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On Baseband Transmission and Reconstruction of Signals

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Abstract: We consider an audio signal which has been sampled synchronously and uniformly at two sampling frequencies, the first equal to its bandwidth, and the second to 150% of its bandwidth. Therefore the two corresponding discrete-time signals are undersampled versions of the original analog signal. First we analytically prove that the original signal can be reconstructed from the two undersampled signals. Then we describe a practical implementation of method by a block diagram and a synthesized circuit. Finally we show an experimental example which validates the approach. The advantage of the proposed method is that baseband audio signal transmission can be done by two parallel transmission channels, whose bandwidth is only half of the original audio signal bandwidth.

Key words: undersampled, audio, signal, reconstruction, aliasing.

INTRODUCTION

In many contexts, processing of discrete – time signals is more flexible and is often preferable to processing of continuous – time signals, in part because of increasing availability of inexpensive, light – weight, programmable and easily reproducible digital and discrete – time systems. For example, in communication systems, continuous signals such as speech are sampled and converted to discrete – time signals and then they are quantized in order to convert to digital signals. Then, digital signals are modulated as PCM, PAM and etc. Later, modulated digital signals are transmitted. Conversely, in destination, again digital signals are converted to discrete – time signals and then, to continuous – time signals. Other example is digital control systems which work in similar manner.

A continuous – time signal is converted to discrete – time signal by sampling and under certain conditions original continuous – time signal can be completely recovered from its sampled signal. This somewhat surprising property follows from a basic result which is referred to as the sampling theorem. Much of the importance of the sampling theorem also lies in its role as a bridge between continuous – time signals and discrete – time signals (Oppenheim, A.V., A.S. Willsky, 1983). Recently, reconstruction of a signal from its samples has attracted considerable attention. Most results addressing this problem available in the signal processing literature (Venkataramani, R. and Y. Bresler, 2003), (Tseng Ching-Hsiang, 2002), (Choi Kwonhue and Lee Joon-Ho, 2004), (Prendergast, R.S., B.C. Levy, 2004). Also there are many works in reconstruction and sampling of audio signals (Cvetkovic, Z. and J.D. Johnston, 2003), (Esquef, P.A.A. and L.W.P. Biscainho, 2006). In this paper, we consider an audio signal *x(t)* which can be sampled synchronously with sampling

frequencies $\omega_{S_1} = \omega_M$ and $\omega_{S_2} = 1.5\omega_M$ Where ω_M is the bandwidth of *x(t)*. As we see, ω_{S_1} and ω_{S_2} are less than $2\omega_M$. So, we have aliasing in the spectra of sampled signals. With considering above

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conditions, a set of essential equations will be proposed for exact reconstruction of $X(\omega)$, where $X(\omega)$ is the spectra of the original signal $x(t)$.

2. Undersampled Signal:

Consider the signal $x(t)$ and the spectra of it $X(\omega)$ which is shown in Fig. 1. As we see $x(t)$ is band-limited

signal with $X(\omega) = 0 \text{ for } |\omega| > \omega_M$. Let us, define the impulse train

$$p(t) = \sum_{m=-\infty}^{+\infty} \delta(t - mT) \tag{1}$$

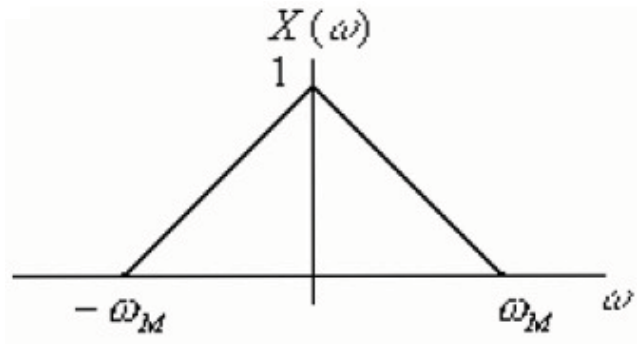

Fig. 1: Spectra of the original signal.

