Australian Journal of Basic and Applied Sciences, 2(4): 1129-1136, 2008
ISSN 1991-8178

Application of Fast Fourier Transform to Ozone Data at Lagos, Nigeria

M.L. Akinyemi

Department of Physics, Covenant University, Ota, Nigeria.

Abstract: Fast Fourier Transform (FFT) is a mathematical tool often used to represent time-varying real data set function *f(t)*. In this paper, the FFT was applied to the measured stratospheric ozone data at Lagos from 1997 to 2002. The total number of data set used was 2^{11} giving 2048. Both first order and second order derivatives of the data set were generated using FFT. The total ozone data over Lagos was latter generated using the derivatives. The simulated ozone data was compared with the observed ones. The mean ozone concentration value from the observed data was 270.8±0.3DU, its standard deviation was 15.3DU and its range 93.1DU. For FFT (N = 2048) generated data, its mean was 271.2±0.3DU, its standard deviation was 15.5DU and its range was 79.8DU. The result of the generated data using FFT of (N = 2048), and that of the observed values had a correlation of 0.99 while that of (N= 1024) had a correlation value of 0.94. The rate of depletion of ozone in January and February was also observed to have average values of -0.14DU per day with an average initial value of about 250DU. Similarly the rate of increase in ozone in July to September was about 0.3DU per day with an average initial value of about 280DU.

Key words: Fast Fourier Transform, Ozone Data, Correlation Coefficient, linear regression.

INTRODUCTION

A function expressed as a sum of sine and cosine terms over a finite interval with no particular periodicity is called the Fast Fourier Transform (FFT). FFT provides a simple representation of a function defined over an interval using data set of $N = 2^m$, where m is an integer. FFT is often used to represent real time-varying data set function *f(t)* (Riley *et al.,* 1999). If therefore a measured data is time varying, one can perform a FFT on the data. To do this, the basic requirement on the function *f* is that

$$K = \left| f'(t) \right| dt \qquad (1)$$

where K is finite.

This implies that the integral over one period of $|f(t)|$ must converge (Riley *et al.,* 1999).

The measured ozone data for Lagos was taken to represent the function *f*, which in turn was represented as a complex Fourier series:

$$f(t) = \sum_{k=0}^{N-1} \tilde{g}(w_k)\left(\cos w_k t + i \sin w_k t\right) \qquad (2)$$

$N = 2^m$, where m is an integer, For this work only two values of N were used:

$N = 2^{10} = 1024 \qquad N = 2^{11} = 2048$

At any time, the value of N used was specifically mentioned.

w_k is a discrete frequency and $\tilde{g}(w_k)$ is the Fast Fourier Transform of *f*.

Equation (2) can be simplified by making use of the fact that

$$\exp(iwt) = \cos wt + i \sin wt$$

Corresponding Author: M.L. Akinyemi, Department of Physics, Covenant University, Ota, Nigeria.
E-mail: marvel.akinyemi@gmail.com

Thus,

$$f(t) = \sum_{k=0}^{N-1} \tilde{g}(w_k) e^{iw_k t} \tag{3}$$

Similarly, we can write

$$\tilde{g}(w_k) = \frac{1}{N} \sum_{k=0}^{N-1} f(t) e^{-iw_k t} \tag{4}$$

This implies that the function $f(t)$ is the inverse FFT of $\tilde{g}(w_k)$.

