Australian Journal of Basic and Applied Sciences, 4(7): 1555-1560, 2010
ISSN 1991-8178

Coordination of Quality Decisions in a Supply Chain

A. Ahmadi Javid and P. Hoseinpour

Department of Industrial Engineering, Amirkabir University of Technology, Iran

Abstract: In this paper we consider a model for coordination of quality decisions in a supply chain. The model is analyzed by game theory, and the Nash and Stackelberg equilibrium points of the model are obtained. Finally, the results are analyzed and several managerial comments are presented.

Keywords: Supply chain, quality management, coordination, game theory.

1. INTRODUCTION

Quality has been defined as fitness for use, or the extent to which a product successfully serves the purpose of consumers (Juran *et al.,* 1974). The case of improving quality is an important factor in achieving competitive advantages for companies, and it is attended extensively in the today's fast-paced and increasingly competitive market (Tellis and Johnson, 2007). Two important aspects of quality are product and service quality. A product and the service offered to the customers purchasing the product must meet or exceed the customers' expectations. The emphasis on improving the quality of products and services has been increased by firms in reaction to enhanced competitive environments. In other words, product and service quality have been recognized as playing crucial rules for success and survival in the today's competitive market. In the following we briefly review papers dealing with product and service quality.

In order to offer the best customer service, efficient mechanisms are investigated in several papers (*e.g.* Hosanagar *et al.,* 2005; Hsu *et al.,* 2008; Ren and Zhou, 2008). In fact, the retailers in a supply chain focus on activities and operations that might give them an advantage over their competitors. Therefore, they invest on service quality as a method to enhance the efficiency of the entire supply chain. Also, there are several papers considering product quality. Some of them are about pay back-warranty contracts that are for the purpose of sharing the costs caused by low quality among various parts in a supply chain (Bachlausa *et al.,* 2009; Barber *et al,* 1996; Reyniers and Tapiero, 1995a; 1995b; Mann and Wissink, 1990; Jarrell and Peltzman, 1985; Balachandran and Radhakrishnan, 2005; Zhu *et al.,* 2007; Rupp, 2004; 2003; Tunca and Johnson, 2007; Lim, 2001). Some papers consider designing quality control processes. These papers present models consisting of determining batch size, order quantity, sampling size which is randomly drawn from a lot, and critical value for accepting or rejecting the lot in order to minimize total cost (Reyniers and Tapiero, 1995b; Tapiero, 2007; Porteus, 1990; Cheung and Leung, 2000; Starbird and Lee, 1996). There are papers studying relationship between product quality and inventory control. They determine the lead time, order quantity and probability of the production processes going out of control for the purpose of minimizing total cost aggregated of setup cost, ordering cost, adjustment of the production processes and the holding cost (EI Saadany and Jaber, 2008; Yang and Pan, 2004). Some papers are related to trade off between price and quality in order to select the best suppliers (Tagaras and Lee, 1996; Lee *et al.,* 2003; Ghodsypour and Obrien, 2001). Other papers investigate the effect of price and quality on the demand, and others show that how demand changes by variation of price, quality, brand diversity and location of factory (Chambers *et al.,* 2006; Piga and Poyago-Theotoky, 2005; Degryse, 1996; Hall and Porteus, 2000; Kim and Lin, 2006; Melumad and Ziv, 2004; Economides, 1989; Gans, 2002; Mukhopadhyay and Setaputra, 2007; Doraszelskiw and Draganskaz, 2006; Choudhary, *et al.,* 2005; Xu, 2009; Sanjo, 2007). Finally, there are papers incorporate product quality in designing supply chain networks (ElMaraghy and Majety, 2008; Bachlausa, 2009; Ramudhin, 2008).

As we have reviewed above, in the literature there is little research that simultaneously considers product and service quality in a supply chain. Here we present a model to consider product and service quality closely in a supply chain with a manufacturer and a retailer. The manufacturer invests in product quality and the retailer compensates to improve service quality. The model is analyzed by game theory under two scenarios of how the supply chain components make decision.