where $p(t)$ is referred to as the sampling function, T as the sampling period, and the fundamental frequency of $p(t)$, $\omega_S = \frac{2\pi}{T}$, as the sampling frequency. In time domain, we define

$$x_p(t) = x(t)p(t) \tag{2}$$

where $x_p(t)$ is an impulse train with the amplitude of impulses equal to the samples of $x(t)$ at intervals spaced by T, that is,

$$x_p(t) = \sum_{m=-\infty}^{+\infty} x(mT)\delta(t - mT) \tag{3}$$

Since, we have

$$X_p(\omega) = \frac{1}{2\pi}[X(\omega) * P(\omega)] \tag{4}$$

and with using the fact that

$$p(\omega) = \frac{2\pi}{T} \sum_{k=-\infty}^{+\infty} \delta(\omega - k\omega_s) \tag{5}$$

it follows that

$$X_p(\omega) = \frac{1}{T} \sum_{k=-\infty}^{+\infty} X(\omega - k\omega_s) \tag{6}$$

That is, $X_P(\omega)$ is a periodic function of frequency consisting of a sum of shifted replicas of $X(\omega)$ scaled by $\frac{1}{T}$ as illustrated in Fig. 2. In Fig. 2(a), $\omega_s > 2\omega_M$ and thus, there is no overlap between the shifted replicas of $X(\omega)$, whereas in Fig. 2(b) with $2 < \omega_s < 2\omega_M$, there is overlap (Oppenheim, A.V., A.S. Willsky, 1983). For the case illustrated in Fig. 2(a), $X(\omega)$ is faithfully reproduced at integer multiples of sampling frequency. Consequently, if $\omega_s > 2\omega_M$, $x(t)$ can be recovered exactly from $x_p(t)$ by means of a

low-pass filter with gain T and a cutoff frequency greater than ω_M and less than $\omega_S - \omega_M$. As we know, this basic result, referred to as the sampling theorem (Oppenheim, A.V., A.S. Willsky, 1983).

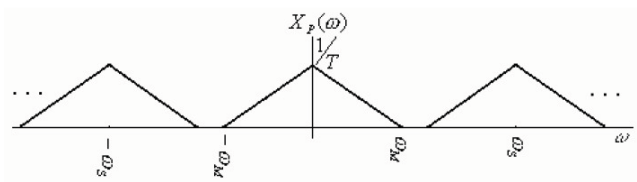

Fig. 2: (a) Spectra of sampled signal with $\omega_s > 2\omega_M$

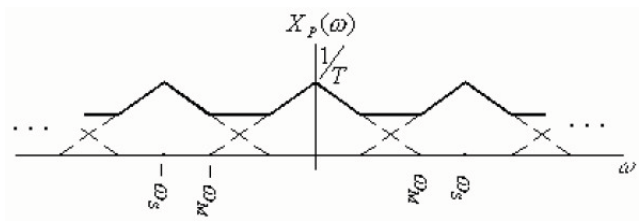

Fig. 2: (b) Spectra of sampled signal with $\omega_s < 2\omega_M$

When $\omega_s < 2\omega_M$, $X(\omega)$ is no longer replicated in $X_p(\omega)$ and thus is no longer recoverable by low-pass filtering. This effect, in which the individual terms in equation (6) overlap, is referred to as aliasing, and in this case, $x_p(t)$ is said the undersampled signal of $x(t)$. Undersampled signals are very important. In practice, they are applied in filtering, radar, sonar and etc (Santraine, A., S. Leprince , 2006), (Prendergast, R.S. and T.Q. Nguyen, 2005), (Zolesio, J.L. and B. Olivier, 1992). For example, blind signal in FM echoes, can be separated with using undersampled signals (Tsuruta, K., K. Teramoto, 2002). Even, in some cases, reconstruction of a signal from other parameters is very important (Sun Wenchang and Zhou Xingwei, 2002).

3. Reconstruction:

Again, consider the band-limited signal $x(t)$ and the spectra of it $X(\omega)$ which is shown in Fig. 1. Suppose that, $x_{p_1}(t)$ and $x_{p_2}(t)$ are samples of $x(t)$ with sampling frequencies $\omega_{s_1} = \omega_M$ and $\omega_{s_2} = 1.5\omega_M$ respectively. So, we have

$$X_{p_1}(\omega) = \frac{1}{T_1}\sum_{k=-\infty}^{+\infty} X(\omega - k\omega_s) \tag{7}$$

and

$$X_{p_2}(\omega) = \frac{1}{T_2} \sum_{k=-\infty}^{+\infty} X[\omega - k(1.5\omega_M)]. \tag{8}$$

