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ECG Denoising Using Singular Value Decomposition

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Abstract: In this work we proposed a new method and a swift one of optimizing electrocardiogram signal (ECG), the basis of this method is achieved through filtering singular values of the signal. The advantage of this proposed method is its capability in enhancing the signal very well. The signals under experiment are added by additive white noise with different levels of noise mixture and after applying this method, the signals according to signal noise ratio (SNR) are evaluated, also time-frequency spectrum are compared with each other. The efficiency of the proposed method is clear and high enough in enhancing ECG.

Key word: ECG enhancement, Singular value decomposition, White noise.

INTRODUCTION

ECG, an indicator of cardio bioelectrical activities is one of the most important criterion, help physician and doctors in recognizing hard diseases and also someone's health. Each kind of variations in the main rhythm of ECG is called arrhythmic. Sometimes ECG is influenced badly by noise and the external appearance of ECG changes in a way that it's possible for a doctor to make a mistake in diagnoses of the rhythm kind. Figure (1) illustrates a sample of ECG without noise. The methods of ECG enhancement are all trying to separate the ECG noise from the main signal as much as not to ruin the signal itself. Reduction of noise effects in increasing the capability of rhythm kind diagnoses via intelligent systems or a doctor is very important issue. Also in a vast domain of sciences related to signal processing (vital signals) is a lot used. On this purpose, many methods for reducing the noise effect in ECG are presented. Among which we can mentioned the method base on adaptive filters, Winer filter and wavelet transform [1, 2, 3, 4, 5]. In this work in order to enhance ECG using SVD new method for noise effect is presented that has a good influence in enhancing the SNR, and also in terms of time-frequency spectrum nice results are achieved. One of the advantageous of this method is its uses in real time and online applications, which enables us to use in the condition in which online noise enhancement is crucially required. In following after researching SVD we will be explaining it and finally we will evaluate this method in terms of quality and quantity.

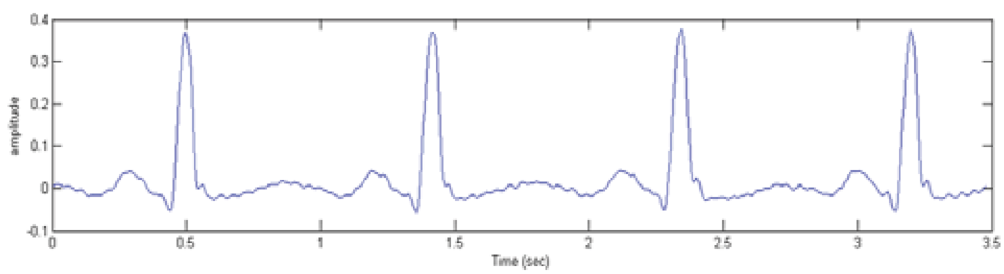

Fig. 1: A sample of clean ECG signal

Singular Value Decomposition (SVD):

The method of SVD is an important tool in signal processing and analyzing the statistical data. SVD is representing signal in time-frequency domain. If there is signal and it's possible to convert it into $m \times n$ matrix, SVD matrix can be written as below:

$$X = USV^T \tag{1}$$

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Here $Um\times m$ and $Vn\times n$ respectively are called left and right singular value matrix. Also S is a diagonal matrix $m\times n$ includes singular values which the elements on main diagonal of it are nonzero and the rest of theelements are zero. Members of the main diagonal of this matrix are like $\sigma_{11} > \sigma_{22} > ... > \sigma_{rr} > 0$ that are so called as the singular values of matrix data of x. singular values of the signal, information like noise level, the amount of signal energy and the number of elements which make up the signal, shows. In addition to that the singular values also present the importance and location of singular values in matrix. However in this way the greater singular values are, more important the corresponding singular values are. Of the most important usages of SVD we can, signal noise reduction and element separation that on this purpose we used singular values and the property of singular vectors perpendicular. Singular values of several different signals can be the same. That is why this parameters of signal processing cannot be reliable. The property of singular vectors being perpendicular can be written as below:

$$UU^T = V^T V = I \tag{2}$$

I is square matrix of unit and V^T is the transposed matrix of V. as said we used SVD to reduce the noise we can call noise reduction kind of elements separation. When signal which is highly correlated and mixed with the one dimensional noise using SVD for mapping to the higher dimensions space ,the data which is more correlated, means the data of the main signal are aligned in one direction ,it means that singular values related to the ECG are greater and the singular values of noise are assigned smaller values as well. In this way, the great and small elements of signal would be separated from each other by singular values. In result, we can separate the noise signal from the main signal. Of course, the singular values of the signal and noise cannot be perfectly separated from each other. In this status, we can do the separating procedure as below:

$$X = USV^T = \begin{pmatrix} U_a & U_{a,n} & U_n \end{pmatrix} \begin{pmatrix} S_a & 0 & 0 \\ 0 & S_{a,n} & 0 \\ 0 & 0 & S_n \end{pmatrix} \begin{pmatrix} V_a^T \\ V_{a,n}{}^T \\ V_n^T \end{pmatrix} \tag{3}$$

Here the a indices for main signal and n indices belongs to the noise. As seen in the above matrix, the singular values of signal and noise are added in the number of the matrix elements. In this work ,we tried to use a method in filtering the singular values of SVD mapping, new way is presented in order to omit noise from the ECG signal. In this process, at first signal is transferred to the higher dimension space by using SVD mapping, Therefore through an algorithm its singular values are optimized and via the reverse mapping to the initial space would be reconverted.

