Australian Journal of Basic and Applied Sciences, 2(3): 381-387, 2008
ISSN 1991-8178

Characterization Of Geometry Of Type $D_{6,4}$

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Abstract: A new point-line geometry $D_{6,4}(F)$ will be presented, F is a finite field, we define the points, the lines, the incidence relation and all the varieties of the geometry as an isomorphic to the classical polar space of type $^{+}(12,F)$. Many important properties will be investigated; moreover we present a characterization of such geometry.

Keyword: parapolar space- classical polar space-Symplecton.

INTRODUCTION

The class of the geometries $D_{n,2}$ ($n\geq 5$), $D_{n,3}$ ($n\geq 6$) and $D_{n,4}$ ($n\geq 7$) had been studied and characterized as a point-line geometries see (Zayda, Abdelsalam, 2002). The above cases does not include the point-line geometries of type $D_{5,3}$ and $D_{6,4}$, recently, a characterization of $D_{5,3}$ has been presented as a dual half-spin geometry see (Zayda, Abdelsalam, 2007). AT. Mohammed and zayda abdelsalam also were able to give the general case of the class of the geometries in (Zayda, Abdelsalam, 2002) by presenting a theorem which characterized, by axioms on points and lines, the geometry $D_{n,k}$ where k³2 and n³k+3 and all properties of such geometry investigated see (AT. Mohammed, AT. and Zayda, Abdelsalam, 2005) and (Zayda, Abdelsalam, 2006). The Half-spin geometry $D_{5,5}(F)$ was characterized as a group isomorphic to $W^{+}(10)$ by B.N. Cooperstein and A.M. Cohen (1977) and (1983). In (Cohen, A.M., 1984) A.M. Cohen gave description of two buildings of spaces of type of $D_{n,1}$ and $D_{4,2}$. Here we present a description of a new point-line geometry of type $D_{6,3}$, construction related to building and properties of such geometry will be investigated and then the characterization of the geometry established. For the grassmannian geometries $A_{n,2}$ and $A_{5,4}$ that are needed to recognized a point line geometry $D_{6,4}$ see (Hanssens, G., 1987).

First we present some definition of terminology's that will be used. For most of the following definitions see (Buekenhout, F., and E.E. Shult, 1974).

Given a set I, a geometry G over I is an ordered triple G=(X,*, D), where X is a set, D is a partition $\{X_i\}$ of X indexed by I, X_i are called components, and * is a symmetric and reflexive relation on X called incidence relation such that:

x*y implies that either x and y belong to distinct components of the partition of X or x = y. Elements of X are called objects of the geometry, and the objects within one component X_i of the partition are called the objects of type *i*. The subscripts that index the components are called types. The obvious mapping t:X→ I, which takes each object to the index of the component of the partition containing it is called the type map t.

A point-line geometry (P, L) is simply a geometry for which ïIï=2, one of the two types is called points; in this notation the points are the members of P, and the other type is called lines. Lines are the members of L. If pÎP and *l*ÎL, then p**l* if and only if pÎ*l*. In point-line geometry (P, L), we say that two points of P are collinear if and only if they are incident with a common line. (We use the symbol ~ for collinear)

The singular rank of a space G is the maximal number n (possibly ¥) for which there exist a chain of distinct subspaces j¹X_0ÌX_1Ì ... ÌX_n such that X_i is singular for each *i*, X_i¹X_j, i ¹j. For example rank(j)=-1, rank({p})=0 where p is a point and rank(*l*)=1 where *l* a line. $x^{\wedge}$ means the set of all points in P collinear with x, including x itself.

A subspace of a point-line geometry G=(P, L) is a subset XÍ P such that any line which has at least two of its incident points in X has all of its incident points in X. <X> means the intersection over all subspaces containing X, where X⊆P. Lines incident with more than two points are called thick lines, those incident with exactly two points are called thin lines. In a point-line geometry G=(P, L), a path of length n is a sequence of n+1 $(x_0,x_1,..,x_n)$ where, (x_i,x_{i+1}) are collinear, x_0 is called the initial point and x_n is called the end point. A geodesic from a point x to a point y is a path of minimal possible length with initial point

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x and end point *y*. We denote this length by $d_G(x, y)$, the length of the geodesic from x to y is called the distance between x and y. The diameter of the geometry is the maximal distance of points.