We can also write

$$\tilde{g}(w_k) = F\left[f(t)\right] = F_k \tag{5}$$

The discrete frequency w_k as noted by Parker (2002) is given by;

$$w_k = \frac{2\pi k}{N\Delta t} \tag{6}$$

In this particular analysis since the measured data were taken daily, which implies

$$\Delta t \approx 1 day$$

Mathematical Procedures and Results:

Total ozone data over Lagos was generated using FFT data set N = 2048 (i.e. k = 0 to k = 2047). Below are the mathematical procedures taken (Riley *et al.,* 1999),

$$F\left[f(t)\right] = \tilde{g}(w_k) = \frac{1}{N} \sum_{k=0}^{N-1} f(t) e^{-iw_k t} \tag{7}$$

On differentiating in equation (7) with respect to time,

$$F\left[\frac{df}{dt}\right] = \frac{d}{dt}\left[\tilde{g}(w_k)\right] = \frac{-iw}{N} \sum_{k=0}^{N-1} f(t) e^{-iw_k t} \tag{8}$$

Thus from equations (7 and 8) we derive the following;

$$F\left[\frac{df}{dt}\right] = \frac{d}{dt}\left[\tilde{g}(w_k)\right] = -iw_k \tilde{g}(w_k) \tag{9}$$

FFT [f(t)] = $\tilde{g}$ (w_k) are listed in Table 1 column 2.

Table 1: Table showing portions of obtained values in simulation of ozone data in Lagos (N=2048), using FFT [f' (t)].

k	FFT [*f(t)*]	FFT [*f '(t)*= {i*w*FFT[*f(t)*]}	IFFT[*f '(t)*]	Re	Im	Si-DE
0	555456.6	0	64.5 + 27.6i	64.5	27.6	-9.2
1	-3105.1 - 948.3i	2.9 - 9.5i	58.9 + 15i	58.9	15	-5
2	5505.4 + 2494i	-15.3 + 33.8i	51.3 + 9.4i	51.3	9.4	-3.1
3	146.2 - 77.4i	0.7 + 1.3i	39.5 - 0.7i	39.5	0.7	0.2
4	-1970.1 -969.5i	11.9 - 24.2i	27.2 + 9.1i	27.2	9.1	-3.0
1020	-165.5 - 8i	25 -517.8i	76.8 + 24.4i	76.8	24.4	-8.1
1021	-12.4 + 375.6i	-1176.6 - 38.8i	63.7 + 14.7i	63.7	14.7	-4.9
1022	-355.8 + 146.2i	-458.4 - 1115.7i	64.2 + 42.0i	64.2	42.0	-14
1023	-275 +190.7i	-598.4 - 863.1i	65 + 11.6i	65	11.6	-3.9
1024	12.6	39.6i	52.1 +16.9i	52.1	16.9	-5.6
1025	-275 - 190.7i	599.6 - 864.8i	53.8 + 36.7i	53.8	36.7	-12.2
1026	-355.8 - 146.2i	460.2 - 1120.1i	50.5 + 12.5i	50.5	12.5	-4.2
1027	-12.4 - 375.6i	1183.5 - 39.1i	48.3 + 43i	48.3	43	-14.3
1028	-165.5 + 8i	-25.2 - 521.8i	56.1 + 32i	56.1	32	-10.7
1029	-101.5 - 183.3i	578.6 - 320.4i	52.2 + 24.1i	52.2	24.1	-8
2043	1374.3 + 1915.7i	-12007.6 + 8614.2i	50.2 + 72.2i	50.2	72.2	-24.1
2044	-1970.1 + 969.5i	-6079.6 - 12354.6i	54.6 + 41.4i	54.6	41.4	-13.8
2045	146.2 + 77.4i	-485.6 + 917.3i	55.8 + 60i	55.8	60	-20
2046	5505.4 – 2494i	15655.2 + 34557.6i	58.3 + 34.5i	58.3	34.5	-11.5
2047	-3105 + 948.3i	-5955.3 - 19500.2i	59.5+48.3i	59.5	48.3	-16.1

f(t) is the original data on which Fast Fourier Transform (FFT) was performed, *f '(t)* is the first order derivative of *f(t)*.
w =2πk/ NΔt = frequency of the wave components, while (i) is the complex number prefix. IFFT is the Inverse FFT.
Re is the real coefficient of IFFT, Im is the imaginary coefficient of IFFT,
Si-DE is the simulated values of ozone deviation from its mean, (Si-DE = Im /-3)

Method of Operation:

The measured satellite data of daily total ozone concentration at Lagos from January 1997 to December 2002 were taken as function *f(t)*, and was Fourier analysed. The measured data were obtained from satellite EPTOMS (EarthProbe Total Ozone Mapping Spectrometer) of NASA Goddard Institute for Space Study, USA (EarthProbe V.7, 2003). The numbers of days first used in the FFT simulation were the first 1024 days of the whole data available. FFT returned the Fourier transform of the data in the form:

$(x + iy)$, where $x, y \in \Re$, $i^2+1 = 0$, $\Re$ - is the set of real numbers.

