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The Analyze of Vibration of Composite Elliptical Shell

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Abstract: One of the important subjects in designing intended structures is the calculation of the natural frequencies. In this article, the composite elliptical shell is modal analyzed by using Finite Element Method (FEM). The composite model made by the use of fiber glass and epoxy that the fibers are scattered randomly and all of the characteristics are similar in every direction. The test is performed by simulating the problem in the polar coordinate system. The determined Frequency Response Function (FRF), curves in modal test is related to the 16 degrees of freedom of model. The natural frequencies of the shell in ANSYS with the FREE_FREE boundary condition are compared with the test. The weight and the vibratory response of the shell have depended on their materials. Also geometrical parameters are effective on the natural frequency and the fiber arrangement with different angles has an important role in the natural frequency. All these results are compared with the test, and then the effects of different parameters on the natural frequencies of the shell are analyzed.

Key words: Elliptical shell-composite- modal test-natural frequency

INTRODUCTION

Composite materials, because of their specific advantages over conventional materials, find extensive applications in aerospace, marine, automotive, civil and other engineering sectors in different structural component and shapes. The manufacturing of composites involves the control of a large number of parameters. The shell shape has some special characteristics in comparison with structural forms such as beams, plates and frames. Some of these are: high strength to weight ratio, very small thickness ratio with span and radius of curvature, high stiffness, high degree of reserve strength and stability, and containment of space.(Yadav and Verma, 1998)

Modal analysis is a method for obtaining a reliable model to describe the dynamic response of different structures through the use of structural characteristics. The response of a structure under load depends on its geometry and the material properties. In conventional structural analysis techniques used for composites and conventional materials, it is common to treat the material properties (elastic modulus, Poisson ratio, etc.) as deterministic quantities.

In modal analysis, the best and most reliable method for extracting the essential modal characteristics is modal test. The modal analysis is used for different purposes including determining and evaluating vibratory phenomena; verifying, correcting and updating analytical dynamic models; creating dynamic models based on the test; evaluating the safety of a structure; diagnosing the defects of a structure; harmonizing the model with other dynamic fields such as acoustic, fatigue, etc.

In the current research, the results of the modal analysis of the composite elliptical shell are compared with those obtained through Finite Element Method (FEM). In addition, the accuracy of the numerical results and the effects of different parameters on the natural frequencies of the shell are analyzed.

Modal Analysis by Experimental Test:

According to Fig.1, making a shell begins with a semi-elliptical wooden model. Then, a composite model is made using fiberglass and epoxy. Finally, the two semi-elliptical are attached for elliptical shell to be ready for the test. Fig.2-a is an overview of the composed shell. The materials of Glass-Epoxy which is used to make the shell are composed of tiny fibers which are scattered randomly and with the features that are similar in every direction.

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The properties of shell: $E = 34 \quad Gpa; \quad \nu = 0.26; \quad \rho = 1754 \quad kg/m^3$

Fig. 1a: Semi-oval wooden model.

Fig. 2a: Overview of the composed shell. **Fig. 2b:** Measuring the response in one point (Point 32).

