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Analysis of Extreme-value Distributions and its Application to Daily Maximum Temperature Measurements

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Abstract: Many conventional probability models had been used for appropriate modelling of various real life situations in the past but the use of a class of extreme-value probability models is presented in this paper. We seek the exact extreme-value distribution that will best fit or model our data (of Daily Maximum temperature measurements) using the maximum likelihood estimate approach and the method of Adeyemi and Ojo (2003) for comparative analysis of distributions.

Key words: Daily maximum temperature, probability model, extreme-value distributions, maximum likelihood estimate.

INTRODUCTION

The importance of elements of weather and climate to nature cannot be overemphasized. As a result, temperature, humidity rainfall and other elements of weather and climate are very important to man.

Temperature which measures the degree of coldness or hotness of a body and or its surroundings controls the activities of cells, tissues, organs and systems in living organism. The major characteristics of living organism viz reproduction, nutrition, growth and digestion all takes place at specific temperatures only.

Again, since human economics activities such as air and water transportation also depends on elements of weather and climate specifically temperature.

Hence, the need for meteorologists who are major 'players' in this business to make error-free weather forecast or predictions becomes a challenge.

Methodology and Model Specification:

A ten year data of daily maximum temperature measurements is taken. We therefore, seek all possible probability models that will give the exact or best fit for the data using the Fisher-Tippet theorem.

Fisher-Tippet Theorem (Fisher-Tippet) Fisher-Tippet Theorem without Proof:

Let $\{x_n\}$ be a sequence of independent and identically distributed random variables. If $\exists$ constants $a_n > 0,\ b_n \in \Re$ (a set of real numbers) and a non-degenerate distribution function $H \ni$ $a_n^{-1}(m_n - b_n) \xrightarrow{d} H.$ Then, it is of the type of the following three distribution functions i.e. the Frechet, the Weibull and the Gumbel distribution

This theorem established the fact that extreme-value measurements follow a class of extreme-value distributions. We will however, fit our data to each of these extreme-value probability models given below

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NAME	LIMIT MAXIMUM DISTRIBUTION	DENSITY FUNCTION
Frechet	$G_1(x,\alpha)=0;\ x\le 0\ ,\ \ \alpha>0$ $=\exp(-x^{\alpha})\ ;\ \ x>0$	$g_1(x,\alpha)=0\ \ ,\ x\le 0\ ,\ \alpha>0$ $=\alpha_{x}^{-\alpha-1}\exp(-x^{-\alpha})\ ,\ x>0$
Weibull	$G_2(x,\alpha)=\exp[-(-x)^{\alpha}]\ ;\ \ x\le 0\ ,\ \alpha>0$ $=\ \ 1\ \ ,\ .x>0$	$g_2(x,\alpha)=\alpha(-(-x))^{\alpha-1}\exp(-(-x)^{\alpha})$ $,x\le 0,\alpha>0$ $=1\ \ ,\ \ x>0$
Gumbel	$G_3(x)=\exp(-e^{-x})\ ,\ -\infty<x<\infty$	$g_3(x)=e^{-x}.\ e^{-e^{X}}\ ,-\infty<x<\infty$

We consider fitting our data to these models using MATLAB, a computer software package. This gives maximum likelihood estimates of the mean and variance.

We will also fit our data to each of these probability models and set up a difference which consists of taking absolute differences of each of these fit and their corresponding relative frequencies (from a frequency distribution table) (Adeyemi and Ojo 2003)

By comparison, the extreme-value probability model with the least absolute value difference will best model the data.

The maximum likelihood estimates for the mean and standard derivation of the Weibull distribution are respectively 23.2395, 3.3788 while for Gumbel distribution we have 23.3087 and 7.2002 respectively.
This is not enough to conclude that the Weibull distribution will fit our data precisely since it has the least standard deviation estimate, since there exist an element of uncertainty in the estimation.