The absolute value of convergence as generated using FFT analysis for data set (N= 1024) of Lagos total ozone concentration, was 305.0.

Fast Fourier Transform was also performed on twice this number i.e. N = 2048. This was the farthest FFT that could be done with the available data, as the next FFT to this will require 4096 data. The absolute value of convergence for FFT at N = 2048 data set of Lagos total ozone concentration respectively was 12.6 (Table 1, column 2). This showed the higher accuracy of the greater N.

RESULTS AND DISCUSSION

Using equation 9, the first order derivatives FFT [$f'(t)$] of total ozone over Lagos were calculated (Table 1 column 3). Re implies the real coefficient of the inverse FFT, Im implies the imaginary coefficient of the inverse FFT. On performing correlation analysis of the Re and Im data each with the observed data, Re had a correlation of less that 0.01 while Im had a correlation of 1.0 respectively (Table 1, columns 5 and 6). Samples of the generated data are shown in Table 1. The first five, middle ten and last five can be seen in the table.

Besides, the relationship between the first order derivatives f' of daily variation ozone concentration at Lagos and the real data series on monthly basis were studied (Figures 1-3). The graph f' against f of ozone concentration at Lagos in January and February had well defined linear regression pattern as revealed in figures 1, 2 and 3 with average negative gradients of – 2.97 for January and -2.62 for February, yielding a general formula of