Corresponding Author: A. Ahmadi Javid, Department of Industrial Engineering, Amirkabir University of Technology, Iran
E-mail: ahmadi_ javid@aut.ac.ir

The paper is organized as follows. The model is presented in Section 2. Sections 3 and 4 give the Nash and Stackelberg equilibriums of the model, respectively. In Section 5, we conclude the paper and we present several interesting managerial comments.

2. MODELING

In this paper we consider a supply chain with one manufacturer and one retailer where the retailer invests in service quality and the manufacturer invests in product quality. The manufacturer's investment in product quality and the retailer's investment in service quality perform different but complementary functions which have positive effects on the ultimate product sales. Saturation may be reached when both or either the investments in product or service quality are increased. The retailer and the manufacturer gross profits are respectively determined as:

$$\pi_r(a,q) = \rho_r S(a,q) - a$$
$$\pi_m(a,q) = \rho_m S(a,q) - q ,$$

where the decision variables, parameters and sale function $S(a,q)$ are defined below.

- **Decision variables:**
  - a is the retailer's investment to improve service quality.
  - q is the manufacturer's investment to improve product quality.
- **Parameters:**
  - ρ_m is the manufacturer's marginal profit for each product unit.
  - ρ_r is the retailer's marginal profit for each product unit.
- **Sale function:** $S(a,q)$ is the one period sale volume which is a function of a and q.

In this paper we consider the following sale function:

$S(a,q) = \max\{0, \alpha - \beta a^{-\gamma} q^{-\delta}\}$ for $a > 0, q > 0$ and $S(a,q) = 0$ for the other points (a,q) with $a \geq 0, q \geq 0$.

The parameters in this function can be interpreted as:

- α is a positive constant and is the sale saturate asymptote. This means that when either or both the product and service quality investments tend to infinity, $S(a,q)$ tends to the constant α.
- γ is a positive constant which is the elasticity of service quality.
- δ is a positive constant which is the elasticity of product quality.
- β is a positive constant and determines the impact of product and service quality investments on the market demand.

In the following two sections we analyze the model under two scenarios. The first scenario is that the manufacturer and retailer sequentially move when the manufacturer is leader and the retailer is follower. This scenario is studied in Section 3 by using game theory, and the related equilibrium point, called Stackelberg equilibrium, is obtained. The second scenario is that the manufacturer and the retailer simultaneously move. This scenario is considered in Section 4 and the associated equilibrium point, called Nash equilibrium, is obtained. In the next two sections, for simplicity we use the auxiliary parameter c defined as:

$$c = \left(\frac{1+\gamma}{\alpha}\right)^{\delta+\gamma+1} \beta \delta^{-\delta} \gamma^{-\gamma} .$$

3. SEQUENTIOAL MOVE

In this section we study the model in the case that the manufacturer and retailer sequentially move when the manufacturer is leader and the retailer is follower. The leader first chooses a strategy about q, and then the follower observes this decision and makes his own strategy choice about a. In this case we can analyze the model by obtaining the associated Stackelberg equilibrium point of the model. To this end, we first find the optimal value of a which optimizes the retailer gross profit, and then by substituting it in the manufacturer gross profit we obtain the optimal value of q.

Let us define $a_r^*(q) = \arg\max_{a \geq 0}(\pi_r(a,q))$, and suppose q^* is the solution of the following problem:

$$\max_q \quad \pi_{st} = \pi_m\left(a_r^*(q), q\right)$$

s.t.

$q \geq 0.$

Then, the Stackelberg equilibrium point can be obtained as $a_S^* = a_r^*(q^*),\ q_S^* = q^*$. In order to determine $a_r^*(q)$, we can write:

$$\pi_r(a,q) = \rho_r S(a,q) - a = \begin{cases} \rho_r\left(\alpha - \beta a^{-\gamma} q^{-\delta}\right) - a & a \geq \tilde{a}(q) \\ -a & 0 \leq a \leq \tilde{a}(q) \end{cases},$$

where $\tilde{a}(q) = \left(\frac{\beta}{\alpha q^{\delta}}\right)^{\frac{1}{\gamma}}$. Let $a_{(1)}^*(q)$ and $a_{(2)}^*(q)$ be the optimum of π_r whose domain restricted to $a \geq \tilde{a}(q)$ and $a \leq \tilde{a}(q)$, respectively. Since π_r is concave for $a \geq \tilde{a}(q)$, we have:

