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H_∞ Control of Robotic Manipulator using Automatic Loop-Shaping (ALS)

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Abstract: H_∞ Control designs are quite effective in robust design for systems under unstructured uncertainty, unknown disturbance and measurement noise. Conventional H_∞ loop-shaping is a reasonable method for designing the robust controller. H_∞ Loop-shaping guarantees closed-loop performance and robustness to generic unstructured dynamic uncertainty, where robustness to uncertainty is incorporated through a μ-synthesis design. At present, computer aided design tools are mostly in use for loop-shaping, and moreover, it is usually based on trial and error approach. In cases, where the plant is somehow complex, proper weight selection for loop shaping may prove to be an extremely hard task for the designer. In particular, when dealing with MIMO systems, it is difficult to set the weights so as to obtain a sufficient robust stability margin and satisfy the control specifications. The remedy to this problem is the automation of weight selection. The proposed H_∞ automatic loop-shaping method blend with advantages of a classical manual loop-shaping method. The main impetus of this paper is on the automatic loop shaping for designing H_∞ control efficiently and effectively. In proposed loop-shaping procedure, the robust stability bounds are used to get the optimized weighting matrices. This methodology helps to automates loop-shaping as well as improves robustness. Proper transformation from the MIMO system to a series of equivalent SISO problems is suggested that motivates an enhanced synthesis procedure. Special consideration is given to the adjustment of design weights in order to meet prescribed bounds. The innovation of this research focuses on the development of compensators augmented with the plant by satisfying the performance and robustness properties with much little effort. In particular, it shows how the specifications in terms of maximum complementary sensitivity is related to the weighted H_∞ introduced by McFarlane and Glover [1]. Finally, the proposed method is applied on the two-link arm to illustrate the effectiveness of the design procedure, as well as performance of the proposed algorithm.

Key words: Robust; Loop-shaping; Unstructured dynamic uncertainty; Robust Performance; complementary sensitivity

INTRODUCTION

Control of robot manipulators has been accomplished with several kinds of controllers, from the conventional PID to modern PID. One such technique was adopted by Khan and Iqbal (Khan and Iqbal, 2007) for the Control of robotic arm PA-10. A standard PID controller performs well under normal operating conditions. When disturbances are introduced, a way to ensure the performance is to increase the gains of the controller. However, this method will only be valid for a limited region of the state space. To reject effectively the disturbance, properly approaching the desired point, and counteract the uncertainties, it is necessary that the designed controller should be *robust.*

One such controller was originally proposed by Zames (Zames, 1981) to emphasize on plants including uncertainties. The solution presented by Doyle (Doyle, 1984) to the so called 2×2 block problems using the Youla *et al.,* (1976) controller parameterization and Nehari/Hankel norm approximation (Glover, 1984) with state space methods as a computational tool. The theory developed is famously known as H_∞ control theory.

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During the past few years, control engineers have been paying increasing attention to the H_∞ control theory. The continuous nonlinear case was extensively studied by van der Schaft (Schaft, 1992) as well as he proposed the solution to the problem. Although many methodologies for solving H_∞ control have been developed, there isn't much effort made on applying this theory to robotic manipulators. The main advantage of robust control is the fact that all kind of uncertainties are incorporated explicitly into the problem hypothesis. Therefore, the issue of how to model uncertainty becomes more important. Doyle (1982) and Safonov (1982) independently introduced similar concepts for systems with special kind of structured uncertainty. In linear time invariant (LTI) controller design, H_∞ control and μ-design are jointly used to assure stability and control performance. Nonami, Ito (Nonami, 1996) and Schönhoff, *et al.,* (2000) treated parametric uncertainties using μ-synthesis. Lanzon and Tsiotras (2005) moved one step further and they combined the approaches of H_∞ loop-shaping and μ-synthesis to design controllers. We have also used μ-synthesis to analyse the robust stability and performance of the shaped plant in the presence of unstructured dynamic uncertainty.

A well known approach to MIMO feedback controller design is the so-called loop shaping approach discussed by Doyle, Stein (1981) and Safonov, *et al.,* (1981). Their work was primarily concerned with the issue that how to achieve the benefits of feedback in the presence of unstructured uncertainty, and how the classical loop-shaping ideas could be generalized to MIMO systems. A design method, known as the loop shaping design procedure (LSDP) was proposed by McFarlane and Glover (1990). The H_∞ control theory based on LSDP is one of those methods, which has proven to be efficient in practical industrial design, considers being one of the effective control methods, and shows a reasonable control performance if appropriate weights are selected.

Hyde and Glover (1993) used H_∞ loop-shaping for LTI controller design. In practical systems design, it is usually an essential step that the plant frequency response be re-shaped in order to meet closed-loop performance requirements. However, it is not an easy job to choose the weights in a manner as to achieve good control performance and robustness especially in the case of MIMO systems. It is also evident that due to lack of knowledge of the loop-shaping concepts, the designed loop-shape system may not be able to achieve sufficiently large robust stability margin. This is the main reason for the problem of automatic loop shaping to be of enormous interest, and thus it has received a considerable attention, especially in the last two decades.

Gera and Horowitz (1980) were among the first one who made an effort towards the goal of achieving an automatic loop-shaping (ALS) algorithm. They described a semi-ALS process based on iterative approach, in which the designer updates the open-loop transfer function after each successive iteration by adding an element, and force it to pass a given straight line in the Nichols chart. Lanzon (2001) proposed an algorithm that automatically design weights W_1 and W_2 for H_∞ loop-shaping design. Lanzon (2005) also adopted an iterative approach for the design of the controllers and weights for the problem of frequency domain optimization for H_∞ loop-shaping. Whidborne, *et al.,* (1994) proposed to incorporate the method of inequalities with LSDP such that it is possible to search for "optimal" weighting functions automatically and to meet more explicit design specifications in both the frequency domain and the time domain. In mixed-sensitivity H_∞ control design techniques a performance criterion is used to ensure that the closed loop achieves desirable behaviour. Various approaches adopted and analyzed by Lundström, *et al.,* (1991) for the selection of performance weight using H_∞.