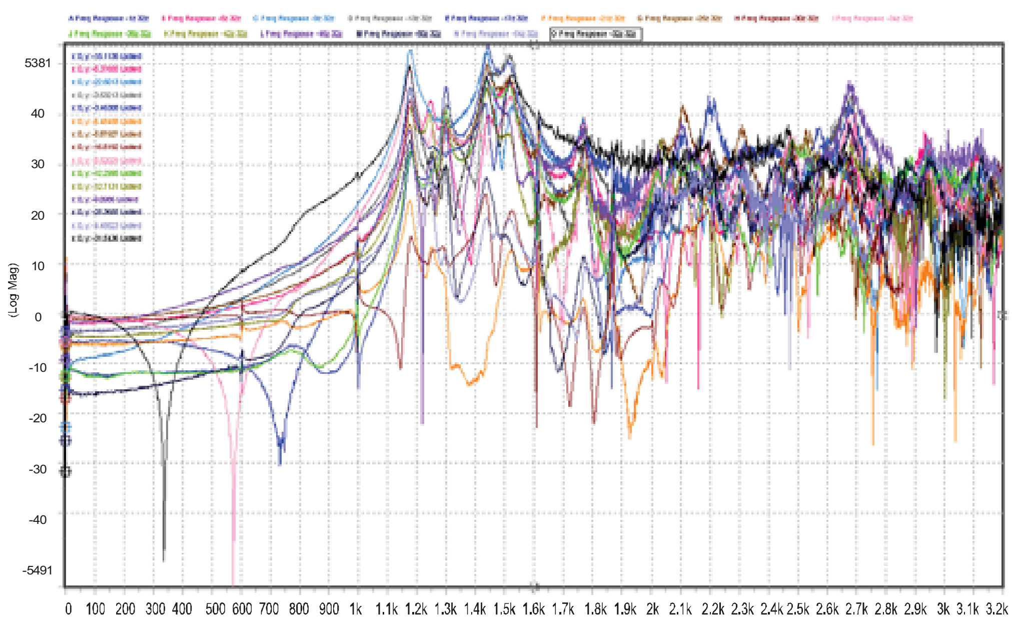


Fig. 3: FRF diagrams measured in modal test related to 16 degrees of freedom.

The measurement in experimental modal analysis includes taking frequency response function (FRF) data from the structure. In the modal test of the elliptical shell, impact impulse method is used. This test is performed by stimulating the polar coordinate system (z, θ, r) at all points and in the normal direction of the shell along with measuring the response in one point (point 32) which is shown is Fig.2-b. FRF is the proportion of the acceleration of response to the stimulation power in the frequency domain or acceleration A_{jk} (ω) that should be analyzed. The analysis of the obtained data in FRF is conducted to find a theoretical model that must be similar to real response of the sample test.

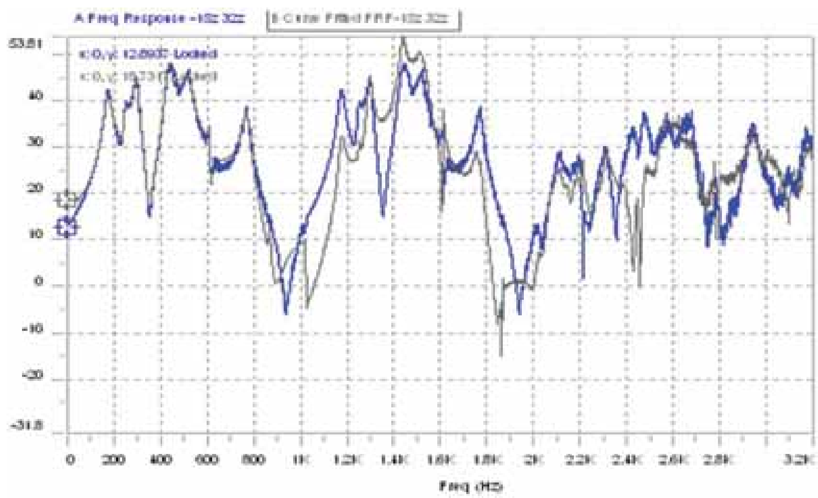


Fig. 4: FRF diagrams related to the point 15 along with fitted polynomial curve.

This process is divided into two parts: 1- Selection of the proper model 2-Determination of parameters of the selected model.

The second part, which is more important, is usually called experimental modal analysis. This part, as the main part of the analysis, consists of fitting the equation (1) for determining the curve's FRF to find proper parameters' modal.

$$A_{jk}(\omega) = \frac{\ddot{x}_j}{f_k} = -\omega^2 \sum_{r=1}^{N} \frac{(r_j^{\phi})(r_k^{\phi})}{\lambda_r^2 - \omega^2} \tag{1}$$

Where λ_r^2 is the Eigen value r th mode and r_j^{ϕ} array j th of eigenvector of the r th mode which is $\{\phi\}_r$ and N is the number of model's degree of freedom. Consequently, the modal characteristics of the sample would be obtained through curve-fitting algorithms and using FRF diagrams. The determined FRF curves in modal test related to the 16 degrees of freedom of the model are presented in Fig.3. The black curve relates to the point FRF which is the frequency response of a point where accelerator meter is installed. After analyzing FRF curves by the curve-fitting algorithm and using polynomial functions of the 30th order for curve-fitting, the modal characteristics of the composite elliptical shell in the range of 1000 to 2000 HZ is obtained according to Table 1. In Fig.4, FRF curve relating to the point 15 along with the fitted polynomial 30th order curve is presented.