$$a_{(1)}^*(q) = \max\left\{\tilde{a}(q), \left(\gamma\rho_r\beta q^{-\delta}\right)^{\frac{1}{\gamma+1}}\right\},$$

and also it is obvious that $a_{(2)}^*(q) = 0$, so we conclude that if $\pi_r\left(a_{(1)}^*(q)\right) \geq \pi_r\left(a_{(2)}^*(q)\right) = 0$, then $a_r^*(q) = a_{(1)}^*(q)$, else $a_r^*(q) = a_{(2)}^*(q) = 0$. In Table 1, the Stackelberg equilibrium point is given.

Table 1. Stackelberg equilibrium point.

Case	Set of conditions	Stackelberg equilibrium point
S-A	$c \leq \left(\frac{\rho_m}{1+\gamma}\right)^{\delta} \rho_r^{\gamma}$ $c \leq \frac{\rho_r^{\gamma}\left(\rho_m(1+\gamma)\right)^{\delta}}{(1+\gamma+\delta)^{\delta+\gamma+1}}$	$q_S^* = \left[\left(\frac{\delta\rho_m}{1+\gamma}\right)^{1+\gamma}(\gamma\rho_r)^{-\gamma}\beta\right]^{\frac{1}{\delta+\gamma+1}}$ $a_S^* = \left[\left(\frac{\delta\rho_m}{1+\gamma}\right)^{-\delta}(\gamma\rho_r)^{\delta+1}\beta\right]^{\frac{1}{\delta+\gamma+1}}$
S-B	$c > \left(\frac{\rho_m}{1+\gamma}\right)^{\delta} \rho_r^{\gamma}$ $c \leq \rho_r^{\gamma}\left(\frac{\gamma\rho_m}{\delta}\right)^{\delta}$	$q_S^* = \left(\frac{1+\gamma}{\alpha}\right)^{\frac{1+\gamma}{\delta}} \beta^{\frac{1}{\delta}} (\gamma\rho_r)^{\frac{-\gamma}{\delta}}$ $a_S^* = \frac{\alpha\gamma\rho_r}{1+\gamma}$
S-C	otherwise	$q_S^* = a_S^* = 0$

4. SIMULTANCE MOVE

In this section we examine the model by considering that the manufacturer and retailer simultaneously move, which means that both the components of supply chain make their strategies in the same time. In this case, we analyze the model by obtaining the Nash equilibrium point of the model. By defining:

$$a_r^*(q) = \arg\max_{a \geq 0}(\pi_r(a,q)) \text{ and } q_m^*(a) = \arg\max_{q \geq 0}(\pi_m(a,q)),$$

the Nash equilibrium point $\left(a_N^*, q_N^*\right)$ is the solution of the following system:

$$a = a_r^*(q)$$
$$q = q_m^*(a)$$
$$a \geq 0, q \geq 0.$$

This system always has a solution, so the Nash equilibrium point exists. Table 2 presents this equilibrium point.

Table 2. Nash equilibrium point.

Case	Condition	Nash equilibrium point
N-A	$c \leq \rho_r^{\gamma} \rho_m^{\delta} \min\left\{1, \frac{1+\gamma}{1+\delta}\right\}$	$q_N^* = \left(\delta\rho_m^{\gamma+1}(\gamma\rho_r)^{-\gamma}\beta\right)^{\frac{1}{\delta+\gamma+1}}$ $a_N^* = \left(\delta\rho_m^{-\delta}(\gamma\rho_r)^{\delta+1}\beta\right)^{\frac{1}{\delta+\gamma+1}}$
N-B	$c > \rho_r^{\gamma} \rho_m^{\delta} \min\left\{1, \frac{1+\gamma}{1+\delta}\right\}$	$q_N^* = a_N^* = 0$