In this paper, to alleviate the problem of manually tuning the weighting matrices, the design methods using PD controller is proposed for Automatic H_∞ loop-shaping method. The paper mainly focuses on blending the advantages of classical loop-shaping with that of automatic loop shaping. The other concern of this article is to achieve robust stability and performance even in the presence of disturbance and unstructured uncertainty. This paper is organized as follows: In Section 2, some preliminaries will be discussed which are very vital in H_∞ control and Loop-shaping design. Proposed Automatic Loop-Shaping technique will be discussed in Section 3. In section 4, case study of a two-link arm is presented, robust stability and performance will also be evaluated in the same section. The last section is devoted to the Conclusion.

2. Robust Control Methodology and Loop-shaping Design:

Given generalized plant *P(s)* and stability measure $\gamma>0$, the H_∞ standard problem is to find admissible controller *K(s)*, such that:

- The controller ensures the internal stability of the closed-loop system as shown in Fig. 1.
- Minimizes the maximum singular value of the transfer function between the input w and output z i.e.

$$\left\|\Omega(P,K)\right\|_\infty < \gamma$$

McFarlane and Glover (1992) introduced a single index, robust stability margin ε. ε is an excellent measure of closed-loop robust stability, as well as it shows the success of the design in satisfying the loop-shaping specifications. Sometimes, $\gamma = 1/\varepsilon$ is also used as stability measure. γ is actually known as the cost function. In order to obtain maximum stability margin i.e. $\varepsilon_{\max}$, as well as to synthesize H_∞ robust stabilizing controller K_∞, it is essential to use the shaped plant G_s. So, loop shaping is often a necessary step before synthesizing controller K_∞. In any plant model G, the shaped plant G_s is formulated as McFarlan and Glover (1992), $G_s = W_2 G W_1$. The details of the shaped plant and the weighting matrices are given in the next section. There are four necessary steps in determining the controller K_∞ to achieve maximum stability margin.

1. Transfer function matrix G_s is used to establish minimal state space model (A,B,C,D) of the shaped plant.

$$G_s \stackrel{s}{=} \begin{bmatrix} A & B \\ C & D \end{bmatrix} \tag{1}$$

2. Solve the generalized control algebraic Riccati Equation

$$\left(A - BS^{-1}D^T C\right)^T X + X\left(A - BS^{-1}D^T C\right) - XBS^{-1}B^T X + C^T R^{-1} C = 0 \tag{2}$$

for X, where

$$R = I + DD^T$$

$$S = I + D^T D$$

Solve the second generalized ARE for Y

$$\left(A - BD^T R^{-1} C\right)Y + Y\left(A - BD^T R^{-1} C\right)^T - YC^T R^{-1} C^T Y + BS^{-1} B^T = 0 \tag{3}$$

where R and S are the same as used in the above equation. Procedures for ARE solving can be found in (Zhou *et al.,* 1996; Maciejowski, 1989).

3. If $X \geq 0$ and $Y \geq 0$ are unique stabilizing solutions to ARE above, we can obtain $\varepsilon_{\max}$ as

$$\varepsilon_{\max} = \frac{1}{\sqrt{1 + \lambda_{\max}(XY)}} \tag{4}$$

where $\lambda_{\max}(XY)$ denotes the maximum Eigen value of the matrix product of XY.

Central or minimum entropy controller, which satisfies that, $\left\|\Omega(P,K)\right\|_\infty < \gamma$ has the state space form McFarlane and Glover (1992)

$$K_\infty \stackrel{s}{=} \begin{bmatrix} A + BF + \gamma^2 \left(L^T\right)^{-1} ZC^T (C + DF) & \gamma^2 \left(L^T\right)^{-1} ZC^T \\ B^T X & -D^T \end{bmatrix} \tag{5}$$

where

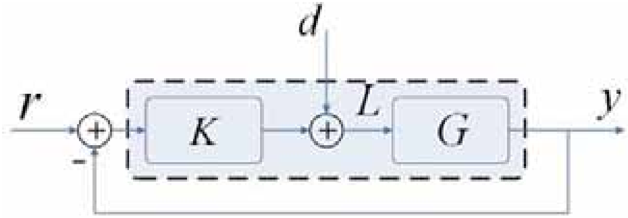

Fig. 1: SISO feedback configuration

$$F = -S^{-1}\left(D^T C + B^T X\right)$$

$$L = \left(1-\gamma^2\right)I + XZ$$

$$\gamma = \varepsilon^{-1}$$

In addition, in contrast to conventional H_∞ control problems, the calculation of γ_{min} does not require bisection search. The reader is referred to (Zhou *et al.,* 1996; Maciejowski, 1989) for details on the H_∞ control problem and related discussions.

Remarks:

Performance of the Glover-Doyle Central Controller is indicated by only a single index ε i.e., robust stability margin. $\varepsilon_o < 0.25$ or $\gamma_o > 4$ indicates that weights W_1 and W_2 are failed to achieve the desired robust stability requirement. The optimal range specified for robust stability margin is between zero and unity i.e. $0 \le \varepsilon \le 1$

Loop-shaping is the process that needs to be done prior designing the robust controller. The main theme of the H_∞ loop-shaping design is to synthesize a controller which provides robustness against uncertainties, and minimizes the signal transmission from disturbance to process input and output. According to standard H_∞ LSDP, the following steps were proposed to design a robust controller McFarlane and Glover (1992).

1. Using a pre-compensator W_1 , and/or a post-compensator W_2 , the singular values of the nominal plant *G(s)* are modified to give a desired loop shape. The nominal plant and the weighting functions W_1 and W_2 are combined to form the shaped plant G_s, which is illustrated in Fig. 2.