Table 1: Modal chrematistics of composite elliptical shell

Frequency (HZ)	Mode
1176.375	1
1266.625	2
1277.060	3
1336.953	4
1516.013	5
1613.776	6

The Numerical Analysis of the Composite Elliptical Shell:

As shown in Fig.5, the shell has the external diameter of 38_{cm}, the internal diameter of 22cm and the thickness of 2mm. Normally, in FEM physics problems are solved through using the differential equations that dominate the system or minimizing the potential energy. In this method the geometrical model is divided into more tiny parts called elements. Each element has a number of nodes for which input values (loading and boundary condition) and output (results) are defined. Additionally, each element has a reaction called mode shape which determines the amount of the degrees of freedom (for example, displacement: U) in each point of the element. The main factor in selecting a suitable mode shape is the satiability of the boundary condition by the function. These functions can be 1st, 2nd or any other order. Finite element softwares usually use polynomial functions, meaning $ax^n + bx^{n-1} + \ldots$. In this project first order shell square element is used to mesh the shell (Fig.6).

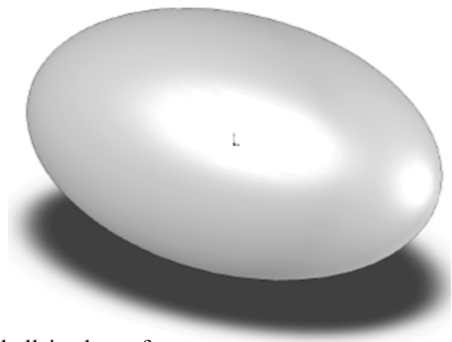


Fig. 5: Manufactured model of shell in the software.

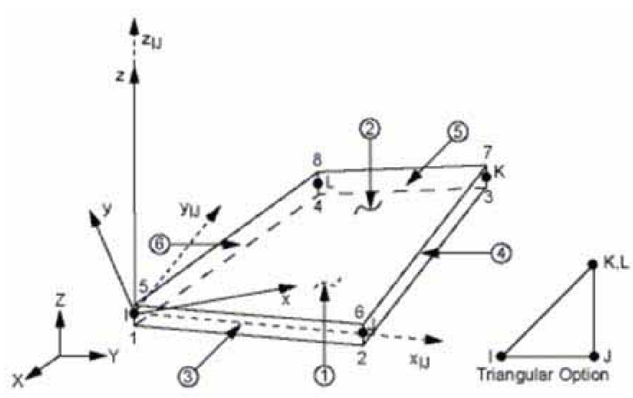


Fig. 6: Schematic model of the element of the shell used for analysis.

By having the values [U^e], the values [U] in all the internal points of the element can be obtained through the equation [U] = [N] [U^e]. In this equation, [N] is the mode shape of each element, and [U^e] in element is computed through FEM. In FEM in order to solve problems, the potential energy and the error of differential equation dominating the system are minimized, yielding a group of equations as many as the degrees of freedom of the whole model. Then, by solving the equations, the amounts of the degrees of freedom U_i, U_j, and so on are obtained. Hence, by assorting and considering the continuous conditions of nearby elements and applying the boundary conditions, the problem can be solved. In this section, 4-node element shell is used. The shell element has 6 degrees of freedom (3 degrees of translation freedom and 3 degrees of rotation freedom) in each node.

Minimizing the Potential Energy:

Minimizing the potential energy, introduced by Courant for the first time, is usually used for solving structural problems. Generally the amount of strain energy in 3D mode for each element is defined by the following equation:

$$\Lambda^e = \frac{1}{2}\int_v (\sigma_{xx}\varepsilon_{xx} + \sigma_{yy}\varepsilon_{yy} + \sigma_{zz}\varepsilon_{zz} + \sigma_{my}\varepsilon_{my} + \sigma_{xz}\varepsilon_{xz} + \sigma_{yz}\varepsilon_{yz})dv \tag{2}$$

Or in matrix mode:

$$\Lambda^e = \frac{1}{2}\int_v \{\sigma\}^T \{\varepsilon\}\, dv \tag{3}$$

Where V is the volume of the element and Λ^e is the strain energy of elements.

$$\begin{aligned} \{\sigma\} &= \{\sigma_{xx} \quad \sigma_{yy} \quad \sigma_{zz} \quad \sigma_{xy} \quad \sigma_{xz} \quad \sigma_{yz}\} \\ \{\varepsilon\} &= \{\varepsilon_{xx} \quad \varepsilon_{yy} \quad \varepsilon_{zz} \quad \varepsilon_{xy} \quad \varepsilon_{xz} \quad \varepsilon_{yz}\} \end{aligned} \tag{4}$$

On the other hand, for solving elastic problems the Generalized Hook Relation in the 3D mode is used:

$$\sigma = D(\{\varepsilon\} - \{\varepsilon_T\}) \tag{5}$$

$[D]_{6\times 6}$ is a symmetrical matrix of the material properties and is based on the main coefficients E (elastic module) and ν (Poisson ratio) and $\varepsilon_T \ = \ .\alpha\, \Delta T$

