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Calculations of Permanent Magnet Using Distributed-Parameters Equivalent Circuit

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Abstract: In the current work a procedure for determination the ratio of the length to the diameter of the permanent magnet, at which the ratio of the leakage flux to the desired flux through the magnet body is minimized, has been developed. The permanent magnet has been modeled by an electric equivalent circuit with distributed parameters, which describe the real physical phenomena in the permanent magnet. A cylindrical permanent magnet has been studied and calculated. In the equivalent circuit, of this cylindrical permanent magnet, is taken into account the difference of magnetic potentials between the magnetic poles. So for cylindrical permanent magnet in order to get less, relatively, leakage flux the permanent magnet must be designed with the ratio of its length to the radius of 1.97 times. The developed procedure enables us to determine the permanent magnet dimensions with higher, relatively, magnetic flux, which makes the magnet more efficient. So this will be very useful in permanent magnet design.

Key words: Leakage flux, hysteresis loop, distributed parameters, cylindrical form

INTRODUCTION

Permanent magnets have many different applications, such as in electrical measuring instruments, magnetic bearing (Andrew J.,*et al.,* 2003; Jiles, D.C.; S.H. Song, 2007; Hawkins, L.A., 1997 and Lee S.J., *et al.,* 2007) and electrical machines (M.A. Rahman, *et al.,* 2006; S. Vaes-Zadeh; A. Hassanpour, 2007; Lukaniszyn M., *et al.,* 2003; Mendrela E.A., *et al.,* 2003; Lukaniszyn M., *et al.,* 2002; Jagiela M.; Wrobel R., 2003; Wrobel R. and Mellor P., 2003) as a source of the magnetic field and in many other applications (Wrobel R. nd Mendrela E.A., 1999; Kenny A., *et al.,* 2001; Lukaniszyn M. and Jagiela M., 2003).

Magnetic field is necessary in operation of many different machines and devices. To get magnetic field may be used electromagnetic circuits or permanent magnets made from hard-iron ferromagnetic materials with high coercive magnetomotive force. Permanent magnets are made from different alloys, such as iron, aluminum, nickel, copper, cobalt, titanium and other materials (Belov K.P., 1987; Osin I.L. and Shakaryan U.G., 1990). Many works focus on the behavior and properties of permanent magnets (Andrew J., *et al.,*2003; Melikhov Y. and Jiles D.C., 2007; Lee S.J., *et al.,* 2007; Altman A.B. and Vernikovsky E.E., 1980; Zimin E.F. and Kochanov E.S., 1985). But the problem here is how to design the permanent magnet and how to make calculations in order to determine its construction with optimal dimensions, mass, cost and other parameters. Therefore, the main objective of the research is to find the ratio of the length to the diameter of the permanent magnet, at which the ratio of the leakage flux to the desired flux through the magnet body is minimized.

Permanent Magnet Calculations:

The main purpose of this paper is to develop a procedure for calculations of the permanent magnet dimensions in order to get minimum leakage flux relative to the useful desired magnetic flux. These calculations are based on the magnetization curve of the used material as shown in figure 1.

Let the permanent magnet has a cylindrical form as shown in figure 2. In real permanent magnet its surface does not have the same magnetomotive force. Therefore, it has different magnetic potentials at different points of its height. For simplification calculations, suppose that the magnetic potential is linear distributed along the magnet.

The total magnetic flux of this magnet is the sum of leakage flux from the side surface and the flux through the magnet itself as shown in figure 2, and may be determined by the following equation:

$$\Phi = \Phi_{mag} + \Phi_{l},$$

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where

Φ - the total magnetic flux of the permanent magnet,
Φ_{mag} - the magnetic flux through the magnet itself,
Φ_l - the leakage magnetic flux from the side surface.

But magnetic flux through the permanent magnet side will vary along it, thus it is better to determine the total magnetic flux with respect to the cross sectional area of the permanent magnet poles. Therefore, the shown in figure 3 equivalent circuit of the permanent magnet may be used, where:

R_i - the magnetic reluctance of the i^{th} element disc with the height dl of the permanent magnet as shown in figure 2, (i = 1, 2, 3, …, n),
ρ_i - the magnetic permeance through the air per unit of length at the permanent magnet surface between two adjacent elemental discs of the permanent magnet, (i = 1, 2, 3, …, n-1).