$$f' = -2.8 f + 750.0 \text{ for January and February} \quad (10)$$

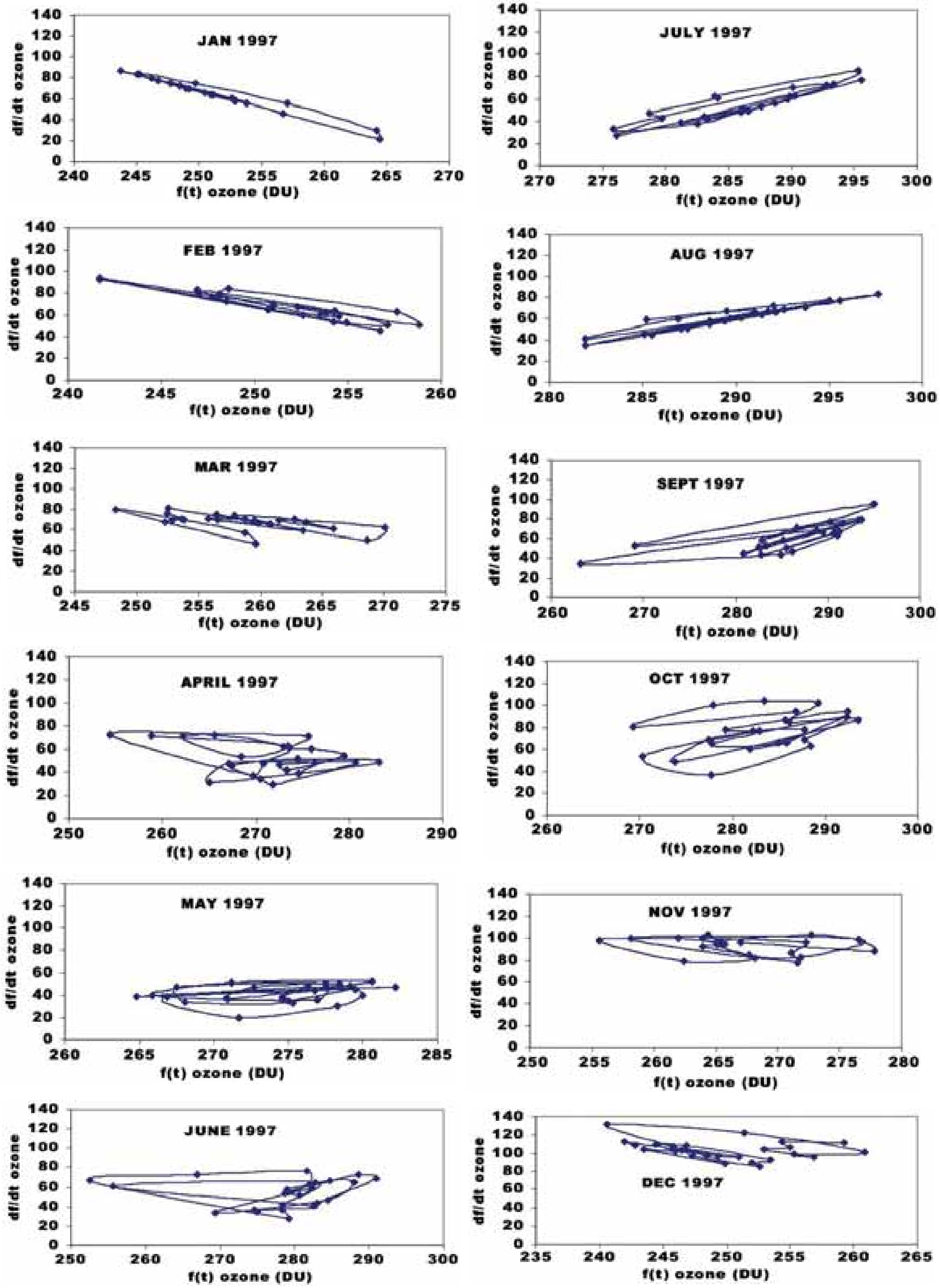

Fig. 1: Comparing the monthly patterns of df/dt of ozone concentration at Lagos (y-axis) with the raw ozone data (x-axis) in 1997.