By combining the above equations:

$$\Lambda^e = \frac{1}{2}\int_v (\{\varepsilon\} - \{\varepsilon_T\})^T [D] \ \ (\{\varepsilon\} - \{\varepsilon_T\}) dv \tag{6}$$

Then, the above equation will be simplified as:

$$\Lambda^e = \frac{1}{2}\int_v \{\varepsilon\}^T [D] \ \ \{\varepsilon\}^T dv - \frac{1}{2}\int_v \{\varepsilon\}^T [D] \ \ \{\varepsilon_T\} dv \tag{7}$$

Now, calculations are based on nodal amount [U^e]. The relation $\varepsilon_{mn} = \dfrac{\partial u_m}{\partial x_n} + \dfrac{\partial u_n}{\partial x_m}$ is converted to the matrix form: $\{\varepsilon\} = \{L\} \ \ \{U\}$

According to [U]= [N][U^e]

$$\{\varepsilon\} = [L] \ [N] \ [U] \tag{8}$$

And [B]=[L][N], the following relation will be obtained $\{\varepsilon\} = [B] \ \ \{U^e\}$; Then, by replacing $\{\varepsilon\}$ in (7) the following relation will be gained:

$$\Lambda^e = \frac{1}{2}\int_v \{U^e\}^T [B]^T [D] \ [B] \ \{U^e\} \ \ dv - \frac{1}{2}\int_v \{U^e\}^T [B]^T \{\varepsilon_T\} dv \tag{9}$$

Also, for external load:

$$W_{body} = \int_v \{U\} \ \begin{Bmatrix} b_x \\ b_y \\ b_z \end{Bmatrix} \ dv = \int_v \{U^e\}^T [N]^T \begin{Bmatrix} b_x \\ b_y \\ b_z \end{Bmatrix} \ dv \tag{10}$$

$$W_{surf} = \int_A \{U\}^T \begin{Bmatrix} p_x \\ p_y \\ p_z \end{Bmatrix} dA = \int_A \left[U^e\right]^T \left[N\right]^T \begin{Bmatrix} p_x \\ p_y \\ p_z \end{Bmatrix} dA \tag{11}$$

Where, W body and W surf are the external energy from the body load and surface load respectively. The total potential for the whole system is determined by the relation: $\pi^e = \Lambda^e - w^{external}$. By deriving from the relation $\frac{\partial}{\partial U^e}(A^e - W_{body} - W_{surf}) = 0$

Finally:

$$k^e U^e = F_{thermal} + F_{body} + F_{surf}$$

$$k^e = \int_v [B]^T [D]\ [B]\ dv \tag{12}$$

$$F_{thermal} = \int_v [B]^T [D]\ \{\varepsilon_T\}\ dv$$

$$F_{surf} = \int_s [N]^T \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} ds$$

(13)

$$F_{body} = \int_v [N]^T \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix} dv$$

By solving the above equation system based on U^e, the amounts of nodal degrees of freedom would be obtained.

Meshing and Presenting the Results:

As mentioned before, the shell element was used for the analysis. To check the accuracy of meshing and the number of elements, a comparison between the number of elements and the obtained frequencies was conducted and is presented in Fig.7. According to Fig.7, by minimizing the elements, the results were converged so that when the elements were minimized to 0.1 or less, the frequencies became equal.

Comparison of the Natural Frequencies Obtained from the Test and ANSYS:

The natural frequencies of the shell in ANSYS with the FREE-FREE boundary conditions were compared with the test (Fig.8). The frequencies were extracted from mode shapes shown in Figure 9 in the test and FEM. As it is apparent in Figure 8, there is a good conformity between the present analysis and the test.

In Table 2, the error percentage between the test and the FEM is presented. It was observed that as (n) increased, the error percentage increased as well. In the first frequency, the percentage of error was very low and acceptable. The natural frequencies begin from the minimum, which was called the base frequency, and then they increased.