5. CONCLUSION

In Sections 3 and 4, we have studied the model presented in Section 2 by presenting the Stackelberg equilibrium point for the scenario of sequential move and the Nash equilibrium point for the scenario of simultaneous move, respectively. This study helps both manufacturer and retailer to choose the suitable strategies about their investment values in product and service quality. Moreover, from this study it could be seen how the parameters of the model affect the investment values in each scenario. For example, for the parameters ρ_m and ρ_r we have:

$$\frac{\partial a_S^*}{\partial \rho_m} \leq 0, \frac{\partial q_S^*}{\partial \rho_m} \geq 0$$
$$\frac{\partial a_S^*}{\partial \rho_r} \geq 0, \frac{\partial q_S^*}{\partial \rho_r} \leq 0$$
$$\frac{\partial a_N^*}{\partial \rho_m} \leq 0, \frac{\partial q_N^*}{\partial \rho_m} \geq 0$$
$$\frac{\partial a_N^*}{\partial \rho_r} \geq 0, \frac{\partial q_N^*}{\partial \rho_r} \leq 0.$$

These show that the manufacturer's investments in quality positively depend on her marginal profit ρ_m. Similarly, the retailer's investments in service quality positively depend on her marginal profit ρ_r. In words, if they want to increase their marginal profits, they cannot decrease their investments in quality. Moreover, in both scenarios each component's investment negatively depends on the other component's marginal profit.

We can also compare the values of retailer's investment in service quality in the two scenarios. It can be easily shown that $a_N^* \leq a_S^*$. This shows that the scenario of sequential move has a more cost for the retailer in comparison to the scenario of simultaneous move. Also, we have $q_N^* \geq q_S^*$, which means the scenario of simultaneous move has a more cost for the manufacturer rather than the scenario of sequential move.

In the scenario of sequential move, it is obvious that the manufacturer can choose the Nash equilibrium point, so the manufacturer is always at least as well off as she would be in the scenario of the simultaneous move. However, the retailer might have better off in the scenario of sequential move depending on the parameters of the model.

REFRENCES

Bachlausa, M., M.K. Tiwarib, F.T.S. Chan, 2009. Multi-objective resource assignment problem in a product-driven supply chain using a Taguchi-based DNA algorithm. International Journal of Production Research, 47: 2345–2371.

Baiman, S., P.E. Fischer, M.V. Rajan, 2000. Information, contracting, and quality costs. Management Science, 46: 776–789.

Baiman, S., P.E. Fischer, M.V. Rajan, 2001. Performance measurement and design in supply chains. Management Science, 47: 173–1 88.

Balachandran, K.R., S. Radhakrishnan, 2005. Quality implications of warranties in supply chain. Management Science, 51: 1266–1277.

Barber, B.M., M.N. Darrough, 1996. Product reliability and firm value: the experience of American and Japanese automakers. Journal of Political Econom, 104: 1084–1099.

Chambers, C., P. Kouvelis, J. Semple, 2006. Quality-based competition, profitability, and variable costs. Management Science, 52: 1884–1895.

Cheung, K.L., K.F. Leung, 2000. Coordinating replenishments in a supply chain with quality control considerations. Production Planning & Control, 11: 697–705.

Choudhary, V., A. Ghose, T. Mukhopadhyay, U. Rajan. 2005. Personalized pricing and quality differentiation. Management Science, 51: 1120–1130.

Degryse, H., 1996. On the interaction between vertical and horizontal product differentiation: An application to banking. The Journal of Industrial Economics, 44: 169–186.

Doraszelskiw, U., M. Draganskaz, 2006. Market segmentation strategies of multiproduct firms. The Journal of Industrial Economics, 54: 125–149.

Economides, N., 1989. Quality variations and maximal variety differentiation. Regional Science and Urban Economics, 19: 21–29.

ElMaraghy, H.A., R. Majety, 2008. Integrated supply chain design using multi-criteria optimization. International Journal of Advance Manufacturing Technology, 37: 371–399.

El Saadany, A.M.A., M.Y. Jaber, 2008. Coordinating a two-level supply chain with production interruptions to restore process quality. Computers & Industrial Engineering, 54: 95–1 09.

Gans, N., 2002. Customer loyalty and supplier quality competition. Management Science, 48: 207–221.

Ghodsypour, S.H., C. O'Brien, 2001. The total cost of logistics in supplier selection, under conditions of multiple sourcing, multiple criteria and capacity constraint. International Journal of Production Economics, 73: 15–27.