The equivalent circuit of figure 3 may be replaced by the simplified equivalent circuit of figure 4, where $F = H \cdot l$,

F - the magnetomotive force,
l - the equivalent length of the permanent magnet,
H - the mean value of demagnetizing magnetic field intensity inside the permanent magnet,
R_{eq} - the equivalent magnetic reluctance of the whole permanent magnet calculated from the equivalent circuit in figure 3.

Demagnetizing magnetic field intensity may be determined using the magnetization curve of the permanent magnet material (the hysteresis loop).

The value of R_{eq} may be calculated as follows (Neyman L.R. and Kalantarov P.L., 1959):

$$R_{eq} = R_o \cdot \tan(\gamma \cdot l),$$

where $R_o = \sqrt{R_i/\rho_i}, \gamma = \sqrt{R_i \cdot \rho_i}$.

Thus $R_{eq} = \sqrt{R_i/\rho_i} \cdot \tan(l \cdot \sqrt{R_i/\rho_i})$.

And from magnetization curve, residual magnetic flux density may be determined. Thus the magnetomotive force can be found, in first approximation, as follows:

$$F = H_1 l = \Phi \cdot R_{eq} = \frac{B_r \cdot \pi \cdot d^2}{4} \sqrt{R_i/\rho_i} \cdot \tan(l \cdot \sqrt{R_i/\rho_i}),$$

where:

H_1 - the first approximation of the mean value of the demagnetizing magnetic field intensity in the permanent magnet body,
B_r - the residual magnetic flux density,
$\frac{\pi \cdot d^2}{4}$ - the cross sectional area of the permanent magnet.

From the last equation yields:

$$H_1 = \frac{B_r \cdot \pi \cdot d^2}{l \cdot 4} \sqrt{R_i / \rho_i} \cdot \tan(l \cdot \sqrt{R_i / \rho_i}).$$

It is well known that B_r is the residual magnetic flux density when the external magnetizing force is removed. Therefore, from the magnetization curve, it is clear that the residual flux density will be reduced to the value B_{r2} as shown in figure 1. As a result of that the magnetic flux decreases:

$$\Phi = B_{r2} \cdot \pi \cdot r^2,$$

where $r = d/2$.

And so the resultant demagnetizing magnetic field intensity decreases also:

$$H_2 = \frac{B_{r2}}{l} \pi \cdot r^2 \cdot \sqrt{R_i / \rho_i} \cdot \tan(l \cdot \sqrt{R_i / \rho_i}).$$

Thus the new value of the mean demagnetizing magnetic field intensity H_2 will be less than the value of H_1. In the other hand, this decrease of H makes the residual flux density to increase to be B_{r1} as shown in figure 1. The last one gives a new smaller H and so on.

The values of the demagnetizing magnetic field intensity H_1 and H_2 are the limit values of the studied case: H_1 is the upper limit and H_2 is the lowest limit. Thus the geometric mean value of H may be found: $H = \sqrt{H_1 \cdot H_2}$.

Using the magnetization curve, the last equation may be rewritten as follows:

$$H = \frac{\sqrt{B_{r2} \cdot B_r}}{l} \pi \cdot r^2 \cdot \sqrt{R_i / \rho_i} \cdot \tan(l \cdot \sqrt{R_i / \rho_i}) \qquad (1)$$

To be able to calculate H the values of R_i and ρ_i must be known. First one may be found by the following equation:

$$R_i = \frac{dl}{\mu \cdot A} = \frac{dl}{\mu \cdot \pi \cdot r^2} \qquad (2)$$

where

$\mu = B/H$ - the magnetic permeability of the permanent magnet material.

The value of the magnetic permeability depends on the point on magnetization curve at which it operates. And so if this point is not known it is impossible to find the value of μ. And if μ is not known then R_i cannot be determined and so the operating point cannot be found.

To avoid this paradox or dead point, instead of the permeability value may be used the ratio B/H at the point of maximum energy $(BH)_{max}$ (Osin I.L. and Shakaryan U.G., 1990).

Thus magnetic reluctance may be calculated using equation (2) assuming that $\mu = B/H$ at the point of maximum energy $(BH)_{max}$ of the magnetization curve of permanent magnet material.