A geometry G is called connected if and only if for any two of its points there is a path connecting them. A subset X of P is said to be convex if X contains all points of all geodesics connecting two points of X.

A polar space is a point-line geometry G=(P, L) satisfying the Buekenhout-Shult axiom:

For each point-line pair (p, *l*) with p not incident with *l*; p is collinear with one or all points of *l*, that is çp^Ç*l*ï=1 or else p^É*l*. Clearly this axiom is equivalent to saying that p^ is a geometric hyperplane of G for every point pÎP.

A point-line geometry G=(P, L) is called a projective plane if and only if it satisfies the following conditions (Zayda, Abdelsalam, 2007):

- G is a linear space; every two distinct points x, y in P lie exactly on one line,
- every two lines intersect in one point,
- there are four points no three of them are on a line.

A point-line geometry G=(P, L) is called a projective space if the following conditions are satisfied:

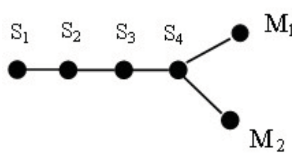
- every two points lie exactly on one line ,
- if l_1, l_2 are two lines l_1Çl_2 ¹Æ, then ál_1, l_2ñ is a projective plane. (ál_1, l_2ñ means the smallest subspace of G containing l_1 and l_2.)

A point-line geometry G=(P, L) is called a parapolar space if and only if it satisfies the following properties:

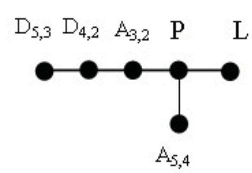
- G is a connected gamma space,
- for every line *l*; *l*^ is not a singular subspace,
- for every pair of non-collinear points x, y; x^Çy^ is either empty, a single point, or a non-degenerate polar space of rank at least 2.

If x, y are distinct points in P, and if êx^Çy^ï=1, then (x, y) is called a special pair, and if x^Çy^ is a polar space, then (x, y) is called a polar pair (or a symplectic pair). A parapolar space is called a strong parapolar space if it has no special pairs.

Construction of $D_{6,4}(F)$:



Polar space Δ



$D_{5,3}$(F)

Consider the classical polar space D=W^+(12, F) that comes from a vector space of dimension 12 over a finite field F= GF(k) with a symmetric hyperbolic bilinear form. The two classes M_1, M_2 consist of maximal totally isotropic 6-dimensional subspaces. Two 6-subspaces fall in the same class if their intersection is of odd dimension.

The geometry of type $D_{6,4}$ (F) is the point-line geometry (P, L), whose set of points P is corresponding to the class S_4 that is: the collection of all totally isotropic 4-dimensional subspaces of the vector space V, and whose lines are corresponding to the collection of all 6-dimensional subspaces of the vector space V that are fall in the class M_1. A point C is incident with a line B if and only if CÌB as a subspaces of V.

To define the collinearity, let C_1 and C_2 be two point (the points are the T.I 4-spaces), then C_1 is collinear to C_2 if and only if the intersection of C_1 and C_2 is a T.I 2-dimensional space. This intersection in addition to the complement of C_1 and C_2 must form a T.I 6-dimensional space. The elements of the class M_2 are geometries of type $A_{5,4}$(F).

The symplecta of $D_{6,4}$ (F) are the Grassmannians of type $A_{3,2}$(F) that are corresponding to the collection of TI 3-dimensional spaces.

Notation:

Let the map Ψ: P® V defined above, i.e., Ψ(p) is the T.I. 4-dimensional subspace corresponding to the point p. We will use Ψ for the rest of the geometry; for example $\Psi(D_{4,2})$ is the T.I. 2-dimensional subspace corresponding to a geometry of type $D_{4,2}$ and $\Psi(D_{5,3})$ is the T.I. 1-dimensional subspace corresponding to a geometry of type $D_{5,3}$. The inverse map Ψ^{-1} will be used for the inverse; for example $\Psi^{-1}(C)$ is the point corresponding to the T.I. 4-dimensional subspace C.