Where $T_1 = \frac{2\pi}{\omega_M}, T_2 = \frac{2\pi}{1.5\omega_M}$ and also $X_{p_1}(\omega)$ and $X_{p_2}(\omega)$ are the spectra of $x_{p_1}(t)$ and $x_{p_2}(t)$ respectively. Equation (8) with using the fact that

$$X(\omega) = 0 \text{ for } |\omega| > \omega_M \tag{9}$$

gives

$$X(\omega) = T_2 X_{p_2}(\omega) \text{for } |\omega| < 0.5\omega_M \tag{10}$$

Also from (7) with noting (9), we have

$$X(\omega = \omega_M) = T_1 X_{p_1}(\omega) - X(\omega) \text{ for } -0.5\omega_M \le \omega \le 0 \tag{11}$$

and

$$X(\omega = \omega_M) = T_1 X_{p_1}(\omega) - X(\omega) \text{ for } 0 \le \omega \le 0.5\omega_M \tag{12}$$

So, $X(\omega)$ can be reconstructed exactly with using (10), (11) and (12).

Consider the block diagram of baseband audio signal transmission system that is shown in Fig. 3. The block diagram has been made with using above proposed method. The system consists of three sections, the transmitter, two parallel transmission channels, and receiver. Where t_d is the delay time of each transmission channel and the block diagram of the receiver has been made based on equations (10), (11) and (12).

As we see, baseband audio signal transmission can be done by two parallel transmission channels that the bandwidth of each channel is only 50% of bandwidth of the audio signal

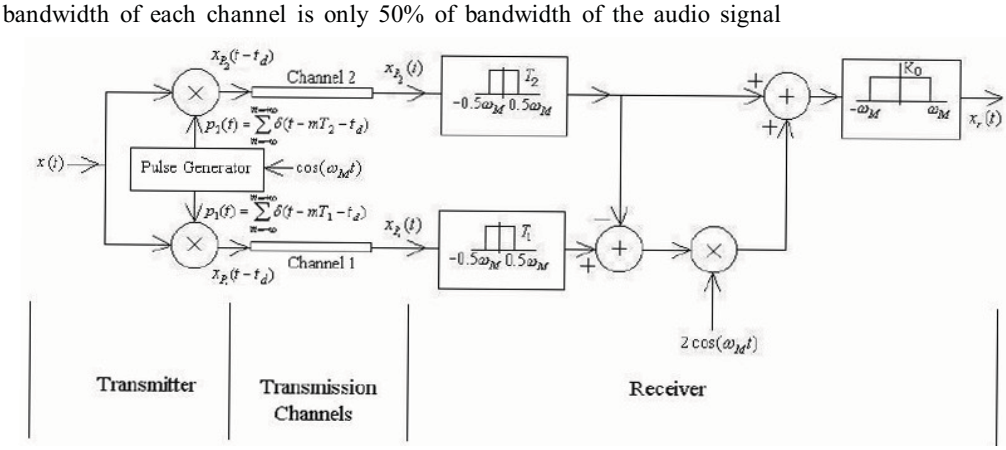

Fig. 3: The block diagram of the system.

4. Approximation and Synthesis:

Again consider the block diagram of the receiver which has been shown in Fig. 3. Since original signal is audio, phase distortion in output spectra or in other word delay time distortion in reconstructed signal is not very important. So we center on the approximation of the amplitude characteristic for all units of the block diagram. For the synthesized and simulated circuit shown in Fig. 4, the audio signal of the telephone line with bandwidth has 3*kHz* been considered as the original signal. The reconstructed signal $x_r(t)$ is the output of the circuit and two order standard low pass transfer functions has been used for the approximation of the amplitude characteristics. So, we have

$$\varpi^M = 2\pi \times 3000 \, {}^{rad}\!/_{\sec}, T_1 = \frac{1}{3000}\sec, T_2 = \frac{1}{4500}\sec$$

and also transfer functions as the form

$$H(s) = \frac{K_0}{(\frac{s}{\varpi_c})^2 + \frac{1}{Q}(\frac{s}{\varpi_c}) + 1} \tag{13}$$

with Q = 0.1, have been used in the designed circuit.