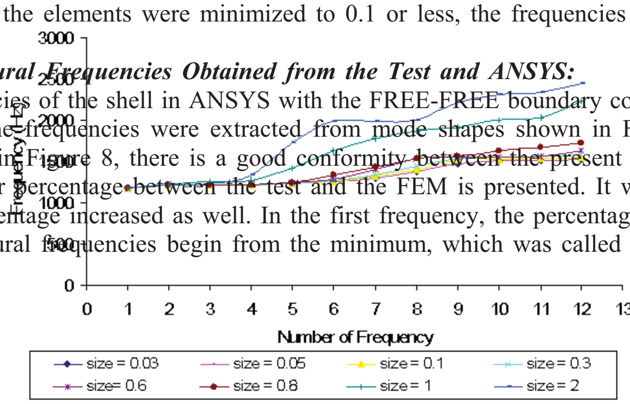


Fig. 7: Analysis of the mesh convergence.

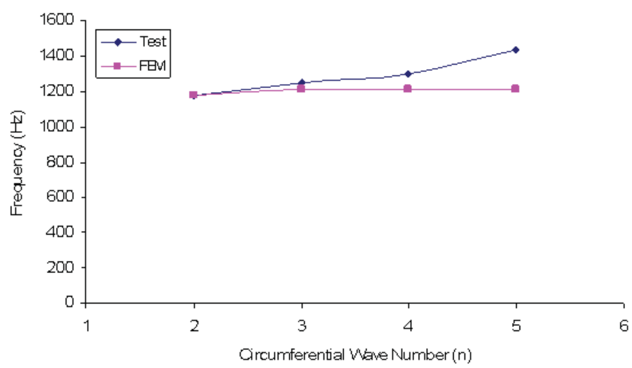


Fig. 8: Comparing the natural frequencies obtained from the test and ANSYS with Free-Free boundary conditions.

Table 2: Percentage of error for the obtained frequencies from FEM and test.

	Percentage of error from the test and FEM
	0.189027707
	2.662068682
	7.000527966
	18.49322874

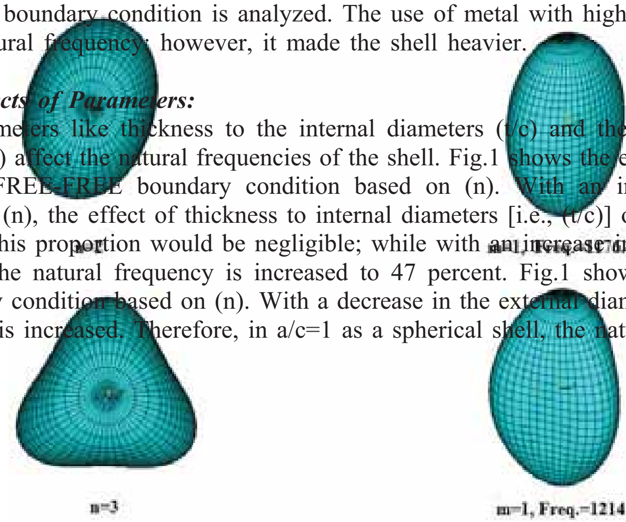
The Analysis of the Effective Parameters on the Natural Frequencies:

The comparison between composite and metallic materials

Since the weight and vibratory response of shells depend on their materials, one of the important issues in making shells is what they are made of. In Fig.9 the effect of different materials on the natural frequency with the FEER-FREE boundary condition is analyzed. The use of metal with high elastic module resulted in an increase in the natural frequency; however, it made the shell heavier.

The Geometrical Effects of Parameters:

Geometrical parameters like thickness to the internal diameters (t/c) and the external diameters to the internal diameters (a/c) affect the natural frequencies of the shell. Fig.1 shows the effect of (t/c) for composite elliptical shell with FREE-FREE boundary condition based on (n). With an increase in the number of circumferential waves (n), the effect of thickness to internal diameters [i.e., (t/c)] on the frequency increases. In n=2, the effect of this proportion would be negligible; while with an increase in the thickness of the shell to 10 times in n=7, the natural frequency is increased to 47 percent. Fig.1 shows the effect of (a/c) with FREE-FREE boundary condition based on (n). With a decrease in the external diameter, the effect of (a/c) on the natural frequency is increased. Therefore, in a/c=1 as a spherical shell, the natural frequency would have its maximum quantity.