The Main Result:

To make a characterization for the geometry $D_{6,4}$ we present the theorem:

Main Theorem:

Let G = (P, L) be a point-line geometry of type $D_{6,4}$, then the following are satisfied:

- (P_1) G is a strong parapolar space of diameter 5.
- (P_2) The symplecta of the geometry are of type $A_{3,2}$.
- (P_3) If (p, S) is a pair of non-incident point-symplecton, then rank($p^{\perp}\cap$ S)= -1 ,0, 2.
- (P_4) If S_1 and S_2 are two different symplecta of $D_{6,4}$, then rank(S_1ÇS_2)=-1, 0.

The proof of (P_1) is the proofs of the Propositions 2.1, 2.2 and 2.3. (P_3) is the Proposition 3.1, (P_4) is the Proposition 3.2 while (P_2) from the above construction and the diagram geometry of $D_{5,3}$.

Propostion:

The diameter of the point-line geometry $D_{6,4}$(F) is equal 5.

Proof:

We prove that max{d(p, q): p, qÎP}= 5. Let p, q be two points in P and $\Psi(p)=\langle x_1, x_2, x_3, x_4\rangle$, $\Psi(q)=\langle y_1, y_2, y_3, y_4\rangle$ be their corresponding. Then we have four cases:

1- $\Psi(p)\cap\Psi(q)$=2-space, say $\langle x, y\rangle$, where $x = x_1= y_1$ and $y= x_2= y_2$. Then $\Psi(p)=\langle x, y, x_3, x_4\rangle$, $\Psi(q)=\langle x, y, y_3, y_4\rangle$, now we have two cases:

i- $x_3^{\perp}\cap\Psi(q)=\Psi(q)$,
$x_4^{\perp}\cap\Psi(q)=\Psi(q)$,

Then the two points p and q collinear in a line *l* such that $\Psi(l)=\langle x, y, x_3, x_4, y_3, y_4\rangle$, so d(p, q)=1.

ii- $x_3^{\perp}\cap\Psi(q)=\langle x, y, y_3\rangle$
$x_4^{\perp}\cap\Psi(q)=\Psi(q)$,
$u^{\perp}\cap\Psi(q)=\langle x, y, y_3\rangle$,
$v^{\perp}\cap\Psi(q)=\langle x, y, y_3\rangle$

Then Ψ(p) contained in a maximal TI 6-space $\langle x, y, x_3, x_4, u, v\rangle$ and we can find a two points r, s such that $\Psi(r)=\langle v, x_3, y_3, x_4\rangle$ and $\Psi(s)=\langle u, v, x, y_3\rangle$. Since $\langle v, x, y, x_3, x_4, y_3\rangle$, $\langle u, x, v, x_3, x_4, y_3\rangle$ and $\langle u, v, x, y, y_3, y_4\rangle$ form TI 6-spaces, they are the lines that connect the points as follow: p is collinear to r, s is collinear to r and q is collinear to s. then d(p, q)= 3.

2- $\Psi(p)\cap\Psi(q)$=3-space, say, $\langle x, y, z\rangle$ where $x=x_1=y_1$, $y=x_2=y_2$, $z= x_3=y_3$ and $x_4^{\perp}\cap\Psi(q)=\Psi(q)$,
then the5-space $\langle x, y, z, x_4, y_4\rangle$ is contained in a maximal TI 6-space, $\langle u, x_4, x, y, z\ y_4\rangle$. Therefore we find a point r such that $\Psi(r)=\langle u, z, x_4, y_4\rangle$, then

$\Psi(r)\cap\Psi(p)$=2-space=$\langle z, x_4\rangle$,
$\Psi(r)\cap\Psi(q)$=2-space=$\langle z, y_4\rangle$

This means that r is collinear to p and is collinear to q. then d(p, q)=2.