2. Minimize max norm of the transfer matrix $\Omega(P,K)$. Select $\varepsilon < \varepsilon_o$ and then synthesize a controller K_∞ . K_∞

is obtained by solving the optimal control problem in the previous section and will satisfy

$$\left\| \begin{bmatrix} I \\ K_\infty \end{bmatrix} \left(I - G_s K_\infty\right)^{-1} \begin{bmatrix} I & G_s \end{bmatrix} \right\|_\infty \le \varepsilon^{-1} \tag{6}$$

3. The Final feedback controller K is the composition of pre- and post-compensators W_1 and W_2 such that $K = W_1 K_\infty W_2$, and is of degree $n_K = n + 2n_w$, where n stands for the degree of the nominal plant and n_w is the combined degree of the shaping compensator W_1 and W_2.

The most important key point in loop-shaping design is to choose appropriate weighting matrices. Usually it is a hard task for the novice engineers, which can often make the situation disastrous, and lead system to instability. The automation technique for the selection of these pre- and post-compensators are discussed in the next section.

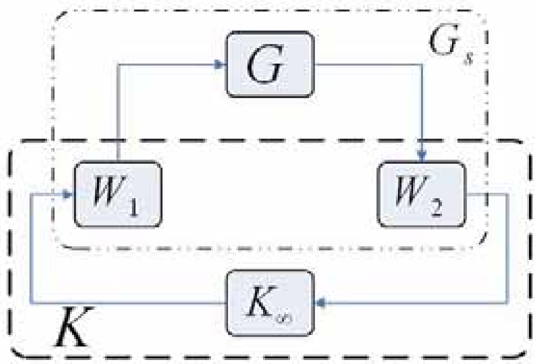

Fig. 2: The Shaped Plant G_s and Controller K

3. Proposed Automatic Loop-shaping Technique:

Loop-shaping is often a key step to help achieving the robust stability in the presence of uncertainties. Loop-shaping is mostly accomplished in a manual fashion using computer-aided control system design (CACSD) tools. Loop-shaping mostly depends upon the experience of the designer, that's why it is considered to be not simple for the non-experienced engineers, who have little or not much knowledge about the technique.

In this section, a design procedure is proposed to automatically tune the PD controller parameters to achieve satisfactory stability robustness. The PD controller is actually defining the pre-compensator W_1. The open-loop transfer function for SISO or MIMO is defined as

$$L(jw) = G(jw)K(jw); \qquad \forall w \geq 0 \tag{7}$$

For real time stability analysis, it is essential that the constraints be applied on closed-loop sensitivity T, which is also known as the complementary sensitivity

$$T(jw) = \frac{L(jw)}{1 + L(jw)} \tag{8}$$

$$\|T\|_\infty = \max_w |T(jw)| \tag{9}$$

Stability robustness for control systems are frequently given in terms of algebraic or functional inequalities, rather than in the minimization of some objective function (Whidborne *et al.,* 1996). The automatic loop shaping problem for SISO or MIMO systems can be described as follows: Find a stabilizing LTI controller *K(s)*, such that the feedback system whose complementary sensitivity transfer function is bounded by

$$\left|\frac{L(jw)}{1 + L(jw)}\right| \leq M_T; \qquad \forall w \geq 0 \tag{10}$$

Note:

It must be kept in mind that we are working on the problem of ALS. In actual, our controller *K(s)* is defining the weights for the pre-compensator W_1. The general MIMO plant will be converted to the equivalent SISO systems, which will be discussed later in the section.

The procedure adopted for designing the pre-compensator is given below:

3.1 Design Procedure:

The design procedure for ALS can be accomplished successfully following these two steps:

- Select an appropriate robust stability margin for each system.
- Design the pre-compensator for the plant's subsystem such that the design bounds are satisfied.

The existence of the solution i.e. controller *K(s)* in our case, depends heavily upon the robustness bound selected. So, careful consideration is needed while specifying the bounds. The automatic determination of the pre-compensator W_1 is discussed in the next section.

3.2 Automatic Weighting Function Selection:

The relationship between controller *K(s)* and plant *G(s)* is readily obtained in complex plane and defined as (Yaniv, 2006)

$$L(s) = a(G_1(s) + bG_2(s)) \tag{11}$$

where G_1 and G_2 are two pair of plants. If we define $s=jw$, the plant *G(s)* in the frequency domain is defined as

$$G(jw) = G_R + jG_I \tag{12}$$

where G_R is the real part of the plant, and G_I is the complex part. Consider the SISO system in Fig.1, where the plant *G(s)* is controlled by the controller *K(s)*. The controller can be either strictly proper or atleast proper. Now, let us first design PD controller.

3.2.1 Derivation of Pd Control Parameters:

The Proportional plus Derivative terms connected in series can be represented using

$$K = K_P\left(1 + \frac{K_D}{K_P}s\right) \tag{13}$$

Now, if we separate controller from plant in Eq. (11), then we have

$$L(jw) = (G_R + jG_I)(a(1 + jbw)) \quad = x + jy \tag{14}$$

Now, the task is to find the scalar values *a* and *b* which are in actual control parameters K_P and K_D. Now Eq. (10) will be of the form

$$\left|\frac{x + jy}{1 + x + jy}\right| \le M_T \tag{15}$$

By doing some back substitutions, we have

$$M_T^2 + a\left(M_T^2\left(2G_R - 2bwG_I\right)\right) + a^2\left\{\left(G_R^2 + G_I^2\right)\left(M_T^2 - 1\right) + b^2\left(w^2G_I^2 + w^2G_R^2\right)\left(M_T^2 - 1\right)\right\} \ge 0 \tag{16}$$

For simplification the above equation can be written like this

$$\alpha + a\left(\beta_1 + b\beta_2\right) + a^2\left(\gamma_1 + b^2\gamma_2\right) \ge 0 \tag{17}$$