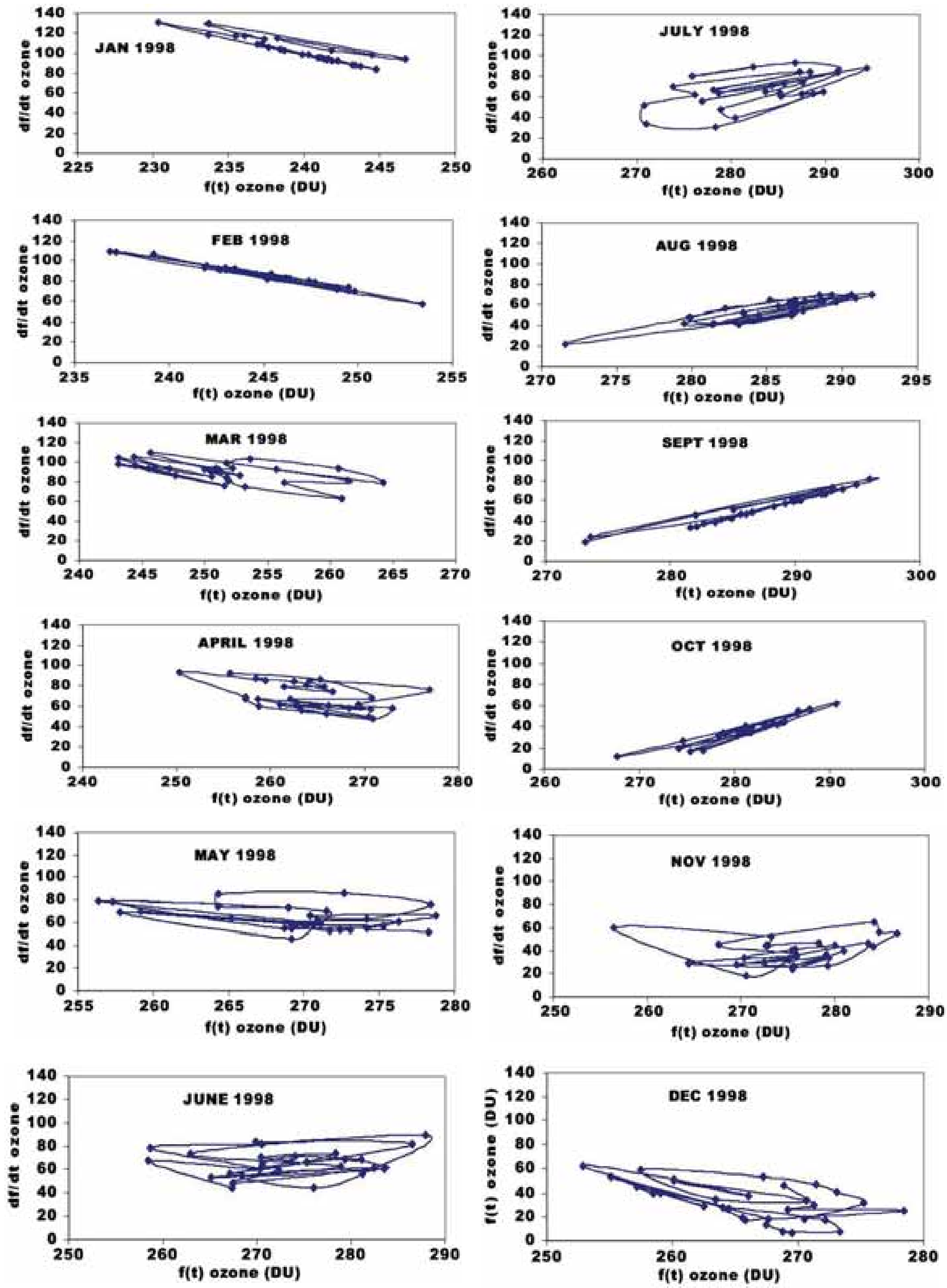

Fig. 2: Comparing the monthly patterns of df/dt of ozone concentration at Lagos (y-axis) with the raw ozone data (x-axis) in 1998.