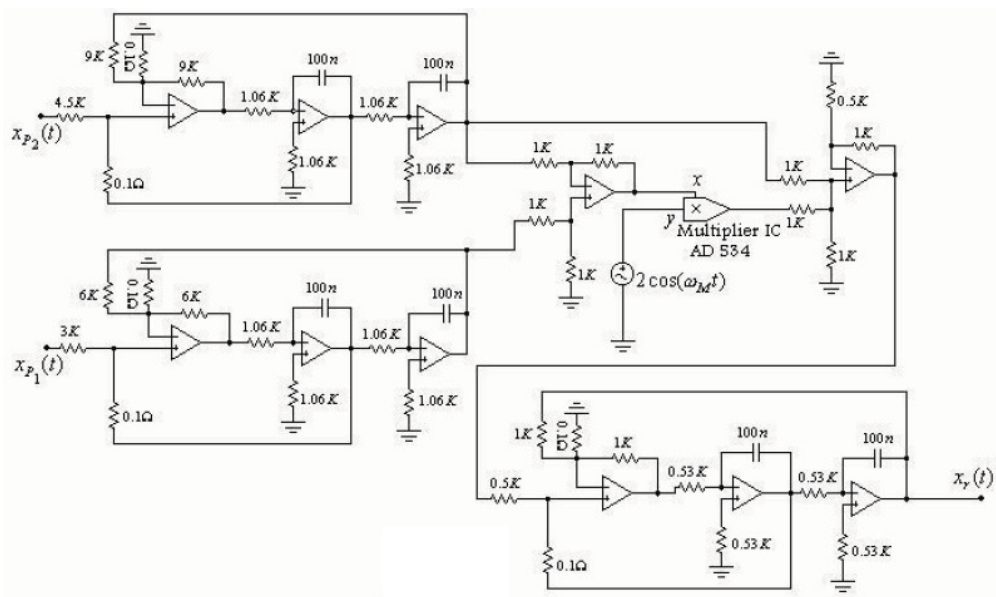

Fig. 4: The synthesized and simulated circuit for *B.W =3kHz.*

Example 1:

For simulation, consider the original audio signal *x(t)* which is shown in Fig. 5(a). This signal has been sampled synchronously with sampling frequencies equal to 3kHz and 4.5kHz ($x_{p_1}(t)$ and $x_{p_2}(t)$ respectively). The output of the above simulated circuit (the reconstructed signal $x_r(t)$) is shown in Fig. 5(b).

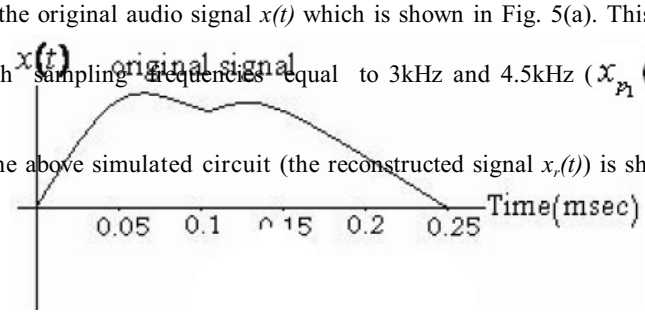

Fig. 5: (a) The original audio signal *x(t)*.

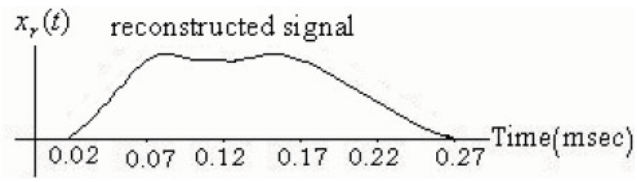

Fig. 5: (b) The reconstructed signal $x_r(t)$.

Example 2:

Consider the simulated circuit which has been shown in Fig. 4. We realized it in lab with using CA3030s as the Op-Amps, V_{DD} = +12V (positive voltage source of the Op-Amps), V_{SS} = -12V (Negative voltage source the Op-Amps). An audio signal was applied as the original (input) signal *x(t)*. It was sampled synchronously

with sampling frequencies equal to 3kHz and 4.5kHz ($x_{p_1}(t)$ and $x_{p_2}(t)$ respectively). The spectra of the

original (input) signal (*X(f)*) and reconstructed (output) signal ($X_r(f)$) are shown in Fig. 6(a) and Fig. 6(b) respectively. The spectra have been measured by spectra analyzer. So, in above two examples, the baseband audio signal was transmitted by two parallel transmission channels that the bandwidth of each channel was 1.5kHz (50% of bandwidth of the audio signal). As we see, the results of simulation (example1) and actual experimental work (example2) validate the presented proposal.