We are interesting in finding the optimal value (a_0 , b_0) that gives the low high frequency gain. There will be more than one (*a,b*) pairs, which satisfies the Eq. (17), but we are interested in the pair, which gives us the lowest high frequency gain, as well as satisfies the specification bounds. All the (*a,b*) pairs needed to be plotted on the frequency plot. At the point the product of the pair is minimum, pick up that pair. When we differentiate Eq. (17) with respect to frequency *w*, we have

$$\dot{\alpha} + a\left(\dot{\beta}_1 + b\dot{\beta}_2\right) + a^2\left(\dot{\gamma}_1 + b^2\dot{\gamma}_2\right) = 0 \tag{18}$$

Solving the equality for *a* gives *b*

$$a = \frac{\alpha\dot{\beta}_1 - \dot{\alpha}\beta_1 + b\left(\alpha\dot{\beta}_2 - \dot{\alpha}\beta_2\right)}{\alpha\dot{\gamma}_1 - \dot{\alpha}\gamma_1 + b^2\left(\alpha\dot{\gamma}_2 - \dot{\alpha}\gamma_2\right)} = \frac{A + bB}{C + b^2D} \tag{19}$$

gives a fourth order polynomial equation for

$$x_4 b^4 + x_3 b^3 + x_2 b^2 + x_1 b^1 + x_0 = 0 \tag{20}$$

whose coefficients are the following functions of w

$$\begin{aligned} x_4 &= \alpha D^2 + BD\beta_2 + B^2\gamma_2 \\ x_3 &= BD\beta_1 + AD\beta_2 + 2\cdot AB\gamma_2 \\ x_2 &= 2\cdot \alpha CD + AD\beta_1 + BC\beta_2 + B^2\gamma_1 + A^2\gamma_2 \\ x_1 &= BC\beta_1 + AC\beta_2 + 2\cdot AB\gamma_1 \\ x_0 &= \alpha C^2 + AC\beta_1 + A^2\gamma_1 \end{aligned} \tag{21}$$

The controller given by Eq. (13) represents an idealized controller. It is useful in many of the cases, but it is improper in real. So that, some modifications need to be made in order to obtain a controller that is practically useful. As we mentioned in the beginning of this section, that the controller needs to be either strictly proper or atleast proper to make it realizable. In general, we can regard the design of controllers of control system as a filter design problem; then there are a large number of possible schemes. The remedy proposed here, is to cascade the differentiator with a first-order, low-pass filter.

$$K = K_P + \frac{K_D s}{1+\zeta K_D s} \tag{22}$$

ζ_k must be chosen less than 0.01. Nevertheless, it has made the improper system proper, which can now be realized easily in any domain. The complete procedure for designing two parameters controller is given below:

3.2.2 Design Steps for Two Parameter Controller:

The two parameter controller can be easily designed following the steps given below:

1. Choose plants G_1 and G_2 for designing PD.
2. The value of ζ is typically<0.01.
3. Find the value of *b* using Eq. (20) for different frequencies w .
4. Using the values of *b*, find the values of a using Eq. (19).
5. This will give you a pair (*a,b*) on different frequencies.
6. Ensure the stability of the closed-loop system using these values of *a* and *b*. Pick up those pairs of *a*,*b* under which system remains stable. This will refine your search for an optimal pair.
7. Find the optimal value $(a_0, b_0) \in (a,b)$, which gives us the minimum high frequency gain. And this you can find using $\inf(ab)\forall[a,b]$.

The same procedure can be applied to MIMO systems with some enhancements, which are discussed in the next section.

3.3 Methodology Extended to Control of MIMO Systems:

Let our MIMO plant has the general form

$$G(s) = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1n} \\ p_{21} & p_{22} & \cdots & p_{2n} \\ . & . & . & . \\ p_{n1} & . & . & p_{nn} \end{bmatrix} \tag{23}$$

where P_{ij} are different systems.

Since, non-iterating control of multivariable system implies one-to-one correspondence between a reference input and a controlled input, it is desired that *L(s)* be a diagonal matrix. Therefore, the off-diagonal element of matrix *L(s)* is zero. Now, we will transform our MIMO system to the most suitably diagonally dominant form for several reasons. Firstly, the pre-compensator designed will have no direct effect on the open-loop shape if the off diagonal terms are dominant. Secondly, it would be much easier to apply stability constraints. The diagonal dominant form can be achieved by pre- and post-multiplying the weighting matrices *U,V* with the plant *G(s)*. *G(s)* is now diagonal and it is easier to control than the original plant. Without loss of generality, it is supposed that controller *K(s)* is a diagonal matrix of the form

$$\mathbf{K} = \begin{bmatrix} k_1 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & k_{k-1} & 0 \\ 0 & \cdots & \cdots & k_k \end{bmatrix} \tag{24}$$

The complementary sensitivity specifications are also applied to the diagonal elements

$$|t_{ii}| < m_i \forall w \tag{25}$$

$$\left|\frac{L_i}{1+L_i}\right| \le m_i; \qquad \forall w \tag{26}$$

Like wise for k*th* plant

$$\left|\frac{k_k \varsigma_k}{1+k_k \varsigma_k}\right| \le m_k \tag{27}$$

ζ_s denotes the diagonal transfer functions of the transformed plant. And, once we have converted our MIMO system problem into the equivalent SISO systems. The rest of the problem solution is the same as defined for the SISO case. The algorithm is capable of searching the coefficients of the pre-compensator for the weighted plant in a very efficient manner, because of its non-iterative nature.