3- $\Psi(p)\cap \Psi(q)$=1-space, $\langle x\rangle$, where $x = x_4= y_4$. Then there are two cases:

i- $y_1^{\perp} \cap \Psi(p)= \Psi(p)$
$y_2^{\perp} \cap \Psi(p)= \Psi(p)$
$y_3^{\perp} \cap \Psi(p)= \langle x, x_2, x_3\rangle$

Then there are two points r and s such that $\Psi(r) =\langle x, x_1, y_1, y_2\rangle$ and $\Psi(r) =\langle x, y_1, x_2, x_3\rangle$. Since $\langle x, x_1, y_1, y_2, x_2, x_3\rangle$, $\langle x, x_1, y_1, y_2, x_2, x_3\rangle$ and $\langle x, y_1, y_2, y_3, x_2, x_3\rangle$ form TI 6-spaces, r is collinear to p, r is collinear to s and s is collinear to q. Then d(p, q)=3.

ii- $\Psi(p)$ contained in a maximal TI 6-space $\langle u, v, x, x_2, x_3, x_4\rangle$ and we can find three points r, s and t such that $\Psi(r)=\langle u, v, x, x_3\rangle$,$\Psi(s)=\langle v, x, x_4, y_4\rangle$ and $\Psi(t)=\langle x, x_2, x_3, y_4\rangle$. The following must be satisfied:
$x_2^{\perp} \cap \Psi(q)= \Psi(q)$
$x_3^{\perp} \cap \Psi(q)= \Psi(q)$
$x_4^{\perp} \cap \Psi(q)= \langle x, y_3, y_4\rangle$

If $u^{\perp} \cap\Psi(q)= \langle x, y_3, y_4\rangle$ and $v^{\perp} \cap\Psi(q)=\langle x, y_3, y_4\rangle$, then p is collinear to r, r is collinear to s, s is collinear to t and t is collinear to q. This means that d(p, q)=4.

4- $\Psi(p)\cap \Psi(q)$=0-space**,** we have the following cases:

i- $x_1^{\perp} \cap \Psi(q)= \Psi(q)$,
$x_2^{\perp} \cap \Psi(q)= \Psi(q)$,
$x_3^{\perp} \cap \Psi(q)= \langle y_1, y_2, y_3\rangle$,
$x_3^{\perp} \cap \Psi(q)= \langle y_1, y_2, y_3\rangle$

Then we have TI 6-space $\langle x_1, x_2, y_1, y_2, y_3, y_4\rangle$ and a point r such that $\Psi(r) =\langle x_1, x_2, y_1, y_2\rangle$ then r is collinear to p. We have also a point s such that $\Psi(s) =\langle x_2, x_3, y_2, y_3\rangle$, since $\langle x_1, x_2, x_3, y_1, y_2, y_3\rangle$ forms a TI 6-space, s is collinear to r. Then there is a point t such that $\Psi(t)=\langle x_3, y_3, x_4, y_4\rangle$ and t is collinear to s and q which means that d(p, q)=4.

ii- $\Psi(q)$ is contained in a maximal TI 6-space $\langle u, v, y_1, y_2, y_3, y_4\rangle$ and the following are satisfied:
$y_1^{\perp} \cap \Psi(p)= \langle x_1, x_2, x_3\rangle$,
$y_2^{\perp} \cap\Psi(p)= \Psi(p)$,
$y_3^{\perp} \cap \Psi(p)= \langle x_1, x_3, x_4\rangle$,
$y_4^{\perp} \cap \Psi(p)= \langle x_2, x_3, x_4\rangle$,

Then we find the points r_1, r_2, r_3 and r_4 such that $\Psi(r_1)= \langle u, v, y_3, y_4\rangle$, $\Psi(r_2)= \langle x_3, x_4, y_3, y_4\rangle$, $\Psi(r_3)= \langle x_3, x_4, u, v\rangle$ and $\Psi(r_4)= \langle x_1, x_4, u, y_2\rangle$. Since $\langle u, v, y_1, y_2, y_3, y_4\rangle$**,** $\langle u, v, x_3, x_4, y_3, y_4\rangle$, $\langle u, v, x_1, x_2, x_3, x_4\rangle$, $\langle u, v, y_2, x_2, x_1, y_4\rangle$ and $\langle u, y_2, x_1, x_2, x_3, x_4\rangle$ form TI 6-spaces, they are considered the lines that connect the different points as follow: r_1 is collinear to q, r_2 is collinear to r_1, r_3 is collinear to r_2, r_4 is collinear to r_3 and r_4 is collinear to p. then d(p, q) =5. So through the above discussion we got the possibilities d(p, q)= 1, 2, 3, 4, 5, then the diameter of $D_{6,4}$ is 5

The following two propositions proved that the geometry $D_{5,3}(F)$ is strong parapolar.

