

## Pricing-Based Primary-Secondary User Power Control Game for Cognitive Radio Networks

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**Abstract:** In cognitive radio networks, if the license spectrum that is allocated to the primary users (PUs) is not utilized, the unutilized spectrum can be used by the secondary users (SUs). This unutilized spectrum is called spectrum hole. However, when the SUs access the spectrum hole, they become source of interference to the others primary and secondary users. Interference temperature is the model, which used to manage the amount of interference that the PU can tolerate. Therefore, efficient power control is crucial to control the interference in the cognitive radio system and promote system quality. Previous works on power control are mainly focus on the SUs SIR maximization as the QoS requirements, ignoring the Primary User (PU). In this paper, PU is considered that has a general utility function, and a linear pricing function is used for SUs utility function. Pricing is an effective tool used to reduce the potential harmful effect of the system and guarantee the PU QoS. We formulated the power control game as a non-cooperative game, in which the first player is the PU. The number of SUs in the system is limited by the status of PU and its ability to achieve its QoS rather than used the interference temperature limits. The numerical results show that the proposed power control algorithm with pricing reduce the power consumed by PU and SUs terminals, and improved the utility functions of PU and SUs.

**Key words:** Cognitive radios, game theory, power control, pricing.

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### INTRODUCTION

An intelligent and efficient technique is of utmost important for improving the spectrum utilization in wireless networks. Cognitive radio, based on software-defined radio, has come to light as a new, smart technique for designing future wireless networks. This is due to its ability to perceive its radio frequency environment, learns, adapt and then reconfigure the system operation to capitalize the radio spectrum and guarantee highly reliable communication (Haykin, 2005). To attain efficient spectrum utilization in cognitive radio networks, dynamic spectrum access (DSA) has been used. In DSA, the license spectrum is assign to the primary users (PUs), but part of this spectrum is unused most of the time. This is a waste! The unused part of spectrum is referred to spectrum hole and can be used by SUs. According to Zhao and Sadler (2007), there are three different dynamic access categories namely (i) common (open sharing) model, (ii) shared-use (hierarchical access) model and (iii) exclusive-use model. In common model, all unlicensed users have the same right to access the spectrum, while in the shared use model the unlicensed users can opportunistically access the spectrum if PU does not use it. Whereas, in exclusive use model, a licensed user can grant the spectrum to the unlicensed users, but the licensed user PU has to guarantee achieving QoS requirement. Several works, in resource allocation for cognitive radio networks, used game theory as a mathematical tool to analyze the interaction among SUs. Xing and Chandramouli (2005) have studied the QoS issue in secondary spectrum sharing subject to an interference temperature constraint. The coexistence between PUs and SUs under interference temperature constraints has been studied in 2006 by Huang *et al.* Jia and Zhang (2007) have recommended the spectrum sharing power control game based on the system model considered in 2005 by Xing and Chandramouli and in 2006 by Huang *et al.*, but the PU was not considered and the SUs tried to maximize its own benefits instead of system utility. Optimal power control with and without interference temperature constraint was investigated by (Wang *et al.*, 2007). Wang *et al.* (2007) has studied the power control game among SUs, which the PU was not considered as a

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decision maker. In all the works (Xing and Chandramouli, 2005; Huang *et al.*, 2006; Wang *et al.*, 2007a; Wang *et al.*, 2007b; Jia and Zhang, 2007), QoS guarantee of SUs was the main goal, leaving behind the QoS of PU unattended. The excellent work of spectrum sharing power control game has been studied by Li and Jayaweera (2008a), in which both the PU and SUs act as decision makers. The system proposed by Li and Jayaweera (2008a) rewarded the PU for allowing SUs to share its licensed spectrum and penalized the PU if the amount of interference more than the Interference Cap (IC). Li and Jayaweera (2008a) used a Matched Filter (MF) detector for primary and secondary receivers. In another work, Li and Jayaweera (2008b) used a Linear Minimum-Mean-Square Error receiver for primary and secondary receivers. Next, Dynamic Spectrum Leasing (DSL) concepts between PU and SUs was proposed by Jayaweera and Li (2009), in which the PU who own the spectrum property right has an incentive to allow SUs to operate in his spectrum band. In this paper, we proposed a power control game, in which both the PU and SUs act as decision makers. However, in this case, we introduced an economic-based utility function for PU as a function of SIR and transmission power, which allowed PU to share its spectrum with SUs and punished the PU if its required transmission quality is not satisfied. We assumed that the system works under the interference temperature limits and all SUs aim to maximize their own benefits (Utility – Pricing) to reach the optimal efficiency Nash equilibrium instead of utility function used in 2008a by Li and Jayaweera. We also assumed a Matched Filter (MF) detector in the primary and secondary receivers.