4. Case Study: a Two-link Robot Manipulator:

In this section, the controller scheme developed in the previous section is tested on an academic example. Fig. 3. shows the scheme of the 2-DOF planar robot considered. It is made of two links and two revolute joints. Each link is characterized by the following parameters: mass (m_i), length (l_i), mass center position (l_{ci}), and inertia (I_i), where i=1,2

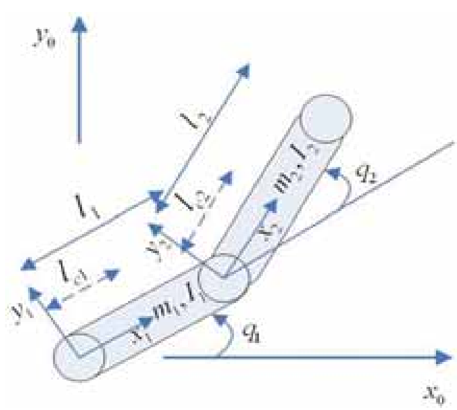

Fig. 3: Scheme of the 2-DOF robot arm

4.1System Description/Nonlinear Mechanical Control System:

The following Euler-Lagrange equation of motion is used to describe the behaviour of a *n* degree-of-freedom (DOF) robot manipulator (see, for example, Lewis *et al*., 2004 and Schilling, 1990):

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + F(\dot{q}) + G(q) + \tau_d = \tau \tag{28}$$

or

$$M(q)\ddot{q} + N(q,\dot{q}) + \tau_d = \tau \tag{29}$$

where the nonlinear terms are

$$N(q,\dot{q}) \equiv C(q,\dot{q})\dot{q} + F(\dot{q}) + G(q)$$

and

$$F(\dot{q}) = F_v\dot{q}$$

where $q = \begin{bmatrix} q_1 & \dots & q_k & \dots & q_n \end{bmatrix}^T \in \Re^n$ is the vector of joint variables (joint positions) and $\dot{q} \in \Re^n$ is the time derivative (joint speeds). It is assumed that these two vectors are available for measurements. Vector $\tau \in \Re^n$ (generalized torques applied on the joint axes) is the torque input vector and τ_d represents the total effect of system modelling errors and external disturbance. $M(q) \in \Re^{n\times n}$ represents inertia associated with the distribution of mass which is also symmetric and positive definite, $C(q,\dot{q}) \in \Re^{n\times n}$ represents interaxis velocity coupling due to centrifugal and Coriolis forces, $F(\dot{q}) \in \Re^n$ represents the friction terms with $F_v \in \Re^{n\times n}$ is the positive definite matrix represents the coefficient matrix of viscous friction and $G(q) \in \Re^n$ denotes the vector of gravitational forces.

The Euler-Lagrange equation defining dynamic model of the robotic arm contains following matrices and vectors:

$$M(q) = \begin{bmatrix} M_{11}(q) & M_{12}(q) \\ M_{12}(q) & M_{22}(q) \end{bmatrix}, \tag{30}$$

where

$$\begin{aligned} M_{11}(q) &= m_1 l_{c1}^2 + I_1 + m_2\left(l_1^2 + l_{c2}^2 + 2l_1 l_{c2}\cos(q_2)\right) + I_2 + m_l\left(l_1^2 + l_2^2 + 2l_1 l_2\cos(q_2)\right) \\ M_{12}(q) &= m_2\left(l_{c2}^2 + l_1 l_{c2}\cos(q_2)\right) + I_2 + m_l\left(l_2^2 + l_1 l_2\cos(q_2)\right) \\ M_{22}(q) &= m_2 l_{c2}^2 + I_2 + m_l l_2^2 \end{aligned} \tag{31}$$

define the manipulator inertia matrix. Velocity coupling matrix is obtained using:

$$V(q,\dot{q}) = \begin{bmatrix} -h\dot{q}_2 & -h(\dot{q}_1 + \dot{q}_2) \\ h\dot{q}_1 & 0 \end{bmatrix}, \textit{where} \quad h = m_2 l_1 l_{c2}\sin(q_2) + m_l l_1 l_2 \sin(q_2) \tag{32}$$

and, the viscous friction is defined as:

(33)

$$F_v(q) = \begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix}, where \quad k_1, k_2 \in \Re$$

Finally, the Gravity term is defined as follows:

$$G(q) = \begin{bmatrix} m_1 g l_{c1} \cos(q_1) + m_2 g (l_1 \cos(q_1) + l_{c2} \cos(q_1 + q_2)) + m_l g (l_1 \cos(q_1) + l_2 \cos(q_1 + q_2)) \\ m_2 g l_{c2} \cos(q_1 + q_2) + m_l g l_2 \cos(q_1 + q_2) \end{bmatrix} \tag{34}$$

with g being the gravity constant.

To facilitate control of the arm, it is helpful to reformulate the equations of motion as a first-order system of $2n$ equations called *state equations*. The key to transforming to the state-space form is the isolation of the acceleration vector $\ddot{q}$. This can be done easily because the manipulator inertia tensor satisfies the Property 1. The constant friction involved in the dynamic equation holds Property 2.

Property 1:

The inertia matrix is symmetric and uniformly positive definite for all $q \in \Re^n$, and therefore non singular.

Property 2:

$F_v \in \Re^{n \times n}$ is positive definite matrix.

The state space description of the two-link planar arm is calculated based on linearization of dynamic equations. The state space form of the dynamic equation can be written as

$$\frac{d}{dt}\begin{bmatrix} q \\ \dot{q} \end{bmatrix} = \begin{bmatrix} \dot{q} \\ -M^{-1} C \dot{q} \end{bmatrix} + \begin{bmatrix} 0 \\ M^{-1} \end{bmatrix} \tau \tag{35}$$

which can also be expressed as

$$\frac{d}{dt}\begin{bmatrix} q \\ \dot{q} \end{bmatrix} = \begin{bmatrix} 0 & I \\ 0 & -M^{-1} C \end{bmatrix} \begin{bmatrix} q \\ \dot{q} \end{bmatrix} + \begin{bmatrix} 0 \\ M^{-1} \end{bmatrix} \tau \tag{36}$$

which is the same as this compact form

$$\dot{x} = Ax + Bu \tag{37}$$

where

$$A = \begin{bmatrix} 0 & I \\ 0 & a \end{bmatrix}, \quad a \in \Re^{2 \times 2}$$
$$B = \begin{bmatrix} 0 \\ b \end{bmatrix}, \quad b \in \Re^{2 \times 2} \tag{38}$$
$$x = \begin{bmatrix} q \\ \dot{q} \end{bmatrix}, \text{and } \ u = \tau$$

Note:

If the robot is upright and located near the surface of the earth, then the gravitational acceleration vector is proportional to $-i^3$, where the constant of the proportionality is $g = \|g\| = 9.8062\, m/s^2$

Table I gives the nominal values for the robot parameters involved in deriving the dynamics of robot manipulator, and are also employed for the controller synthesis.

