Australian Journal of Basic and Applied Sciences
2019 July; 13(7): pages 28-34
DOI: 10.22587/ajbas.2019.13.7.5
Original paper
AENSI Publications
Journal home page: www.ajbasweb.com

Nonlinear Bending Analysis of Composite Cantilever Beams

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Received date: 20 May 2019, Accepted date: 20 July 2019, Online date: 28 July 2019

Abstract

The problem of large deflection of a composite cantilever beam under a vertical concentrated load at free end is studied in this paper. In the formulation of the problem, the nonlinear differential equation is expressed in term of an elliptic integral, which solved numerically by using Simpson method and compared with the obtained results in the literature. Several bending experiments were performed on composite cantilever beams, whose material proprieties were obtained through a series of static tensile tests. The obtained results compare well with the proposed analytical model.

Keywords: Composite beams, Large deflection, Nonlinear bending, Experimental

INTRODUCTION

The minimum weight criterion in the design of some structures in the various fields of industry (aeronautics, aerospace, civil constructions…. etc.), in addition to the use of certain materials that can undergo large displacement. Dus to the geometry of their deformation, the behavior of such structures is highly non-linear, and the solution of such problems becomes very complex.

Because of the practical importance of beam's applications, large deflection of elastic beams has been the focus of many types of research, since the 1940s. The large deflection problem of a cantilever beam with a vertical load at the free end was studied classically for the first time in 1945 by (Bisshopp and Drucker, 1945). (Wang, 1986, 1969) Proposed a simple numerical method for analyzing nonlinear bending of the beam under tip concentrated and uniformly distributed loads respectively. Numerical evaluations of elliptic integral solutions of some large deflection beam and frame problems are presented by (Mattiasson,1981). (Rao et al.,1988) studied in detail large deflections of uniform and non-uniform cantilever beams under tip rotational loads using the elliptic function solution. (Beléndez et al., 2002, 2003) I also studied the same problem, both theoretically and experimentally. (Lee, 2002) Investigated numerically the large deflection of cantilever beams made of a Ludwick type material subjected to combined, concentrated vertical tip point and uniformly distributed forces. The same problem was recently studied by (Solano-Carrillo, 2009) with a bending moment formulation. (Kumar et al., 2004) Suggested genetic algorithm-based search strategies in the context of direct numerical solution of governing differential equation and the principle of stationary of the energy functional in the equilibrium state. A new technique was developed by (Dado and Al-Sadder, 2005) to analyze large deflection of the non-prismatic beam when the angle of rotation was represented by polynomial function on the position variable along the deflected beam axis. (Shvartsman, 2007) solved numerically large deflection of non- uniform cantilever beam under tip concentrated follower load. (Wang et al., 2006) applied the homotopy analysis method (HAM) to investigate the same problem. (Banerjee et al.,2008) employed non-linear shooting and Adomian's decomposition methods to approximate large deflection solution a cantilever beam under arbitrary loading. (Mutyalarao et al., 2010). Using numerical integration to solve large deflection of a uniform beam under tip concentrated follower load, which the effect of inclination of load in relation with the angle of rotation was studied. A new integral approach is proposed by (Li Chen, 2010) to solve the large deflection cantilever beam problems. (Nallathambi et al.,2010) studied large deflection of a curved cantilever beam by fourth-order Runge-Kutta method, whereas (Shvartsman, 2013) studied the same problem by direct numerical method. A new perturbation method was proposed by (He et al.,2013) to solve nonlinear large deflection problem of initially curved beams under two different boundary conditions. (Tari, 2013) used a recently developed automatic Taylor expansion technique (ATET) to analyze the large deflection of cantilever beams subjected to a combined tip point loading including a moment and an inclined

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force. (Ghuku and Saha.,2016) studied a theoretical and experimental study on large deflection behavior of initially curved cantilever beams subjected to various types of loadings.

The present research aims to introduce an analytical formulation for large deflection analysis of composite cantilever beam under a concentrated vertical load at a free end. The obtained results have been compared with the analytical studies from the literature. Furthermore, several bending experiments were performed for composite cantilever beams. The obtained experimental results have been confronted with those obtained using the present analytical formulation.