**System Model:**

We considered a cognitive radio system (Fig. 1), with unlicensed secondary users located in the range of 3G cell and communicated with their SUs base station using DS-CDMA scheme. There was one PU used the licensed spectrum to communicate with a PU base station. PU allowed SUs to access the licensed spectrum using the exclusive use model.  $c_{jk}$ ,  $c_{k0}$ , and  $c_{0k}$  are the code correlation coefficients between the signaling waveforms between different users, which  $j, k$  indicates to the different SUs and 0 indicates to the PU. The path gains between  $k$ -th SU and SUs base station and between  $k$ -th SU and PU base station are  $h_{sk}$ ,  $h_{pk}$  respectively. Similarly, the path gains between PU and PU base station and between PU and SU base station are  $h_{p0}$ ,  $h_{s0}$  respectively. For simplicity, we will assume  $c_{0k} = c_{k0} = c_{jk}$  for all  $k \in \{1, \dots, k\}$ .

Li and Jayaweera (2008a) proposed that the PU adapt Interference Cap (IC) equal to the maximum interference that PU can tolerate. However, in this paper, we assumed that the system works under interference limit and the PU adjusts its transmission power depending on the amount of interference until reaching the value

$P_0^{max}$  in order to achieve its QoS requirements. When the PU power reaches the maximum power value and cannot achieve its QoS requirements, the PU will be punished and its utility will fall down. The advantage of this procedure was to ensure that the PU and all SUs could achieve their minimum QoS requirements. On the other hand, SUs adapted their transmit power to maximize the net utility instead of utility function used by Li and Jayaweera in 2008a in order to reduce the total amount of interference and reduce the power consumed by their terminals and PU terminal. For simplicity, other cells and other primary users interferences were neglected, thus the PU received SINR can be written as:

$$\gamma_0 = \rho_0 \frac{h_{p,0}^2 p_0}{\sum_{j=1}^k h_{pj}^2 c_{j0}^2 p_j + \sigma^2} \tag{1}$$

where  $p_0$ ,  $\sigma^2$  are the PU transmission power (w) and Additive White Gaussian Noise power (w) respectively.

$\rho_0$  is a parameter for power control in  $[0, P_0^{max}]$ .

Similarly, the SINR of the  $k$ -th SU at the SU receiver is given by

$$\gamma_k = \rho_k \frac{h_{sk}^2 p_k}{\sum_{j \neq k}^K h_{sj}^2 c_{j,k}^2 p_j + h_{s0}^2 c_{ps}^2 p_0 + \sigma^2} \quad \forall k \in K \tag{2}$$

A parameter  $\rho_k$  is used in this paper for power control between  $[0, P_k^{max}]$ .

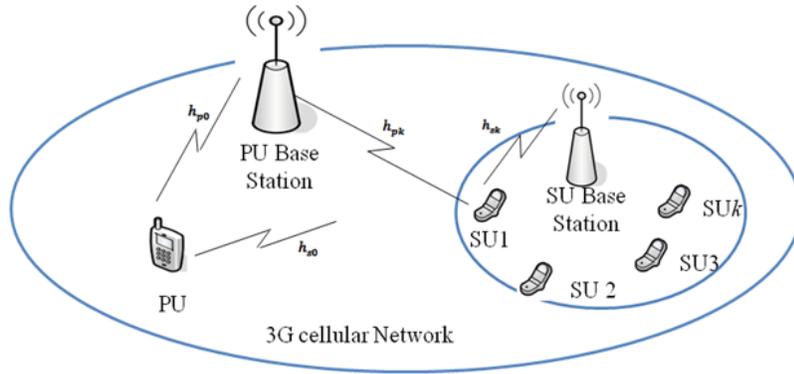


Fig. 1: Cognitive radio networks

**Utility Functions:**

**A-Primary User Utility Function:**

PU utility function has been defined as a function of interference and interference cap (Li and Jayaweera, 2008a). In this paper, we introduced an economic-based utility function for PU as a function of its transmission power and SIR, which had the physical meaning of the QoS represented by SIR, and the energy cost represented by power consumed  $p_0$ . Thus, we could define the PU utility function as in Ji and Huang (1998):

$$u_0(p_0, \gamma_0) = (1 - e^{-\alpha(p_0^{max} - p_0)}) + \mu (1 - e^{-\beta(\gamma_0 - \gamma_0^{min})}) \tag{3}$$

where  $p_0^{max}$  is PU maximum transmission power,  $\gamma_0^{min}$  is the minimum target SIR of PU,  $\mu$  is the relative importance of either PU transmit power or SIR,  $\alpha, \beta$  are constants of transmit power and SIR. The behavior of PU can be explained as follows:

1. If  $p_0 < p_0^{max}$ , PU can achieve its minimum target SIR requirement without using maximum transmit power.
2. If  $p_0 = p_0^{max}$ , in this case, when the amount of interference increase PU needs to reach the maximum transmit power to achieve its QoS, and then the first term of equation will be equal to zero.
3. If  $p_0 = p_0^{max}$  and  $\gamma_0 < \gamma_0^{min}$ , in this case PU cannot achieve its target SIR, and the system punish PU, and then the PU utility function will fall down. Fig. 2 explains the quasi-concavity of PU utility function as a function of transmits power.