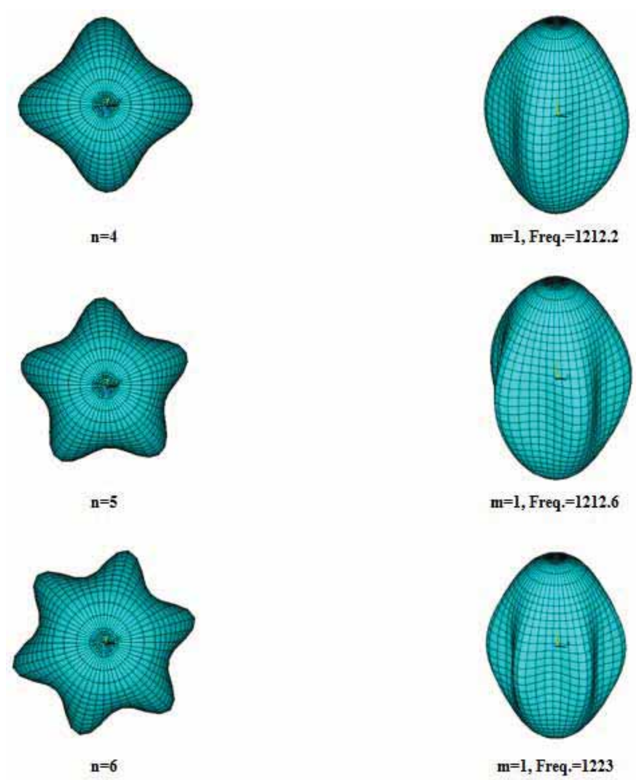
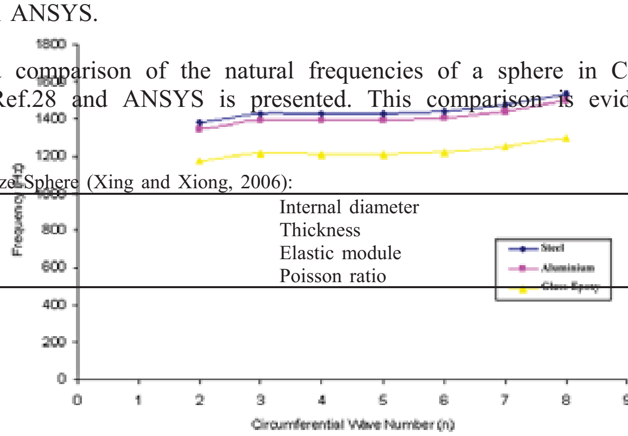


Fig. 9: Mode shapes from ANSYS.

In Figs.13 and 14, a comparison of the natural frequencies of a sphere in CLAMPED-CLAMPED boundary conditions in Ref.28 and ANSYS is presented. This comparison is evidence to the obtained frequencies from ANSYS.



The Characteristics of the Analyze Sphere (Xing and Xiong, 2006):

X=29.44 cm	Internal diameter
t=0.28cm	Thickness
E=3.1Gpa	Elastic module
V=0.31	Poisson ratio

Fig. 10: Effect of materials on the natural frequency.

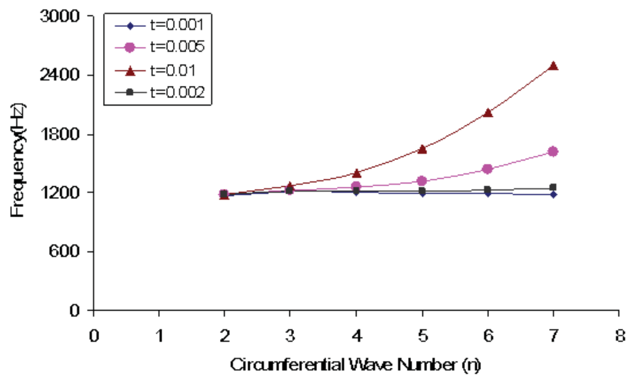


Fig. 11: Effect of (t/c) parameter on the natural frequency with Free-free boundary condition a/c=1.7272

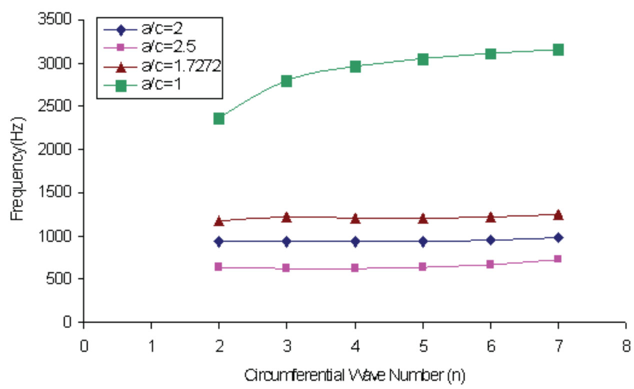


Fig. 12: Effect of a/c parameter on the natural frequency with Free-free boundary condition, m=1 t/c =0.009

The Effect of Fiber Arrangement in Glass-Epoxy on the Natural Frequency:

Arranging fiber with different angles has an important role in the natural frequency. Fibers can either be in a special direction (special angle) or in all directions. Such arrangements lead to an increase or decrease in the natural frequencies. In Fig.15, the use of fibers in all directions resulted in an increase in the natural frequency to a special direction. In the figure below, a comparison between arrangement in all directions and 90 degree in a single layer is shown.