SVD Method Over ECG:

In this work in order to apply SVD on ECG (Sampling rate 360 Hz), initially one segment of signal in length of 4000 samples is selected and then windowing with the length of 40 samples and overlapping of 44% upon this segment of signal is applied and by placing each window of signal in one column of a 2 dimensional matrix, the desirable matrix for applying the SVD is achieved. Using the overlapping is in order to omit the edge effects. Conclusively, we have:

$$N_i = [x_1, x_2, ..., x_n] \rightarrow \begin{pmatrix} x_1 & x_{w-\alpha} & x_{2w-2\alpha} & \cdots \\ x_2 & x_{w-\alpha+1} & x_{2w-2\alpha+1} & \cdots \\ \vdots & \vdots & \vdots & \vdots \\ x_w & x_{2w-\alpha} & x_{3w-2\alpha} & \cdots \end{pmatrix} \tag{4}$$

Here N_i is a segment of the main signal in the length of m, X_i is the samples of signal and w is the length of one window and α is also representing the overlapping. After applying SVD upon the above matrix, through investigating the related singular values to the clean signals from the library of MIT-BIH and the noised signals, are observed that singular values of the clean signals under the effect of noise in comparison with the state of without noise, some changes upon each elements of it are happening. The idea is based on optimizing the singular values related to the noisy signal in order to reduce the effect of noise, these values to be multiplied into a function and remapped to the initial space by reverse transform procedure.

The New Method of Signal Enhancement:

In this procedure, after applying SVD on noisy signal, the singular values matrix is achieved. As described in above, these values are as below:

$$S = \begin{pmatrix} S_a & 0 & 0 \\ 0 & S_{a+n} & 0 \\ 0 & 0 & S_n \end{pmatrix} \tag{5}$$

In which, S_{a+n} is a diagonal matrix contains the combination of singular values of the clean signal and noise signal ,whose values are added together. In order to separate these values from each other, in this work a single function is proposed that if applied on the singular values of noisy signal, their values can approach to the singular value of the clean signal, in the other hand, the singular value of noise signal are omitted from the singular value matrix by applying the proposed function and in this way, the singular value of noisy signal will be optimized. Therefore by the inverse SVD upon the optimized singular value and the right and left singular vectors of noisy signal, that segment of the signal will be enhanced and rebuilt. This process continues till the end of the signal. While in the process, whenever required, zero padding method will be used. The proposed equation is as below:

$$\alpha_i = \begin{cases} 1 & 1 \le i \le 3 \\ e^{-(i-4)/4.5} & 4 \le i \le 15 \\ 0 & 16 \le i \le 40 \end{cases} \tag{6}$$

In which, α_i is a coefficient that whenever is multiplied into the singular value of ith element of noisy signal, its value will be enhanced. The number of singular values in singular value matrix are based on the chosen length of the window, because of what in this work the length of the enhanced window is achieved 40, so that the number of the singular value are 40 too. So the proposed method for 40 coefficients is considered.

Investigating the Procedure and Results:

The Measurements Criterion:

The first measuring method to investigate the efficiency of the proposed method is SNR, so we have:

$$SNR = 10\log_{10}\left(\frac{\sum x^2_{org}}{\sum (x_{org} - x_{est})^2}\right) \tag{7}$$

In which x_{org} is indicating clean signal and x is indicating the enhanced one. In addition of using SNR, the time-frequency graphs are also investigated as other measuring parameters.

Ten signal are selected from standard database MIT-BIH which are usually used in researches and are mixed with the white additive noise, and for each one of them the method is five times applied and the average is taken out from the whole achieved values. The SNR varied from 0dB to 15 dB, and the average of the results is illustrated in the table 1.

Table 1: SNR of ECG enhancement by proposed method

Input SNR	Output SNR
6.8	0
11.9	5
16.6	10
20.8	15

Also as an example, in figure 2 one ECG signal (clean ECG signal) with a noisy signal with the ratio of (SNR 10 dB) and the enhanced signal are illustrated and in figure 3 the related time-frequency graphs are illustrated. As in figure 2 is shown, the main elements of the initial signal are preserved in the enhanced signal, while the occurred changes in signal (because of getting noisy) are obviously eliminated. In the rest, spectrograms related to this signal are shown for further researches. As shown in figure 3, the occurred changes in the noisy signal versus to the main and clean one are approximately omitted. And the fundamental frequency elements are in two figures a and c resembling each other so much.