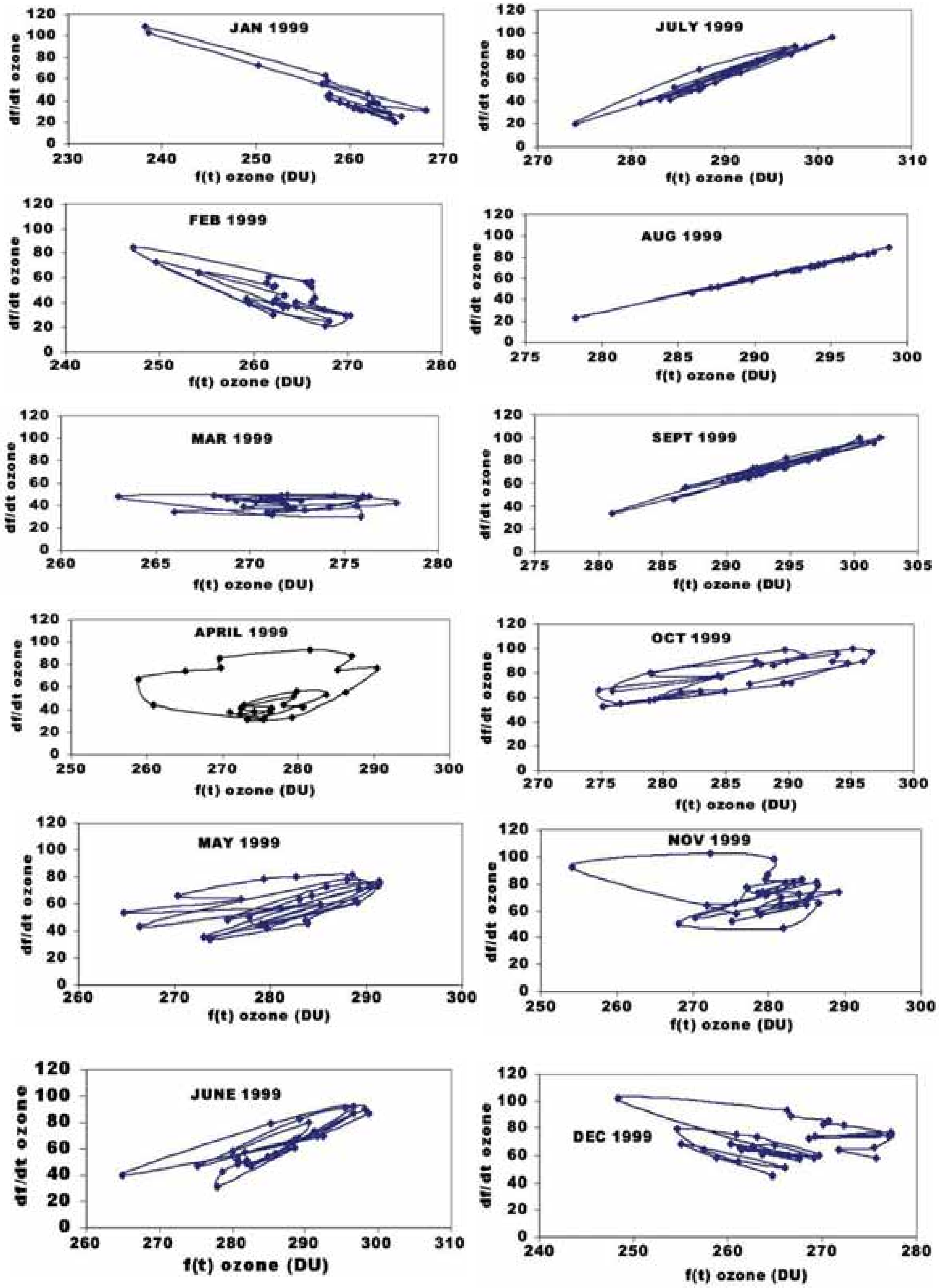

Fig. 3: Comparing the monthly patterns of df/dt of ozone concentration at Lagos (y-axis) with the raw ozone data (x-axis) in 1999.

Also the linear relationship of f*(t)* to *t* for the same period was

and $f = -0.14t + 251.2$ (11)

This showed that the rate of depletion of ozone concentration in January and February was about -0.14DU per day. Some reasons for the observation may be that, in the absence of any other factors, it is expected that total ozone concentrations would be highest over the tropics because of the greater intensity of solar ultraviolet radiation in equatorial regions, which makes the regions the major source of stratospheric ozone. However, most of the ozone produced in the tropical stratosphere is transported to the middle and high latitudes of the northern and southern hemispheres by the stratospheric circulation, known as the Brewer-Dobson circulation (Stolarski, R., 2003). The Brewer-Dobson circulation transports rich ozone air from the tropics pole ward and downward to the lower stratosphere of the high latitudes.

There is also the effect of the extra-tropical stratosphere and mesosphere action upon the tropical stratosphere on a continuous basis as a kind of global scale fluid-dynamical suction pump (Holton *et al.,* 1995). It was discovered that the most important of these eddy motions called "the breaking Rossby Waves", have a strong tendency to give a persistently one-way suction pumping action, whose strength varies with season and also inter annually. With this eddy effect, air is gradually withdrawn from the tropical stratosphere and pushed pole ward and downward (Dickson, 1968; Holton *et al.,* 1995). It can therefore be concluded from figures 1-3 that the resultant effects of these stratospheric circulations and the extra-tropical suction pump action were highest in the months of January and February at Lagos, and that the rate at which they transport ozone rich air from over the station is proportional to the rate at which ozone is produced over the station by equations 10 and 11 above.