Proposition:

$D_{6,4}(F)$ is a strong geometry.

Proof: We prove that the geometry has no special pair. Let p, q be two points where, $\Psi(p) =\langle x_1, x_2, x_3, x_4\rangle$and $\Psi(q) =\langle y_1, y_2, y_3, y_4\rangle$, referring to Proposition 2.1, we showed through the different cases that d(p, q)= 1, 2, 3, 4 or 5. Now for the cases in which d(p, q)=1, 3, 4, 5 and $p^{\perp}\cap q^{\perp}= \varphi$, i.,e., (p, q) is not a special pair.

To complete the proof we show that (p, q) is not a special pair at the remaining case d(p, q)=2. This case is obtained when the intersection $\Psi(p)\cap \Psi(q)$=3-space, say, $\langle x, y, z\rangle$ where $x=x_1=y_1$, $y=x_2=y_2$, $z= x_3=y_3$ and $x_4^{\perp}\cap\Psi(q)= \Psi(q)$, then the TI 5-space $\langle x, y, z, x_4, y_4 \rangle$ is contained in a maximal TI 6-space, $\langle u, x_4, x, y, z, y_4\rangle$. Therefore we can find more than one point, r and s such that $\Psi(r)= \langle u, z, x_4, y_4\rangle$ and $\Psi(s)= \langle u, y, x_4, y_4\rangle$ satisfying the conditions:

$\Psi(r) \cap \Psi(p)$=2-space=<z, x_4>,
$\Psi(r) \cap \Psi(q)$=2-space=<z, y_4>,
$\Psi(s) \cap \Psi(p)$=2-space=<y, x_4>,
$\Psi(s) \cap \Psi(q)$=2-space=<y, y_4>,

Then each of the points r and s are collinear to the points p and q, so $|p^{\perp} \cap q^{\perp}|>1$ which means that $D_{6,4}$ has no a special pair. Therefore $D_{6,4}$ is a strong geometry

Proposition:

$D_{6,4}(F)$ is a parapolar geometry.

Proof:

The geometry $D_{6,4}$ is connected, see the proof of Proposition 2.1, to show that $D_{6,4}$ is a gamma space, let (p, *l*) be a non-incidence pair of a point p and a line *l* such that $\Psi(p)$=<x_1, x_2, x_3, x_4> and $\Psi(l)$=<u_1, u_2, u_3, u_4, u_5, u_6>. To be specified we must identify two points r and s that define the line *l* say, $\Psi(r)$=<u_1, u_2, u_3, u_4> and $\Psi(s)$= <u_1, u_2, u_5, u_6>. Then the intersection $\Psi(p) \cap \Psi(l)$ has three cases:

1 If $\Psi(p) \cap \Psi(l)$=0-space or $\Psi(p) \cap \Psi(l)$=1-space, then there is no any 4-space contained in $\Psi(l)$ and intersect $\Psi(p)$ in 2-space which means that $p^{\perp} \cap\ l = \varphi$.
2 $\Psi(p) \cap \Psi(l)$=2-space = <x, y>, $x=x_1=u_1$, $y=x_2=u_2$. Then $x_3^{\perp} \cap\ \Psi(l)$=4-space=<x, y, u_3, u_4>. Since $\Psi(r)$ Í $\Psi(l)$, $\Psi(p) \cap \Psi(r)$=<x, y> and <x, y, x_3, x_4, u_3, u_4> is a TI 6-space, p~r i.e., $p^{\perp} \cap\ l=\{r\}$, mean while <x, y, x_3, x_4, u_5, u_6> is not TI 6-space, then p is not collinear to s.
3 $\Psi(p) \cap \Psi(l)$=3-space = <x, y, z>, $x=x_1=u_1$, $y=x_2=u_2$, $z=x_3=u_3$ and $x_1^{\perp} \cap \Psi(l)$=5-space=<x, y, z, u_4, u_5>. In this case the points r and s became $\Psi(r)$= <x, y, z, u_4> and $\Psi(s)$= <x, y, u_5, u_6>. Then there is a unique point, say, t incident to the line *l* such that $\Psi(t)$=<x, y, u_4, u_5>. Since $\Psi(t) \cap \Psi(p)$=2-space=<x, y> and <x_4, x, y, z, u_4, u_5> forms a TI 6-space, t is collinear to p i.e., $p^{\perp} \cap\ l=\{t\}$. Then according to the a above cases $D_{6,4}$ is gamma space.