Hall, J.M., E.L. Porteus, 2000. Customer service competition in capacitated systems. Manufacturing & Service Operations Management, 2: 144–165.

Hosanagar, K., R. Krishnan, V. Choudhary, 2005. Pricing and resource allocation in caching services with multiple levels of quality of service. Management Science, 51: 1844-1859.

Hsu, V.N., S.H. Xu, B. Jukic, 2009. Optimal scheduling and incentive compatible pricing for a service system with quality of service guarantees. Manufacturing & Service Operations Management, 11: 375–396.

Jarrell, G., S. Peltzman, 1985. The impact of recalls on the wealth of sellers. Journal of Political Economic, 43: 467–473.

Juran, J.M., F.M.J. Gryna, R.S. Bingham, 1974. Quality Control Handbook (3rd Ed.). McGraw-Hill Book Co, New York, NY.

Kim, K., D. Lin, 2006. Optimization of multiple responses considering both location and dispersion effects. European Journal of Operational Research, 169: 133–145.

Lee, M.S., Y.H. Lee, C.S. Jeong, 2003. A high-quality-supplier selection model for supply chain management and ISO 9001 system. Production Planning & Control, 14: 225–232.

Lim, W.S., 2001. Producer-supplier contracts with incomplete information. Management Science, 47: 709–715.

Mann, D.P., J.P. Wissink, 1990. Money-back warranties vs. replacement warranties: A simple comparison. American Economic Review, 80; 432–436.

Melumad, N.D., A. Ziv, 2004. Reduced quality and an unlevel playing field could make consumers happier. Management Science, 50: 1646–1659.

Mukhopadhyay, S.K., R. Setaputra, 2007. A dynamic model for optimal design quality and return policies. European Journal of Operational Research, 180: 1144–1154.

Piga, C., J. Poyago-Theotoky, 2005. Endogenous R&D spillovers and locational choice. Regional Science and Urban Economics, 35: 127–139.

Porteus, E.L., 1990. The impact of inspection delay on process and inspection lot sizing. Management Science, 36: 999–1007.

Ramudhin, A., C. Alzaman,, A.A. Bulgak, 2008. Incorporating the cost of quality in supply chain design. Journal of Quality in Maintenance Engineering, 14; 71–86.

Ren, Z.J., Y.P. Zhou, 2008. Call center outsourcing: Coordinating staffing level and service quality. Management Science, 54: 369–3 83.

Reyniers, D.J., C. Tapiero, 1995a. The delivery and control of quality in supplier-producer contracts. Management Science, 41: 158 1–1589.

Reyniers, D.J., C. Tapiero, 1995b. Contract design and the control of quality in a conflictual environment. European Journal of Operational Research, 82: 373– 382.

Rupp, N.G., 2004. The attributes of a costly recall: Evidence from the automotive industry. Review of Industrial Organization, 25: 21–44.

Sanjo, Y., 2007. Hotelling's location model with quality choice in mixed duopoly. Economics Bulletin, 18: 1–11.

Starbird, S.A., 1997. Acceptance sampling, imperfect production and the optimality of zero defects, Naval Research Logistics Quarterly, 44: 515–530.

Tagaras, G., H.L. Lee, 1996. Economic models for vendor evaluation with quality cost analysis. Management Science, 42: 1531–1543.

Tapiero, C.S., 2007. Consumers risk and quality control in a collaborative supply chain. European Journal of Operational Research, 182: 683–694.

Tellis, G.J., J. Johnson, 2007. The value of quality. Marketing Science, 26: 758-773.

Tunca, T.I., S.A. Zenios, 2006. Supply auctions and relational contracts for procurement. Manufacturing & Service Operations Management, 8: 43–67.

Xu, X., 2009. Optimal price and product quality decisions in a distribution channel. Management Science, 55: 1347–1352.

Yang, J.S., J.C. Pan, 2004. Just-in-time purchasing: an integrated inventory model involving deterministic variable lead time and quality improvement investment. International Journal of Production Research, 42: 853–863.

Zhu, K., R.Q. Zang, F. Tsung, 2007. Pushing quality along the supply chain. Management Science, 53: 421–436.