Between July and September the slope is observed to reverse to positive with an average gradient value of 2.7. This can be interpreted from the explanations above that the strength of both the Brewer-Dobson's circulations and the extra-tropical suction pump action gradually diminishes and reaches minimum between the months of July to September over Lagos. This also coincides with the time of minimum temperature over the location.

It can also be added that the period of maximum lifting of ozone rich air from over Lagos by both the Brewer-Dobson circulation and the extra-tropical suction pump coincided with the harmattan season, while the period of minimum lifting coincided with the raining season. . It was also observed that the divergence of the points from the observed linearity already mentioned began in March, likewise the gentle rotation from the negative gradient slope to positive.

This trend revealed the useful application of FFT as a mathematical tool to time series data. Also the gradual unfolding of the linear patterns from both the negative gradient form in early part of the year and the positive gradient form in the latter part of the year was observed to follow some definite order (Figures 1-3).

By proceeding from the values of the first order derivatives, new set of daily ozone data were generated for Lagos. This is represented by Figure 4, comparing simulated and observed ozone data at Lagos (using f'' at N=2048). There is a strong link between the observed and the simulated data. The figure also showed the seasonality of ozone concentration over Lagos with a period of between 340 days, which can be termed as annul seasonal variation.

The means, the standard deviations and ranges of the raw data and the simulated data were also compared. The mean from the raw data was 270.8±0.3DU, its standard deviation was 15.3DU and its range 93.1DU, while the mean from FFT of (N = 1024) simulated data was 272.2±0.5DU, standard deviation was 16.2DU and the range was 75DU. The mean from FFT (N = 2048) simulated was 271.2±0.3DU, its standard deviation was 15.5DU and its range was 79.8DU.

Conclusion:

FFT confirms the sinusoidal nature of Lagos ozone data. This was seen from the high level of correlation between the FFT generated data and those of the observed data. The correlation coefficient of the FFT set N = 1024 with the observed data was 0.94, while those of N = 2048 with the observed was 0.99. Also the imaginary values of the inverse FFT for the first order derivatives had high correlation with the observed data, whereas for the second order derivatives, it was the real values of it's the inverse FFT that had high correlation with the observed data.

The graph f' versus f of ozone concentration at Lagos in January and February had well defined linear regression pattern with average negative gradients of – 2.8 for January and February, yielding a general formula of $f' = -2.8f + 750.0$. Between July and September the slope is observed to reverse to positive with an average value of 2.60 in 1997, 2.45 in 1998 and 3.05 in 1999. The effects of the Brewer-Dobson's circulation and the extra-tropical suction pump were strongest over Lagos in January to February and least in July to September which also coincided with the harmattan season and raining season respectively over the station.

Thus FFT had proven a good mathematical tool that can be applied to time series data.

REFERENCES

Dickson, R.E., 1969. Theory of Planetry Wave-Zonal Flow Interaction. J. Atmos. Sci., 26: 73-81.

Earth Probe Total Ozone Mapping Spectrometer (EPTOMS) of NASA, USA V.7 (2003).

Holton, J.R., P.H. Haynes, M.E. McyInty, A.R. Douglas, R.B. Rood and L. Pfister, 1995. Stratosphere Troposphere Exchange. Review of Geophy., 33(4): 403-439.

Parker, D., 2002. Lecture Series on Environmental Physics: ENVI 3260, Numerical Project 3 – Fourier Transforms. http://D.\SUSO 2\envi 3260 lecture.htm Accessed February, 2002.

Riley, K.F., M.P. Hobson and S.J. Benice, 1999. Mathematical Methods for Physics and Engineering. *Cambridge University Press*, U.K.

Stolarski, R., 2003. Stratospheric Ozone Variability. http://hyperion.gsfc.nasa.gov/ code 916 Accessed November, 2003.