MATHEMATICAL FORMULATION

The curvature of the bent beam is given in rectangular coordinates by the well-known relation from analytical geometry, to analyze the large deflection of the composite cantilever beam, subjected to point load applied at its free end.

$$\frac{1}{\rho} = -\frac{\frac{d^2y}{dx^2}}{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}} \quad (1)$$

Where ρ is curvature radius

In the case of large displacement, the Bernoulli's kinematics can be expressed in the following way

$$\begin{aligned} u(x,y) &= u_0(x) - y\sin\varphi \\ w(x,y) &= w_0(x) \end{aligned} \quad (2)$$

Where u_0 and w_0 are the axial and transverse displacements of a point on the neutral axis

The strain-displacement relationship is given by

$$E_{xx} = \frac{du_0}{dx} - y\frac{d\varphi}{ds} \quad (3)$$

This expression can be rewritten as follows

$$E_x = E_{xx}^0 + yk_x \quad (4)$$

The constitutive equations for a k[th] layer of a laminated can be written in terms of stresses-strains relationships in the global coordinates systems as follows

$$\begin{Bmatrix} \sigma_{xx}^k \\ \sigma_{yy}^k \\ \sigma_{xy}^k \end{Bmatrix} = \begin{bmatrix} \bar{Q}_{11}^k & \bar{Q}_{12}^k & \bar{Q}_{16}^k \\ \bar{Q}_{12}^k & \bar{Q}_{22}^k & \bar{Q}_{26}^k \\ \bar{Q}_{16}^k & \bar{Q}_{26}^k & \bar{Q}_{66}^k \end{bmatrix} \begin{Bmatrix} E_{xx} \\ E_{yy} \\ E_{xy} \end{Bmatrix} \quad (5)$$

This equation can be rewritten as

$$\{\sigma^k\} = [\bar{Q}^k]\{E\} \quad (6)$$

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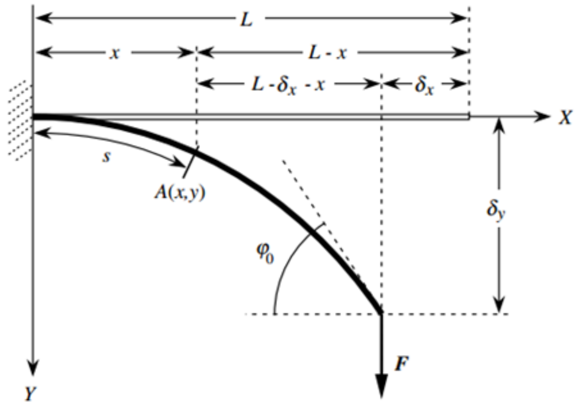

Figure 1: schematic view of a cantilever.

Laminated cantilever beam subjected to a concentrated load F at the free end is shown in figure 1. To determine the horizontal and vertical displacements δ_x and δ_y respectively, of the free end of the beam, the bending moment-curvature relation is used by (Beléndez et al., 2002).

The relation is given by

$$D'\frac{d\varphi}{ds} = -M(x) \quad (7)$$

$M(x)$ is the bending moment at location x, which can be expressed as

$$M(x) = -F(L - \delta_x - x) \quad (8)$$

The derivation of equation (7) concerning s, yields to

$$D'\frac{d^2\varphi}{ds} = -\frac{dM(x)}{ds} \quad (9)$$

After some mathematical manipulation's equation (9) becomes

$$\left(\frac{d\varphi}{ds}\right)^2 = \frac{2}{D'}\left(Fsin\,\varphi_0 - F\sin\varphi\right) \quad (10)$$

By integrating Eq (10), assuming that the beam's length will not change during bending, we can have

$$\sqrt{\frac{2F}{D'}}\int_0^L ds = \int_0^{\varphi_0}\sqrt{sin\,\varphi_0 - sin\,\varphi}\,d\varphi \quad (11)$$