Table 1: Nominal Robot Parameters

Parameter	*Nominal value*
Lengths	$l_1 = 0.6\, m, l_2 = 0.4\, m$
Position of mass center	$l_{c1} = 0.3 m, l_{c2} = 0.2 m$
Masses	$m_1 = 2\, kg, m_2 = 1\, kg$
Inertia moments	$I_1 = I_2 = 0.5\, kgm^2$
Load mass	$m_l = 0$
Frictional terms	$k_1 = 20, k_2 = 30$

In order to evaluate the proposed robot control scheme, extensive computer simulations were performed on a two dof planar robot manipulator using a fourth-order Runge-Kutta numerical integration algorithm because of the non-linearities involved. The state-space matrices *A,B,C,D* of the nominal plant are given as:

$$A = \begin{bmatrix} 0.00 & 0.00 & 1.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 1.00 \\ 0.00 & 0.00 & -19.73 & 36.18 \\ 0.00 & 0.00 & 24.12 & -99.78 \end{bmatrix} \tag{39}$$

$$B = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & 0.00 \\ 0.98 & -1.20 \\ -1.20 & 3.32 \end{bmatrix} \tag{40}$$

$$C = \begin{bmatrix} 1.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 1.00 & 0.00 & 0.00 \end{bmatrix} \tag{41}$$

$$D \approx [0] \tag{42}$$

4.2 Loop-Shaping:

Prior to shape the open-loop singular values of the plant, the MIMO system will be transformed to the diagonally dominant form. The pre-compensator *U* and post-compensator *V* used for the transformation are given as

$$U = \begin{bmatrix} 1 & 0 \\ \dfrac{1.206 \times 10^5 s}{s^3 + 119.5 s^2 + 9.978 \times 10^4 s + 5.482 \times 10^6} & 1 \end{bmatrix} \tag{43}$$

$$V = \begin{bmatrix} 1 & \dfrac{1.206 \times 10^5 s}{s^3 + 119.5 s^2 + 9.978 \times 10^4 s + 5.482 \times 10^6} \\ 0 & 1 \end{bmatrix} \tag{44}$$

After the MIMO system is transformed in the desirable form, the next step is to reshape the open-loop singular values. For loop-shaping, it is desired that roll-off rates of approximately 20db/decade must be achieved at the desired bandwidth, with higher rates at high frequencies. Integral action is also added explicitly to improve the low frequency response and to perform a zero steady-state error. The main goal of the pre-compensator W_1 is to obtain high gain, which in turn yields good reference tracking behaviour and good disturbance rejection properties. The pre-compensator W_1 obtained using proposed ALS technique is given as:

$$W_1 = \begin{bmatrix} \frac{1}{s}\left(\frac{145.9s+545.7}{0.226s+1}\right) & 0 \\ 0 & \frac{1}{s}\left(\frac{546.3s+1089}{0.4597s+1}\right) \end{bmatrix} \tag{45}$$

W_2 is usually chosen as a constant, reflecting the relative importance of the outputs to be controlled and other measurements being fed back to the controller (Skogestad and Postlethwaite, 2001; Sanchez-Pena and Szanier, 1998). Scale factors of 0.01 were used for both channels respectively, resulting in a pose compensator W_2 of the form:

$$W_2 = diag\left(0.01, 0.01\right) \tag{46}$$

The open-loop singular values plot of the shaped plant is illustrated in Fig. 4. It can be observed that the open-loop plot well satisfies the defined bounds.

Proposition:

$W_1, W_1^{-1}, W_2, W_2^{-1}$ are stable, minimum phase, and proper. The closed-loop system composed of G and K_∞ is internally stable. And most importantly, W_1 and W_2 are such that G_s contains no hidden unstable modes.

This constraint is necessary to guarantee the internal stability of the system.

4.3 Control design:

The state-space matrices of the shaped plant are used to formulate the generalized plant. The Robust Control Toolbox and Mu-Analysis & Synthesis Toolbox were used to perform the necessary computations. Performance index γ of 2.565 is achieved using the control scheme.

To determine robust performance, it is assumed that unstructured uncertainty is entering the system in an input multiplicative form, which is illustrated in Fig. 5. This source of uncertainty is usually considered being a part of a real system. To derive perturbed plant model G_P, we use multiplicative uncertainty of the form

$$G_P = G(s)\left[1 + W_\Delta(s)\Delta\right]; \quad \Delta \in \mathrm{H}_\infty, \quad \left\|\Delta(s)\right\|_\infty \le 1 \tag{47}$$

where $G(s)$ is the nominal plant model. The unstructured uncertainties that will be used in the sequel are

$$\Delta \triangleq \left\{ \begin{bmatrix} \Delta_1 & & 0 \\ & \ddots & \\ 0 & & \Delta_m \end{bmatrix} \middle| \Delta_i \in C^{r_i \times r_i}; i = 1, \ldots, m \right\} \tag{48}$$