For magnetic permeance determination ρ_i, the condition of linear magnetic potential along the permanent magnet surface may be used (Altman A.B. and Vernikovsky E.E., 1980). And by using the spheroid equivalent of the cylindrical permanent magnet (Mizyuk L.Ya., 1984) permeance may be found:

$$\rho_i = \frac{\pi\cdot\mu_o\cdot(1-\frac{r^2}{l}\ln\frac{2l}{r})}{3\left[\ln(\frac{2l}{r})-1\right]} \tag{3}$$

Substituting equation (2) and (3) in (1) yields:

$$H = \frac{\sqrt{B_{r2}\cdot B_r}}{l}\pi r^2\sqrt{\frac{3\left[\ln\frac{2l}{r}-1\right]}{\mu\pi^2 r^2\mu_o\left[1-\frac{r^2}{l^2}\ln\frac{2l}{r}\right]}}\tan\left[\sqrt{\frac{\mu_o\left[1-\frac{r^2}{l^2}\ln\frac{2l}{r}\right]}{3\mu r^2\left[\ln\frac{2l}{r}-1\right]}}\right]$$

At $r << l$ the last equation may be simplified:

$$H = \frac{\sqrt{B_{r2}\cdot B_r}}{l}A\sqrt{\frac{3\left[\ln\frac{2l}{r}-1\right]}{A\cdot\pi\cdot\mu\cdot\mu_o}}\tan\left[\sqrt{\frac{\mu_o\cdot\pi}{3\mu A\left[\ln\frac{2l}{r}-1\right]}}\right]$$

where

$A = \pi r^2$ - the cross sectional area of the permanent magnet.

The distributed parameters R_i and ρ_i give ability for determination of the optimal ratio of the permanent magnet length (l) and radius (r). Assuming that $\mu >> \mu_0$ the total permeance of the permanent magnet body and the permeance for leakage flux may be calculated as follows:

$$\rho_b = \frac{1}{R_i\cdot l} = \frac{\mu\cdot A}{l} \tag{6}$$

$$\rho_{lf} = \rho_i\,l = \frac{\pi\cdot\mu_o\cdot l\,(1-\frac{r^2}{l^2}\ln\frac{2l}{r})}{3\,(\ln(\frac{2l}{r})-1)} \tag{7}$$

where:

ρ_b - the total permeance of the permanent magnet body,
ρ_{if} - the total permeance for leakage flux.

By dividing equation (7) by equation (6) yields:

$$\frac{\rho_{\gamma}}{\rho_{b}} = \frac{\mu_o\left[\frac{l^2}{r^2} - \ln(2\frac{l}{r})\right]}{3 \cdot \mu \cdot \left[\ln(2\frac{l}{r}) - 1\right]} \tag{8}$$

Assuming that $X = \frac{l}{r}$ - the ratio of the permanent magnet length to its radius, the last equation becomes:

$$\frac{\rho_{\gamma}}{\rho_{b}} = \frac{\mu_o\left[X^2 - \ln 2X\right]}{3 \cdot \mu \cdot \left[\ln(2X) - 1\right]} \tag{9}$$

To get minimum ratio of the total permeance of the leakage flux to the total permeance of the permanent magnet body, it is useful to find the derivation of this ratio and equal it to zero.

$$\frac{d}{dX}\left(\frac{\rho_{\gamma}}{\rho_{b}}\right) = 0 \ .$$

And finally the last equation gives:

$$2 \cdot \ln(2X) - 3 + \frac{1}{X^2} = 0$$

This equation has two solutions: $X \approx 1.97$ $X \approx 0.626$. Considering the assumed condition r << *l*, t first solution only is acceptable. Therefore, the ratio of the permanent magnet length to its radius, which gives minimum ratio of leakage flux to the flux through the body of the permanent magnet itself, is equal to 1.97 (*l/r*=1.97). And at this value of (*l/r*) the ratio of the total permeance for leakage flux to the total permeance of the permanent magnet body will be:

$$\frac{\rho_{\gamma}}{\rho_{b}} = 2.254\frac{\mu_o}{\mu}.$$

Therefore, the developed procedure for determination the permanent magnet dimensions, finally, gives simple equations for calculations and avoids the complexity in other works. But for short-length permanent magnet the ratio of the magnetic permeance through side surface to the magnetic permeance through magnet pole faces using the last approximated equation will be:

$$\frac{\rho_{\pi}}{\rho_{m}} = \frac{\pi^2 \cdot \mu_o \cdot r}{8 \cdot \mu_o \cdot r} \approx constant.$$

This study was conducted in Tafila Technical University in Jordan and in Yanbu Industrial College in KSA in 2006-2007.