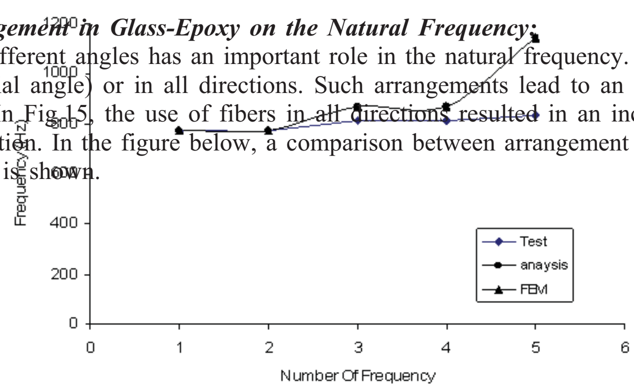


Fig. 13: Comparison of the natural frequency between ANSYS. Ref (28)

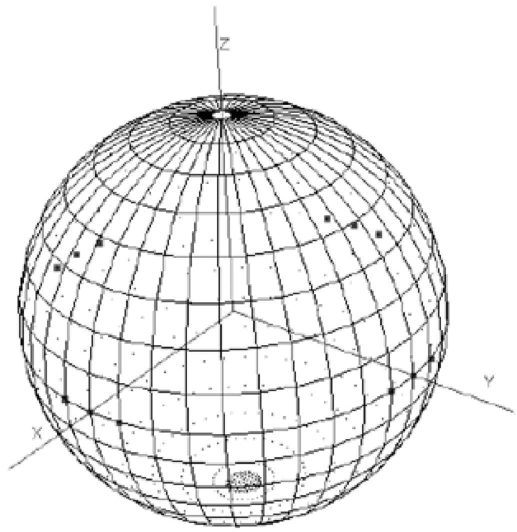


Fig. 14: Sphere under analysis with clamped - clamped boundary conditions. Ref (28)

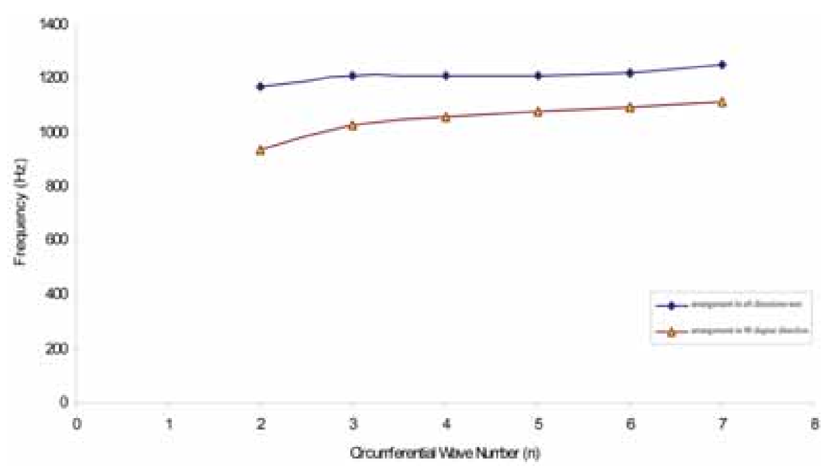


Fig. 15: Comparison between arrangement in all directions and 90 degree in a single layer with Free-Free boundary condition; m=1, t/c =0. 009, a/c=1.7272

The Effect of Layering on Orthotropic Mode

As shown in Fig.16, by equalizing thickness and fiber angle, the number of layers was increased. It was also observed that there was no change in the natural frequency diagram and the base frequency position.

The Effect of Fiber Angle on a Single Layer Orthotropic Mode:

One of the influential factors in vibratory response of shells is the fiber angle in each layer. In Fig.17, for the arrangement of fixed number of layers, the effect of fiber angles on the natural frequency of the shell by FREE-FREE boundary conditions is analyzed. As the figure shows, the fiber angle affects the natural frequencies.

The Effect of Different Composite Materials on the Natural Frequencies in 90 Degree Angle:

Fig.18 presents the effect of composite materials on the natural frequency based on the number of circumferential waves. By increasing E1/E2, the natural frequencies increased, but the base frequency position did not change.