Once the rotation angle φ_0 is obtained, we can calculate the horizontal and vertical displacements δ_x and δ_y by using the following equations

$$\delta_x = L - 2\sqrt{\frac{D'}{2F}}\left[sin\,\varphi_0\right] \quad (12)$$

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$$\delta_y = \sqrt{\frac{D^{'}}{2F}} \int_0^{\varphi_0} \frac{\sin\varphi d\varphi}{\sqrt{\sin\varphi_0 - \sin\varphi}} \qquad (13)$$

EXPERIMENTAL PROCEDURES

Several bending tests were carried out on composite cantilever beams to confronting the analytical formulation presented in the previous section. In this work, a composite of random short fiber mat with unsaturated polyester resin (RSM) is used for experiments. The mechanical proprieties were obtained through tensile tests using a universal testing machine **(INSTRON-5969)**figure2. From the results of the test, the tensile modulus of elasticity, failure stress and strain were determined, where the obtained results are summarized in table (1).

Figure 3 Show the experimental setup of composite (RSM) beams, where several bending tests were performed.

Table 1: Mechanical properties of the (RSM) specimens

Property	Specimen 1	Specimen 2	Specimen 3	Mean	Standard deviation
Failure tensile stress (MPa)	60.6905	50.4734	68.0778	59.747	12.501
Failure tensile strain(mm/mm)	0.01514	0.01211	0.01609	0.0144	0.0029
Modulus of elasticity (MPa)	5437.64	5517.84	6179.37	5711.6	575.65

Figure 2: Static tensile tests of (RSM) specimens.

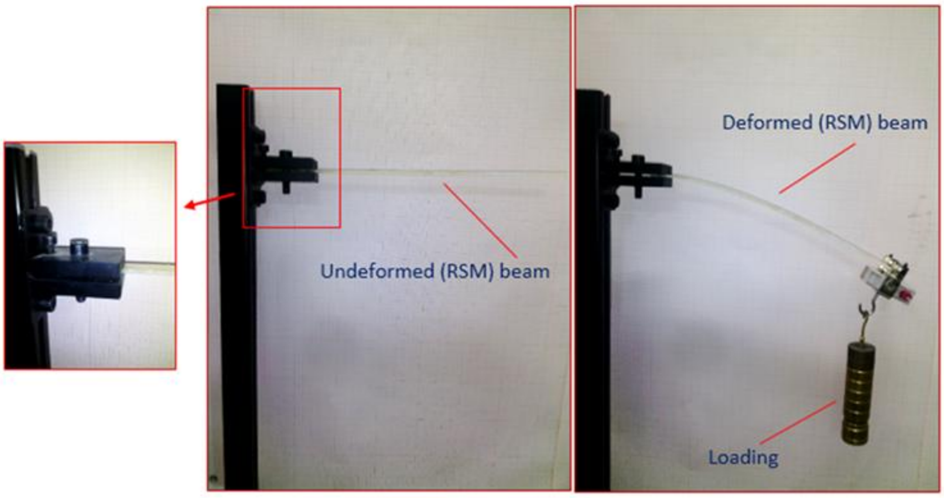

Figure 3: Bending experiments for (RSM) beams.

RESULTS AND DISCUSSION

In order to compare the analytical formulation, an isotropic cantilever beam subjected to vertical concentrated load at free end is considered. The horizontal and vertical deflections are computed for deferent load parameter value where The results are shown in figure 4. The curves show the results are in good agreement with the results obtained by of (Mattiasson,1981) and (Kumar et al., 2004).

