delta^{IL} \in [0, 1].
\end{align*}
$$

The maximum "satisfactory level" ($\delta^{IL} = 1$) is achieved for the solution $x_1^{IL} = 1.234568, x_2^{IL} = 1.234568$.

- Obtain PIS and NIS payoff tables for problem $p^{III}$.

**Table 10:** PIS payoff table for problem $p^{III}$.

<table>
<thead>
<tr>
<th>$F_1^{III}(X)$</th>
<th>$F_2^{III}(X)$</th>
<th>$x_1$</th>
<th>$x_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>385</td>
<td>283.8462492</td>
<td>10.38462</td>
<td>0.7692308</td>
</tr>
</tbody>
</table>

PIS: $f^{III} = (385, 20)$

**Table 11:** NIS payoff table for problem $p^{III}$.

<table>
<thead>
<tr>
<th>$F_1^{III}(X)$</th>
<th>$F_2^{III}(X)$</th>
<th>$x_1$</th>
<th>$x_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>35</td>
<td>20</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

NIS: $f^{III} = (35, 443.4667)$

- Next, construct equation and obtain the following equations:

$$
\begin{align*}
d^{III}_{p^{IH}}^p & = \left[ w_1 \left( \frac{385 - F_1^{III}(X)}{385 - 35} \right)^p + w_2 \left( \frac{F_2^{III}(X) - 20}{443.4667 - 20} \right)^p \right]^{1/p}.
\end{align*}
$$
Thus, problem (14) is obtained. In order to get numerical solutions, assume that \( w_1 = w_2 = 0.5 \) and \( p = 2 \).

Table 12: PIS payoff table of problem (14) when \( p = 2 \).

<table>
<thead>
<tr>
<th>Objective</th>
<th>Proposed TOPSIS Algorithm method (p=2)</th>
<th>Ideal Objective Vector</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p^L )</td>
<td>( F_1^L ) 29.445896</td>
<td>( F_1^L ) 151.5385</td>
</tr>
<tr>
<td>( p^L )</td>
<td>( F_2^L ) 73.714321</td>
<td>( F_2^L ) 168.9333</td>
</tr>
<tr>
<td>( p^H )</td>
<td>( F_1^H ) 50.617288</td>
<td>( F_1^H ) 233.4615</td>
</tr>
<tr>
<td>( p^H )</td>
<td>( F_2^H ) 72.835912</td>
<td>( F_2^H ) 235.8</td>
</tr>
<tr>
<td>( p^H )</td>
<td>( F_3^H ) 18.51852</td>
<td>( F_3^H ) 78.84615</td>
</tr>
<tr>
<td>( p^H )</td>
<td>( F_3^H ) 33.3336</td>
<td>( F_3^H ) 102.0667</td>
</tr>
<tr>
<td>( p^H )</td>
<td>( F_3^H ) 146.913592</td>
<td>( F_3^H ) 443.4667</td>
</tr>
</tbody>
</table>

Conclusions:
This paper extended TOPSIS approach to find compromise solutions for the RLMODM of mixed (Maximize/Minimize)-type. A new algorithm is presented for this proposed TOPSIS approach for solving these types of mathematical programming problems. Also, an illustrative numerical example is solved and compared the compromise solutions of the proposed algorithm with the vector of ideal solutions. Thus, the proposed TOPSIS algorithm gives good compromise solutions.

Abbreviations:
DM: Decision Maker,
CP: Compromise Programming,
PIS: Positive Ideal Solution,
NIS: Negative Ideal Solution,
MODM: Multiple Objective Decision Making,
TOPSIS: Technique for Order Preference by Similarity Ideal Solution,
RLMODM: Linear Multiple Objective Decision Making Problem with Rough parameters in the objective functions.

REFERENCES
Pawlak, Z., "Rough Sets", Springer,