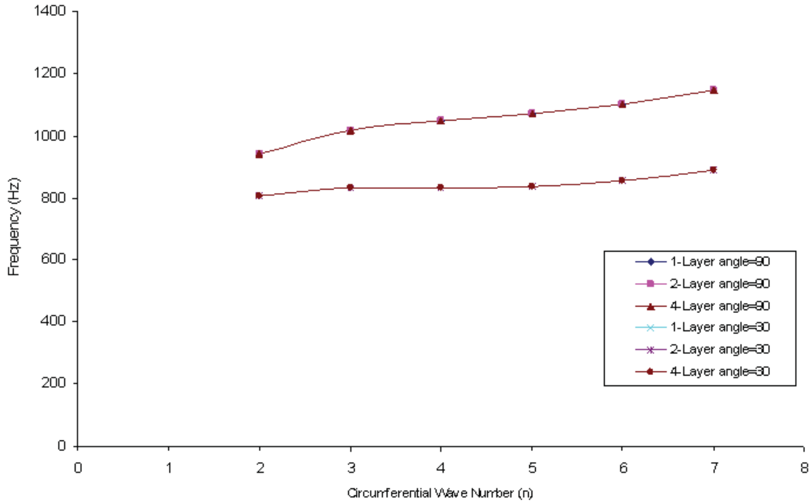


Fig. 16: Effect of arrangement in 90 & 30 degree on the natural frequency with Free-free boundary condition m=1, t/c=0.009, a/c=1.7272

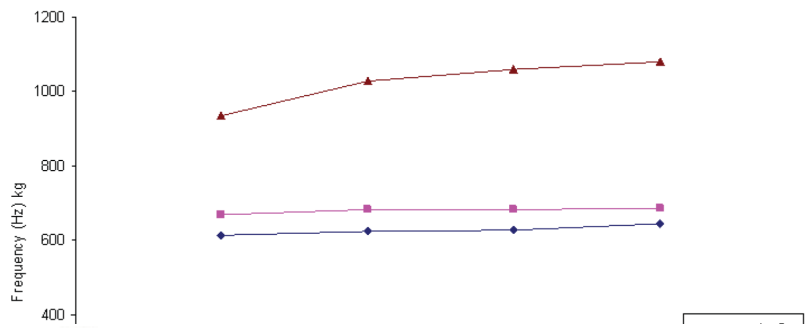


Fig. 17: Effect of fiber angles on a single layer on the natural frequency.

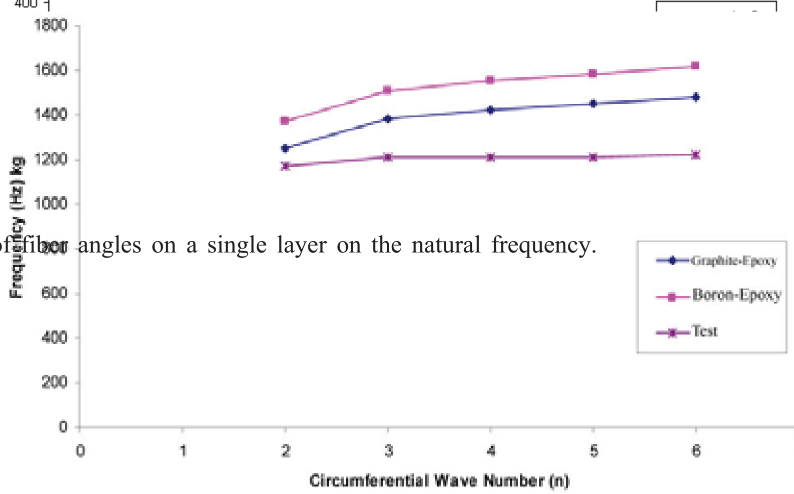


Fig. 18: Effect of composite materials on the natural frequency in 90 degree angle with Free-free boundary condition m=1, t/c=0.009, a/c=1.7272

Conclusion:

By increasing the number of circumferential waves, the effect of thickness to internal diameter on the natural frequency was increased. In addition, it was observed that when (n) increased, the error percentage increased as well.

The use of metal with high elastic module would increase the natural frequency but it would make the shell heavier too. The use of fibers in all directions caused an increase in the natural frequency to a special direction.

By equalizing thickness and fiber angle and increasing the number of layers, it was observed that there was no change in the natural frequency. The fiber angle affected the natural frequency.

By increasing E1/E2, the natural frequency increased but the base frequency position did not change.

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