The weighting function $W_\Delta(s)$ represents the "dynamics" of the uncertainty, or in other words its "frequency distribution". One may also view $W_\Delta(s)$ as a weight which is introduced in order to normalize the perturbation to be less than unity in magnitude over the whole frequency range. Thus, only the magnitude of the weight matters, and in order to avoid unnecessary problems we always restrict $W_\Delta(s)$ to be stable and minimum phase. Also, for the sake of simplicity, the weights for stability are chosen as diagonal i.e. $W_\Delta = w_\Delta I$. Uncertainty is usually higher at high frequencies so that $|w_\Delta(jw)| >> 1$ for large w and ~ 0 for low frequencies. The weighting function $W_\Delta(s)$ used for normalizing the perturbation is:

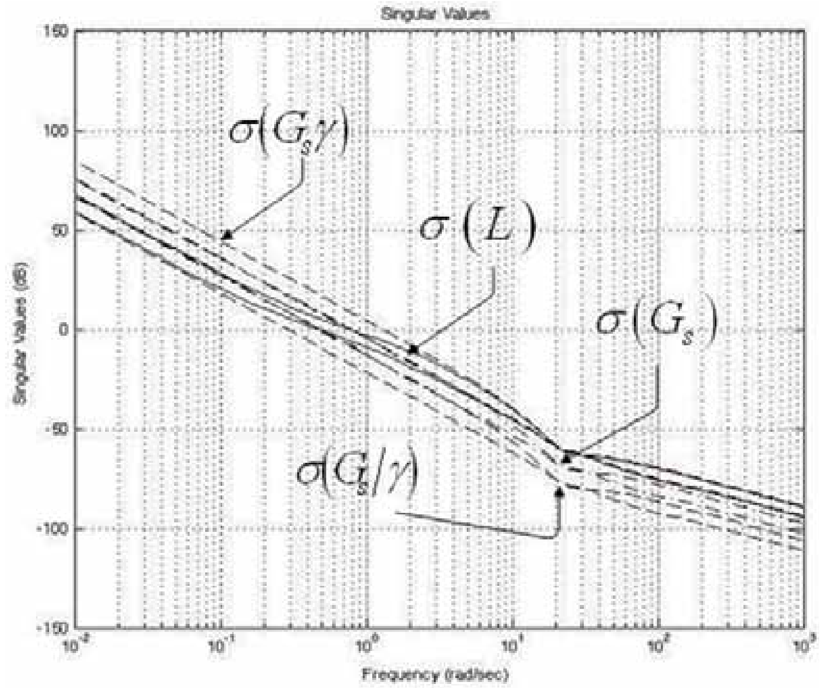

Fig. 4: Open-Loop Singular values plot of the Shaped Plant

$$W_\Delta = \begin{bmatrix} \dfrac{3s+2.843}{s+28.43} & 0 \\ 0 & \dfrac{3s+2.843}{s+28.43} \end{bmatrix} \tag{49}$$

To quantify stability and performance robustness, it is recommended to transform your system model using Linear Fraction Transformation (LFT) and analyze via the μ-analysis. With a given controller *K(s)*, we determine whether the system remains stable and shows satisfactory performance for all plants in the uncertainty set. Robust stability is a measure of how efficiently the controller can counteract the dynamic uncertainties inside the plant. We want to determine the stability of the uncertain feedback system in Fig. 5. The RS of the system can be guaranteed if μ determined remains less than unity over the whole frequency range, which can be illustrated through Fig. 6. By analyzing RP, we actually tend to observe that the performance objective is satisfied for all possible plants in the uncertainty set, even the worst-case plant. To better analyze RP, an additional uncertainty block is included, usually called the *performance block* or *fictitious uncertainty block*. Robust performance is also guaranteed according to the μ-analysis in Fig. 6, since the upper bound stays below the unity over all the frequency range

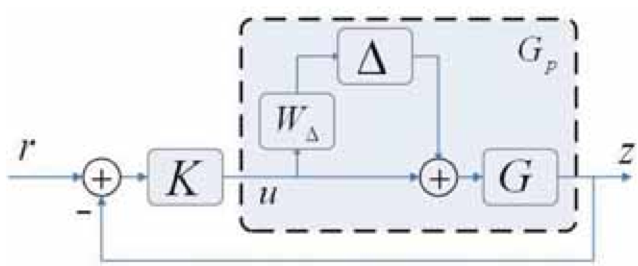

Fig. 5: Feedback System with multiplicative input uncertainty

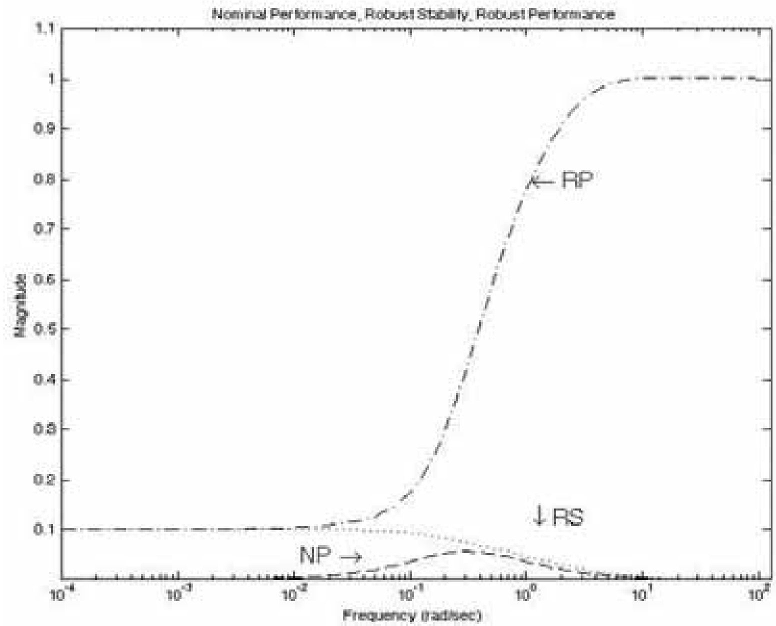

Fig. 6: plots for determining RP, RS, and NS

In addition to uncertainty, some disturbance is also introduced into the system, we applied very large continued disturbance of 100 sin (100*t*) at both the joints, whose simulation result is shown in Fig. 7. The persistent torque of 100 Nm is also applied at joints 1 and 2, the error plot between the reference and actual trajectory is shown in Fig. 8. The system achieves zero steady state error at 90 sec, whereas in case of constant disturbance the error reaches zero at 30 sec. It is hard to tackle disturbance which is variant with time, whereas constant disturbance can be easily dealt with. In the simulation presented in this section, the reference position $[q_1 \quad q_2] = [0 \quad 0]$ rad, to the final position equal to $[\pi/4 \quad \pi/2]$ rad, with initial and final speeds and acceleration equal to zero.