The remaining part of the proof is to show that for any two non-collinear points p and q, $p^{\perp} \cap\ q^{\perp}$ is either empty, a single point, or a non-degenerate polar space of rank at least 2. By Proposition 2.1 and 2.2 we showed that for any pair of non-collinear points p and q, d(p, q)= 3 or 4 and we proved that for d(p, q)=2, $p^{\perp} \cap\ q^{\perp}$ is a non degenerate polar space and then for any line *l*, $l^{\perp}$ is not singular subspace. This completes the proof.

Properties of $D_{6,4}(F)$:

The most important properties that will be used for characterization are related to the relations between the different varieties of the geometry. The relations between the points and the symplecta and the relation between symplecta themselves will be investigated.

Proposition:

Let (p, S) be a non-incidence pair of a point p and a symplecton S. Then rank($p^{\perp} \cap$ S)= -1 ,0 or 2.

Proof:

Let p, S be a point and a symplecton such that $\Psi(p)$=<x_1, x_2, x_3, x_4> and $\Psi(S)$=<y_1, y_2, y_3> be the corresponding TI 4-space and 3-space to the point p and the symplecta S respectively. Then following cases for the intersection $\Psi(p) \cap \Psi(S)$ are satisfied:

1- $\Psi(p) \cap \Psi(S)$=2-space=<x, y>, where $x=x_1=y_1$, $y=x_2=y_2$ and this include two cases:

i- $y_3^{\perp} \cap \Psi(p)=\Psi(p)$. Then <$x_3$, x_4, x, y, y_3> can be extended to a maximal TI 6-space, <u, x_3, x_4, x, y, y_3>. Then we select a point r such that $\Psi(r)$=<u, x, y, y_3> and:
$\Psi(S)$Í$\Psi(r)$,
$\Psi(p) \cap \Psi(r)$=2-space =<x, y>

Then rÎS, r~p and $p^{\perp} \cap$ S={r}. i.e., rank($p^{\perp} \cap$ S)=0.

ii- $y_3^{\perp} \cap \Psi(p)$= <x, y, x_3>. This means algebraically $B(y_3, x_4) \neq 0$ (B is the symmetric hyperbolic bilinear form). Then we cannot find any point in S and collinear to the point p, that means $p^{\perp} \cap$ S= φ, i.,e. rank($p^{\perp} \cap$ S)= -1.

2- $\Psi(p)\cap\Psi(S)$=1-space=$<x>$, where $x=x_1=y_1$, there are two cases:

i- $y_2^{\perp}\cap\Psi(p)=\Psi(p)$ and $y_3^{\perp}\cap\Psi(p)=\Psi(p)$, then we find a three points r, s and t such that $\Psi(r)=<x, x_4, y_2, y_3>$, $\Psi(s)=<x, x_3, y_2, y_3>$ and $\Psi(t)=<x, x_2, y_2, y_3>$. Since these points constitute TI 6- space with $\Psi(p)$ that is $<x, x_2, x_3, x_4, y_2, y_3>$ and each of them contains $\Psi(S)$, then r, s and t are points in S and they are collinear to the point p. Then $p^{\perp}\cap S$ is a plane i.e., rank$(p^{\perp}\cap S)$= 2

ii- $y_2^{\perp}\cap\Psi(p)=\Psi(p)$ and $y_3^{\perp}\cap\Psi(p)=<x, x_2, x_3>$, then by choosing any point in S we show that its not collinear to the point p, so $p^{\perp}\cap S=\varphi$, i.,e. rank$(p^{\perp}\cap S)$= -1.