Frequency Distribution Table

CLASS INTERVAL	CLASS MIDPOINT	FREQUENCY	CUMMULATIVE FREQUENCY	RELATIVE FREQUENCY
7.0- 0.8	7.9	5	5	0.00136986
8.8 –10.6	9.7	21	26	0.00575342
10.6 –12.4	11.5	141	167	0.03863014
12.4 – 14.2	13.3	404	571	0.11068493
14.2 – 16.0	15.1	494	1065	0.13534247
16.0 – 17.8	16.9	465	1530	0.12739726
17.8 – 19.6	18.7	469	1999	0.12849315
19.6 – 21.4	20.5	428	2427	0.11726027
21.4 – 23.2	22.3	319	2746	0.08739726
23.2 – 25.0	24.1	207	2953	0.05671233
25.0 – 26.8	25.9	175	3128	0.04794521
26.8 – 28.6	27.7	134	3262	0.03671233
26.8 – 30.4	29.5	106	3368	0.0290411
30.4 – 32.2	31.3	75	3443	0.02054795
32.2 – 34.0	33.1	84	3527	0.0230137
34.0 – 35.8	34.9	42	3569	0.01150685
35.8 – 37.6	36.7	40	3609	0.0109589
37.6 – 39.4	38.5	29	3638	0.00794521
39.4 – 41.2	40.3	5	3643	0.00136986
41.2 – 43.0	42.1	4	3647	0.00109589
43.0 – 44.8	43.9	3	3650	0.00082192

Table of fit using the Density Functions

MIDPOINT	FREQUENCY	RELATIVE FREQUENCY	FUNCTION 1 (FRECHET)	FUNCTION 2 (WEIBULL)	FUNCTION 3 (GUMBEL)
7.9	5	0.00136986	2.6326E-20	0.020775	0.00024025
9.7	21	0.00575342	9.0974E-21	0.028102	0.001226331
11.5	141	0.03863014	1.04443E-17	0.04008	0.004129938
13.3	404	0.11068493	6.1906E-18	0.061624	0.010060529
15.1	494	0.13534247	2.30015E-18	0.106265	0.019043745

Table Continue

16.9	465	0.12739726	7.18334E-20	0.222818	0.029618903
18.7	469	0.12849315	4.02681E-21	0.699361	0.039531364
20.5	428	0.11726027	4.8681E-21	0.768752	0.046834704
22.3	319	0.08739726	1.11657E-17	5.39E-22	0.050570007
24.1	207	0.05671233	1.2774E-17	0.158894	0.050796263
25.9	175	0.04794521	5.97295E-20	0.10018	0.048231029
27.7	134	0.03671233	1.86534E-21	0.061013	0.043831281
29.5	106	0.0290411	7.35818E-22	0.040227	0.038495997
31.3	75	0.02054795	3.46617E-21	0.028331	0.032923061
33.1	84	0.0230137	5.05797E-21	0.020974	0.027579867
34.9	42	0.01150685	3.46617E-21	0.016131	0.022734246
36.7	40	0.0109589	2.39939E-21	0.012783	0.018506084
38.5	29	0.00794521	2.44199E-18	0.010374	0.014917655
40.3	5	0.00136986	3.12023E-19	0.008585	0.011933787
42.1	4	0.00109589	2.07654E-19	0.007221	0.00949029
43.9	3	0.00082192	5.44838E-18	0.006157	0.007512319

Differences Table

DIFFERENCE 1	DIFFERENCE 2	DIFFERENCE 3	MINIMUM DIFFERENCES
0.00137	0.019406	0.00113	min 1
0.005753	0.022348	0.004527	
0.03863	0.001449	0.0345	0.000822
0.110685	0.049061	0.100624	
0.135342	0.029077	0.116299	min 2
0.127397	0.095421	0.097778	
0.128493	0.570868	0.088962	0.001449
0.11726	0.651492	0.070426	
0.087397	0.087397	0.036827	min 3
0.056712	0.102182	0.005916	
0.047945	0.052234	0.000286	0.000286
0.036712	0.024301	0.007119	
0.029041	0.011186	0.009455	
0.020548	0.007783	0.012375	
0.023014	0.00204	0.004566	
0.011507	0.004625	0.011227	
0.010959	0.001824	0.007547	
0.007945	0.002429	0.006972	
0.00137	0.007215	0.010564	
0.001096	0.006125	0.008394	
0.000822	0.005335	0.00669	

The difference table shows that the Gumbel distribution yields the least minimum difference of 0.000286 Discussion.

From the above difference table, it obvious that the Gumbel distribution gave the least absolute value difference of 0.000286

Conclusion and Recommendation:

The Gumbel distribution is the best model for the data (of daily maximum temperature measurements) It is therefore recommended that any extreme-value measurements (data) will best be modelled by extreme-value distributions: Frechet, Gumbel and Weibull distribution.

More importantly, maximum temperature measurements and other maximum measurements of elements of weather and climate quantities will best be modelled by the Gumbel distribution.

Hence, we prefer an important solution to problems of weather prediction in the field of meteorology

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