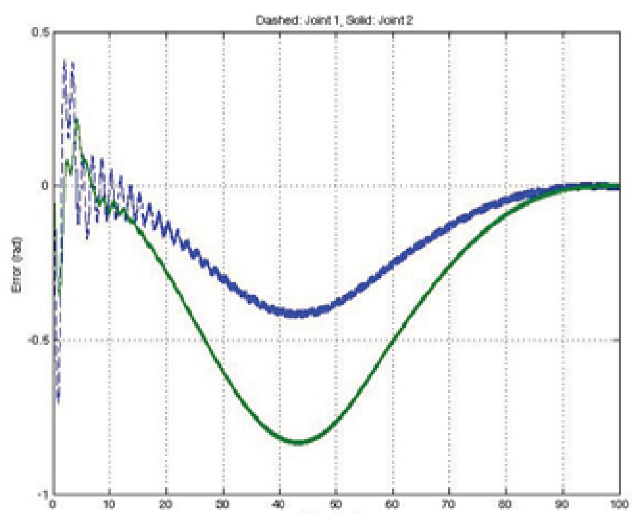

Fig. 7: Error plot for joint 1 and 2 when disturbance is time-varying

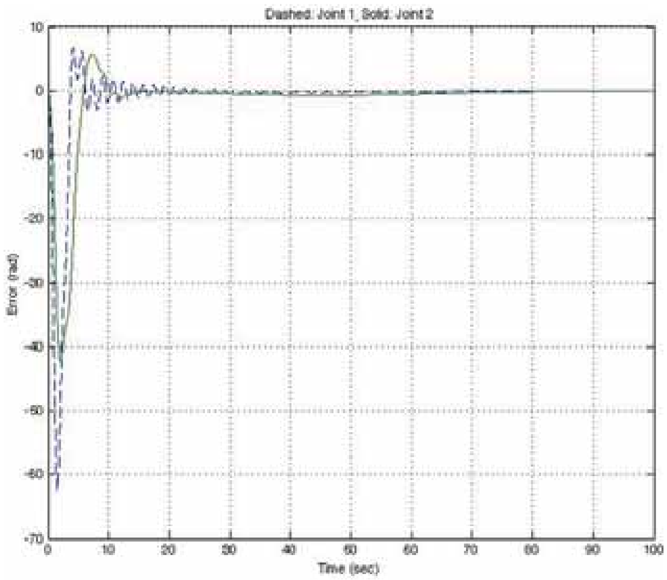

Fig. 8: Error plots for joint 1 and 2 when constant disturbance applied

The maximum singular values sensitivity and complementary sensitivity function plot of the shaped plant is shown in Fig. 9. For successfully attenuating the disturbance effect, it is desired that both *S(s)* and *T(s)* to be small in some sense. Because of the trade-off involved and the unity relationship between the both, we cannot have both small at a same time. This is the reason that when *S(s)* is nearly zero, then *T(s)* is nearly the identity; and conversely, when we have *T(s)* nearly zero, then *S(s)* will be nearly the identity. This fact can be better viewed, as illustrated in Fig. 9.

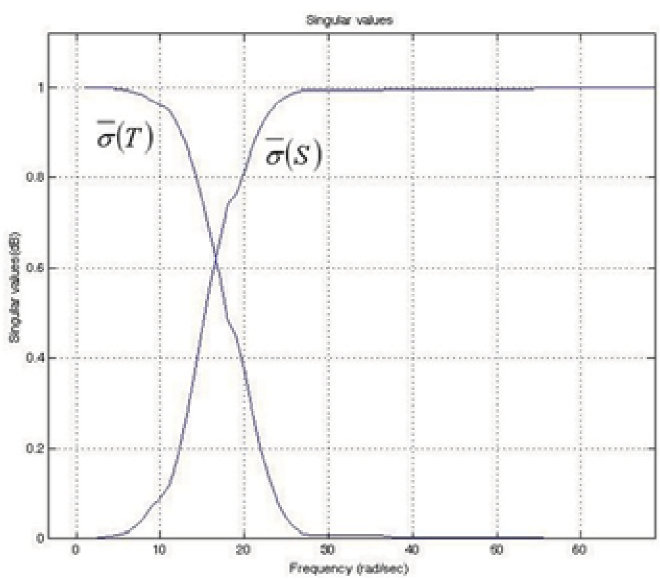

Fig. 9: Upper singular values plot of Sensitivity (*S*) and Complementary Sensitivity (*T*)

Conclusion:

A novel based method for automatic loop-shaping using H_∞ control is proposed. The proposed method uses robust stability requirement in order to generate the frequency weighting function. The aim was to design a controller with good robustness properties in presence of uncertainties in the model. No need of any computer aided control system design (CACSD) tool in designing loop-shaping. As seen from the illustrative case studies, that the proposed method helps to achieve good robust stability and performance by applying two specified weighting functions. To date, the method has been successfully applied using single dof control. Extension to two dof control would be useful; however, it is important not to understate the value of the single dof control case. The algorithm is also considered to be quick because of its non-iterative nature, and capable of producing the optimal solutions very efficiently.

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