3- $\Psi(p)\cap\Psi(S)$=0-space

i- $y_1^{\perp}\cap\Psi(p)=<x_1, x_2, x_3>$, $y_2^{\perp}\cap\Psi(p)=<x_2, x_3, x_4>$ and $y_3^{\perp}\cap\Psi(p)=<x_1, x_3, x_4>$. Then no point in S collinear to p which means that rank$(p^{\perp}\cap S)$= -1.

ii- $y_1^{\perp}\cap\Psi(p)=\Psi(p)$, $y_2^{\perp}\cap\Psi(p)=<x_2, x_3, x_4>$ and $y_3^{\perp}\cap\Psi(p)=<x_1, x_3, x_4>$ or the case in which $y_1^{\perp}\cap\Psi(p)=\Psi(p)$, $y_2^{\perp}\cap\Psi(p)=\Psi(p)$ and $y_3^{\perp}\cap\Psi(p)=<x_1, x_3, x_4>$, its impossible to find a 4-space that contains $\Psi(S)$ and intersect with $\Psi(p)$ in a 2-space because $\Psi(p)\cap\Psi(S)$=0-space, so in those cases we have rank$(p^{\perp}\cap S)$= -1. The above cases gave $p^{\perp}\cap S$ is either empty a point or a plane i.e., rank$(p^{\perp}\cap S)$= -1, 0 or 2.

Proposition:

Let S_1 and S_2 be two symplecta in $D_{6,4}(F)$. Then rank$(S_1\cap S_2)$=-1 or 0 .

Proof:

Let S_1, S_2 be two symplecta such that $\Psi(S_1)=<x_1, x_2, x_3>$ and $\Psi(S_2)=<y_1, y_2, y_3>$ are the corresponding TI3-spaces to the symplecta S_1 and S_2 respectively. Then there are three cases for the intersection $\Psi(S_2)\cap\Psi(S_2)$:

1 $\Psi(S_2)\cap\Psi(S_2)$=2-space=$<x, y>$, where $x=x_1=y_1$ and $y=x_2=y_2$ and $y_3^{\perp}\cap\Psi(S_1)=\Psi(S_1)$. Then we find just one point, p, such that $\Psi(p)=<x_3, x, y, y_3>$ and satisfies the condition:
$\Psi(S_1)$ Í $<x_3, x, y, y_3>$,
$\Psi(S_2)$ Í $<x_3, x, y, y_3>$
Then $S_1\cap S_2$ is a point i.e., rank$(S_1\cap S_2)$=0. Now If $y_3^{\perp}\cap\Psi(S_1)\neq\Psi(S_1)$ $(B(x_3, y_3)\neq 0)$, then the only 4-space $<x_3, x, y, y_3>$ is not Totally isotropic. This means that $S_1\cap S_2=\varphi$,i.e., rank$(S_1\cap S_2)$=-1.

2 $\Psi(S_2)\cap\Psi(S_2)$=1-space=$<x>$, where $x=x_1=y_1$. Then any choice of 4-space contains either $\Psi(S_1)=<x, x_2, x_3>$ or $\Psi(S_2)=<x, y_2, y_3>$ but not both, so $S_1\cap S_2=\varphi$ and rank$(S_1\cap S_2)$=-1.

3 $\Psi(S_2)\cap\Psi(S_2)$=0-space, we cannot find any TI 4-space that contains both $\Psi(S_2)$ and $\Psi(S_2)$, so rank$(S_1\cap S_2)$=-1. Then we have rank$(S_1\cap S_2)$=-1or 0.

Conclusion:

Firstly, the point line geometry $D_{5,3}$ was proved that it is a strong parapolar space with diameter equal 5 see (Zayda, Abdelsalam, 2007). We have proved here that the point line geometry $D_{6,4}$ is a strong parapolar space with diameter equal 6. So, we have the following interesting problems:

1- Can we present a general case for the geometries $D_{5,3}$ and $D_{6,4}$ that is: $D_{n,n-2}$, $n\geq 5$?.

2- Is $D_{n,n-2}$ embedded in a projective space? Describe the hyperplanes of such geometry.

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