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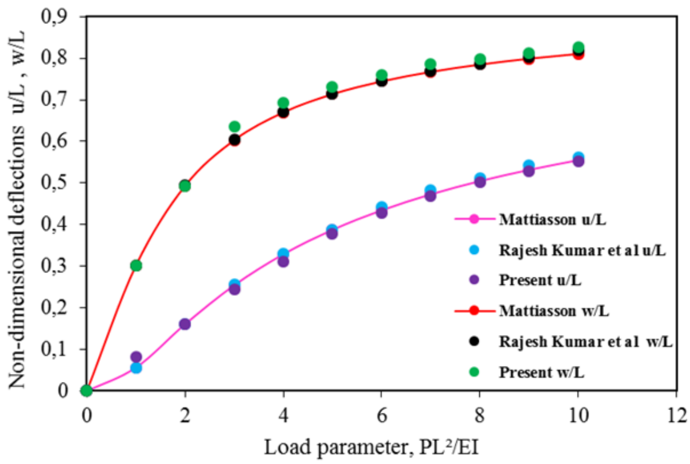

Figure 4: Non-dimensional deflections as a function of the load parameter

Once the analytical formulation has been compared, the results obtained with those obtained analytically in the literature, we proceed to an experimental analysis of the bending of (RSM) beams, to determine the horizontal and the vertical displacements measured at the tip of the beams. For confrontation purpose, the experimental results in terms of horizontal and vertical movements and those obtained using the present analytical formulation for (RSM) beams, are presented in table (2).

Figure 4 depicts the comparison between the load-displacements curves of the analytical and the experimental analysis for (RSM) cantilever beam, which as it can be seen, the analytical curves are in very good agreement with the experimental ones.

Table 2: Horizontal and vertical displacements of (RSM) cantilever beam end

	Present analytical		**Present experimental**	
Load (N)	U(mm)	W(mm)	U(mm)	W(mm)
0.75	5.71	40.54	7	36
1,5	12,53	72.85	16	64.6
1,6	14.26	75.61	17.5	69
1.8	17.93	83.81	21	76
2	20,34	92,19	23,5	82
2,2	24,24	98,14	27	89
2,4	27,71	104,3	30	94
2,9	36,31	117,6	37	105
3,4	44,32	129,4	44	117
3,9	52,72	138,7	51	130
4,4	59,79	144,9	57	137
4,9	65,93	153,1	64	145
5,4	72,44	158,2	71	152
5,9	78,17	163,1	75	157
6,4	83,29	168,6	83	164

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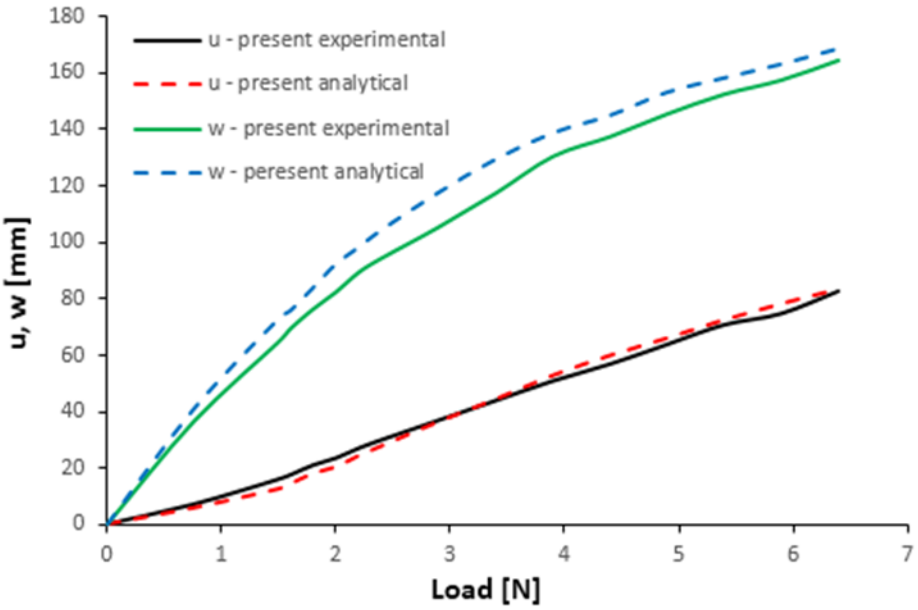

Figure 5: Load-displacements curves for an (RSM) cantilever beam.