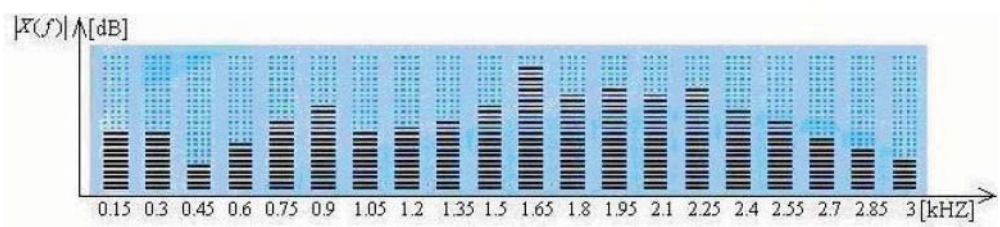

Fig. 6: (a) The spectra of the original audio signal *X(f)*.

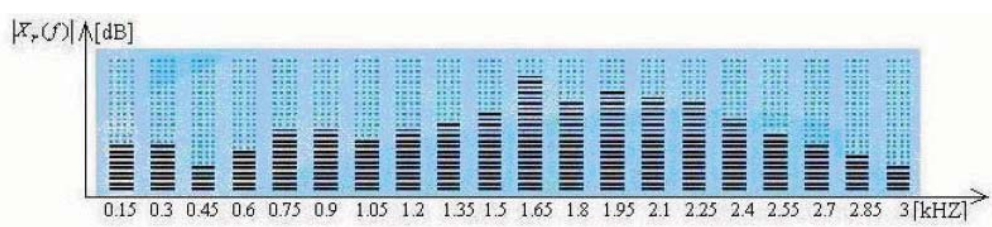

Fig. 6: (b) The spectra of the reconstructed signal $X_r(f)$.

5. Conclusions:

In this paper, we considered an audio signal which was sampled synchronously and uniformly at two

sampling frequencies. Then equations (10), (11) and (12) proposed to reconstruct the original signal. The advantage of the proposed method was that baseband audio signal transmission can be done by two parallel transmission channels, whose bandwidth is only half of the original audio signal bandwidth. The results of simulation and actual experimental work validated the proposed signal reconstruction scheme.

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REFERENCES

Choi Kwonhue and Lee Joon-Ho, 2004. Phase discontinuity-free sampling timing control for IF sampling receiver. IEEE Signal Processing Letters, (11): 810 – 812.

Cvetkovic, Z. and J.D. Johnston, 2003. Nonuniform oversampled filter banks for audio signal processing. IEEE Trans. Speech and Audio Processing, (11): 393-399.

Esquef, P.A.A. and L.W.P. Biscainho, 2006. An efficient model-based multirate method for reconstruction of audio signals across long gaps. IEEE Trans. Audio, Speech and Language Processing, (14): 1391-1400.

Oppenheim, A.V., A.S. Willsky and I.T. Young, 1983. Signals and Systems. Prentice-Hall: Englewood Cliffs, NJ.

Prendergast, R.S., B.C. Levy and P.J. Hurst, 2004. Reconstruction of band-limited periodic nonuniformly sampled signals through multirate filter banks. IEEE Trans. Circuit and Systems I., (51): 1612-1622.

Prendergast, R.S. and T.Q. Nguyen, 2005. Optimal filter bank reconstruction of periodically undersampled signals. In the Proceedings of IEEE Conference Acoustics, Speech, Signal Processing, 4: 201–204.

Santraine, A., S. Leprince and F. Taylor, 2006. Multiplier-free bandpass channelizer for undersampled applications. IEEE Signal Processing Letters, (11): 904-907.

Sun Wenchang and Zhou Xingwei, 2002. Reconstruction of band-limited signals from local averages. IEEE Trans. Information Theory, (48): 2955-2963.

Tseng Ching-Hsiang, 2002. Bandpass sampling criteria for nonlinear systems. IEEE Trans. Signal Processing, (50): 568 – 577.

Tsuruta, K., K. Teramoto, K. Kido and H. Mori, 2002. Blind signal separation with undersampled signals for frequency modulated echoes. In the Proceedings of 41st SICE Annual Conference, 5: 3232–3235.

Venkataramani, R. and Y. Bresler, 2003. Sampling theorems for uniform and periodic nonuniform MIMO sampling of multiband signals. IEEE Trans. Signal Processing, (51): 3152 – 3163.

Zolesio, J.L. and B. Olivier, 1992. Signal reconstruction via under-sampled signals. In the Proceedings of International Conference Radar, 92: 304–307.