RESULTS AND DISCUSSION

Results:

In this research has been developed a procedure for determination the ratio of the length to the diameter of the permanent magnet, at which the ratio of the leakage flux to the desired flux through the magnet body

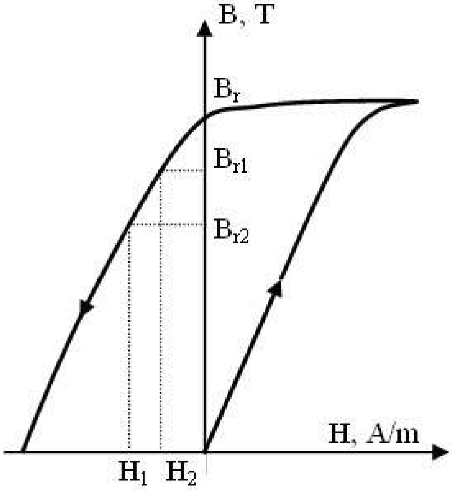

Fig. 1: Magnetization and demagnetization curves of the permanent magnet material.

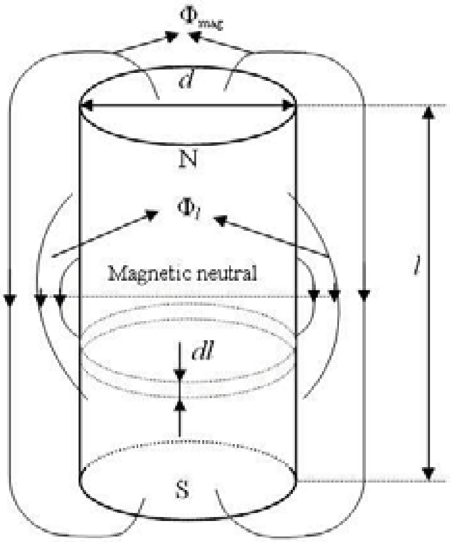

Fig. 2: The cylindrical permanent magnet.

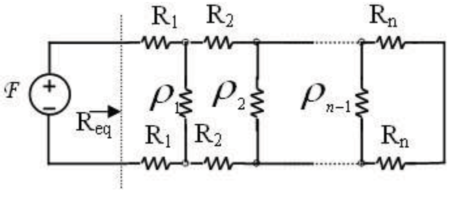

Fig. 3: The equivalent circuit of the permanent magnet with distributed parameters.

is minimized. For cylindrical permanent magnet to get less, relatively, leakage flux the ratio (l / r) must be equal to 1.97. Then ratio of the total permeance for leakage flux to the total permeance of the permanent magnet inside will be 2.254 μo /μ. Therefore, these result are restricted to the cylindrical permanent magnet with $l >> r$. But for short-length permanent magnet, unlike for long permanent magnet, the value of permeance through the useful (desired) magnetic flux path and through the leakage magnetic flux path do not depend on the length of the permanent magnet. But they depend only on the magnet diameter.

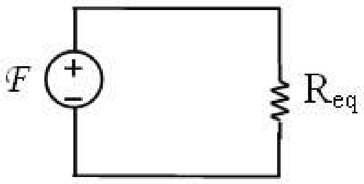

Fig. 4: The equivalent circuit of the petrmanent magnet model.

Discussion:

The permanent magnet has been modeled by an electric equivalent circuit with distributed parameters, which describes the real physical phenomena in the permanent magnet. A procedure for permanent magnet calculations has been developed. Such a procedure enables one to determine the permanent magnet dimensions, particularly the ratio between the length and the diameter, at which the ratio of the leakage magnetic flux to the desired flux through the magnet body is minimized. This will be very useful in permanent magnet design.

For a permanent magnet with cylindrical form the ratio of the permanent magnet length to its radius, which gives minimum ratio of leakage magnetic flux to the flux through the body of the permanent magnet itself is equal to 1.97 ($l / r = 1.97$). These results may be useful in permanent magnet design.

But in this research the final results are restricted to the cylindrical form permanent magnet. So for permanent magnets with other shapes additional study is required. The significance of these results, if compare with other works, is in using the distributed parameters equivalent circuit for getting the optimal ratio (l / r) for the permanent magnet.

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