CONCLUSION

Large deflection of a composite cantilever beam under a concentrated vertical load at free end has been studied. To obtained deflections, we expressed the nonlinear differential equation in term an elliptic integral, which solved numerically using the Simpson method. The analytical formulation has been compared with the results obtained analytically in the literature. A series of bending experiments were carried out on composite (RSM) cantilever beams. The experimental results were compared well and demonstrated a good agreement with the analytical results

REFERENCES

Banerjee, A., Bhattacharya, B., & Mallik, A. K, 2008. Large deflection of cantilever beams with geometric non-linearity: Analytical and numerical approaches. *International Journal of Non-Linear Mechanics*, *43*(5),366-376.

Beléndez, T. B, Neipp, C, Beléndez, A, 2002. Large and small deflections of a cantilever beam, *Eur J Phys*. 23(3): 371-379.

Beléndez, T. B, Neipp,C, Beléndez, A,2003 . Numerical and Experimental Analysis of a Cantilever Beam:A Laboratory Project to Introduce Geometric Nonlinearity in Mechanics of Materials. *Int J Eng Educ* 19, 885-892.

Bisshopp, K. E, Drucker, D.C,1945. Large Deflection of Cantilever Beams. *Quarterly of Applied Math* 3(1): 272-275.

Chen, L, 2010. An integral approach for large deflection cantilever beams. *International Journal of Non-Linear Mechanics*, *45*(3), 301-305.17.

Ghuku, S., & Saha, K. N,2016. A theoretical and experimental study on geometric nonlinearity of initially curved cantilever beams. *Engineering Science and Technology, an International Journal*, *19*(1), 135-146.

He, X. T., Cao, L., Li, Z. Y., Hu, X. J., & Sun, J. Y. (2013). Nonlinear large deflection problems of beams with gradient: A biparametric perturbation method. *Applied Mathematics and Computation*, 219(14), 7493-7513.

Kumar, R., Ramachandra, L. S., & Roy, D,2004. Techniques based on genetic algorithms for large deflection analysis of beams. *Sadhana*, 29(6), 589-604.11

Lee, K.(2002).Large defections of cantilever beams of non-linear elastic material under a combined loading. *Int J Non Linear Mech*, 37(3): 439-443.

Matiasson, K, 1981. Numerical Results from Large Deflection Beam and Frame Problems Analysed By Elliptic Integrals. *Short Commun*,17(1): 145-153.

Mutyalarao, M., Bharathi, D., & Rao, B. N, 2010. On the uniqueness of large deflections of a uniform cantilever beam under a tip-concentrated rotational load. *International Journal of Non-Linear Mechanics*, *45*(4), 433-441.

Nageswara Rao, B, Nagesh Babu, G. L.,Venkateswara Rao, G, 1988 . On the large deflection of a spring-hinged cantilever beam with end rotational load. *Res Mech*, 67(10):24-26.

Nallathambi, A. K., Rao, C. L., & Srinivasan, S. M, 2010. Large deflection of constant curvature cantilever beam under follower load. *International Journal of Mechanical Sciences*, 52(3), 440-445.18.

Shvartsman, B. S, 2007. Large deflections of a cantilever beam subjected to a follower force. *Journal of Sound and Vibration*, *304*(3-5), 969-973.

Shvartsman, B. S, 2013. Analysis of large deflections of a curved cantilever subjected to a tip-concentrated follower force. *International Journal of Non-Linear Mechanics*, *50*, 75-80.

Solano-Carrillo,E, 2009. Semi-exact solutions for large deflections of cantilever beams of non-linear elastic behavior. *Int J Non Linear Mech* , 44(2): 253-25.

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DOI: 10.22587/ajbas.2019.13.7.5

Tari, H, 2013. On the parametric large deflection study of Euler–Bernoulli cantilever beams subjected to combined tip point loading. *International Journal of Non-Linear Mechanics*, *49*, 90-99.
Wang,T. M, 1969 . Non-Linear Bending of Beams with Uniformly Distributed Loads. *Int J Non Linear Mech* ,4(4): 389-395.
Wang, T. M, 1986. Nonlinear bending of beams with concentrated loads. *J Franklin Inst*, 285(5): 386-390.