Conclusion:

In this work, one new method is presented based on enhancing and optimizing the SV for omitting noise from ECG. Using different quantitative and qualitative parameters, the efficacy of this new method is evaluated. Through using the stated parameters, it is indicated that this method can be utilized as one of the most powerful tools of noise enhancement in ECG signal compared to the other methods. In addition of the qualitative and quantitative capability of the proposed method , fast running and applying this method let us substitute it in on-line and real-time applications instead of the ordinary nowadays methods.

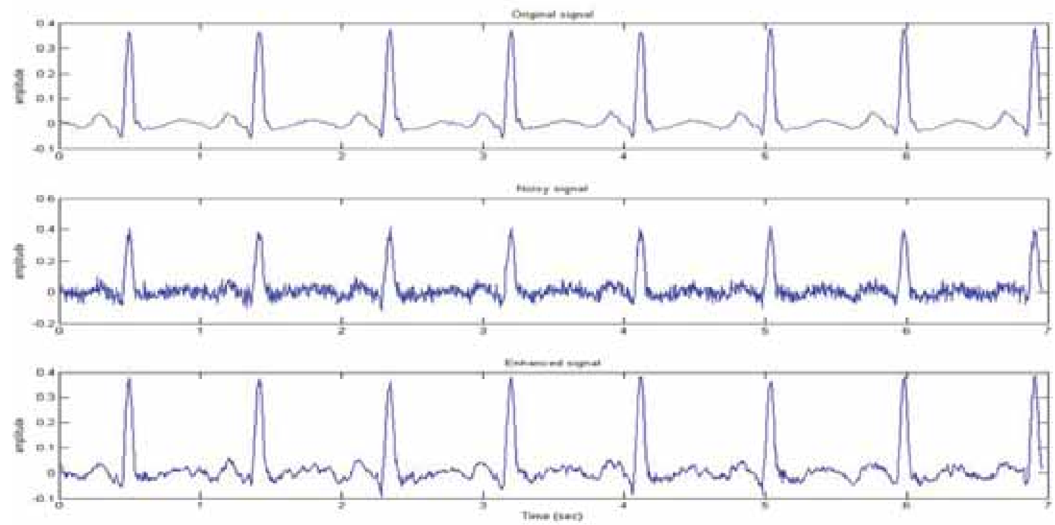

Fig. 2: ECG enhancement results: Clean ECG signal, noisy ECG signal (SNR=10db), enhanced ECG signal (SNR=16.8db)

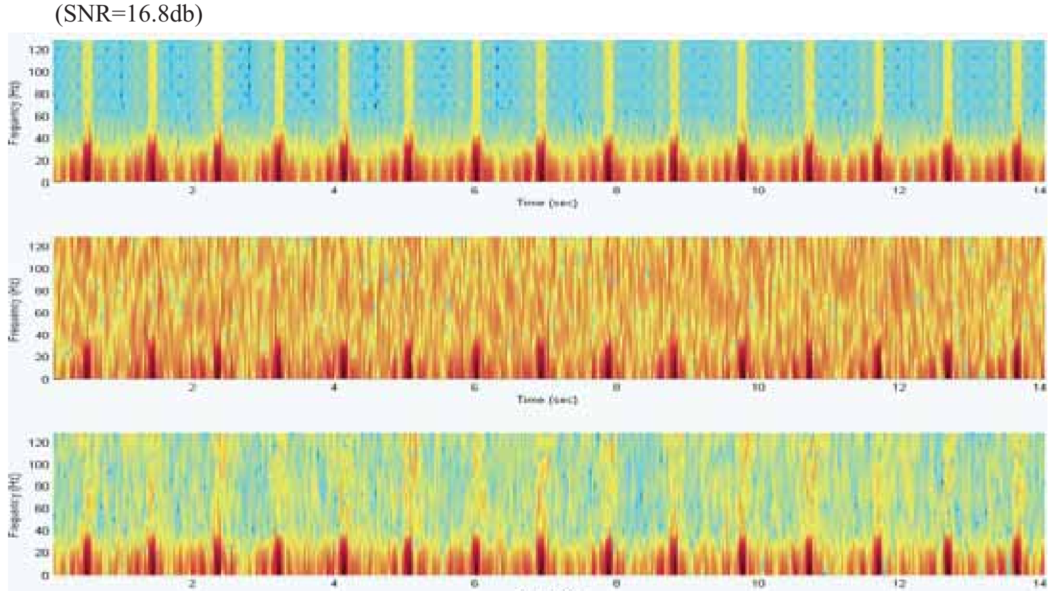

Fig. 3: ECG enhancement spectrograms: clean ECG signal, noisy ECG signal (SNR=10db) and enhanced ECG signal (SNR=16.8db)

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