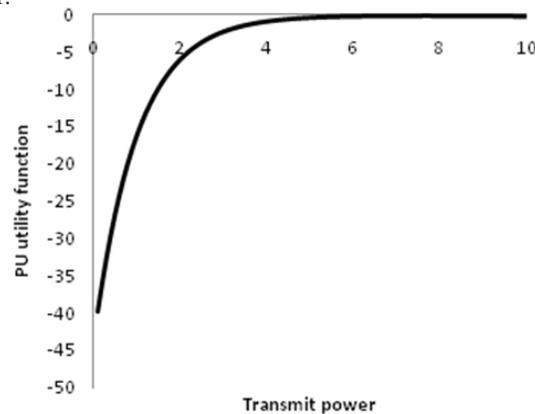


Fig. 2: Quasi-concavity PU utility function.

**B-secondary User Utility Function:**

Each SU entered the cognitive radio system and accessed the licensed spectrum will be a source of interference to PU and other SUs. SUs maximized its own utility by adjusting its own powers to achieve best transmission quality (Li and Jayaweera, 2008a), but it ignored the cost (or harm) it imposes on other SUs terminals by the interference it generates (Saraydar *et al.*, 2002). The Nash equilibrium result of non-cooperative power control was inefficient. In this paper, we introduced a pricing function, which represented the effective tool to guide SUs toward a more efficient operating point and reduced harmful effect. In this case SUs can maximize its own benefit or net utility (utility – price) by adjusting its own power. Thus, net utility function for the  $k$ -th SU is given as follows (Saraydar *et al.*, 2002):

$$u'_k(p_k, p_{-k}) = \frac{R_k f(\gamma_k)}{p_k} - v_k p_k \tag{4}$$

where  $R_k$  is the transmission rate and  $f(\gamma_k) = (1 - \exp(-0.5\gamma_k))^M$  is the efficiency function,  $\gamma_k$  is the SUs SIR,  $v_k$  and  $p_k$  are the  $k$ -th user pricing factor and transmission power, respectively,  $M$  is the length of packet.

The utility function in (4) represents the number of information bits that can be transmitted successfully per joule of energy consumed.

**Game Model:**

The competition for cognitive radio networks resources by PU and SUs could be modeled using game theory. Let  $G = [K, \{P_{0k}\}, \{u_0(\cdot), u'_k(\cdot)\}]$  denoted to the non-cooperative power control game for cognitive radio networks, where  $K = \{0, 1, 2, \dots, k\}$  is the index set for the PU and SUs users, which 0 indicates to the PU.  $P_{0k}$  is the power strategy set of PU and the  $k$ -th SUs.  $u_0(\cdot)$  is the utility function of PU, and  $u'_k(\cdot)$  is the

net utility of the  $k$ -th SU. PU selects a power level  $p_0$  such that  $p_0 = [0, P_0^{max}]$ , where  $P_0^{max}$  is the PU maximum power. Each SU selected a power level  $p_k = [0, P_k^{max}]$ , where  $P_k^{max}$  is the maximum power of SUs and the action space is  $P_{0k} = P_0 \times P_1 \times P_2 \times \dots \times P_k$ .

In this game, PU choose its  $p_0$  in such a way to guarantee its QoS requirement and maximize its utility  $u_0$ . So the optimization problem can be formally expressed as:

$$\max_{p_0 \in P_0} u_0(p_0, p_{-0}) \tag{5}$$

Similarly, each SU also chooses its  $p_k$  to maximize its net utility  $u'_k$ , and the optimization problem expressed as:

$$\max_{p_k \in P_k} u'_k(p_j, p_{-k}) \quad \forall k \in K \tag{6}$$

where  $u'_k(p_j, p_{-k}) = u_k(p_j, p_{-k}) - v_k(p_j, p_{-k})$ .

**Existence and Uniqueness of the Nash Equilibrium:**

Regarding to equation (4), we introduced a pricing function for the SUs utility function, because the Nash

equilibrium for the SUs utility without pricing is inefficient. Self-optimizing behavior of the SUs degrades the performance of cognitive radio networks. Saraydar *et al.* (2002) proved that the Nash equilibrium is inefficient and he used the pricing tool to decrease the users' power. In our study, we noted that the SUs net utility  $u'_k$  is not concave in  $p_k$ . The analysis for this game was proposed in 2002 by Saraydar *et al.* using the super-modular game theory and the result obtained proved that the non-cooperative power control game with pricing is a super-modular game over power strategy space  $[p_k^{min}, p_k^{max}]$ , where  $p_k^{min}$  is derived from  $\gamma_k^{min} \geq 2 \ln M$ . Hence, we only need to study the existence and uniqueness of PU Nash equilibrium.

First, the power action set of the PU is closed subset of  $\mathbf{R}$ , and PU utility function is continuous in  $\mathbf{P}$ . Second, Quasi-concavity of the PU utility function can be obtained by taking the second order derivative:

$$\text{Let } S_0 = \rho_0 \frac{h_{p_0}^2}{I_0}, I_0 = \sum_{j=1}^k h_{p_j}^2 c_{s_0}^2 p_j + \sigma^2$$

then equation (3) could be written as

$$u_0(p_0, \gamma_0) = (1 - e^{-\alpha(p_0^{max} - p_0)}) + \mu (1 - e^{-\beta(p_0 * S_0 - \gamma_0^{min})})$$

All parameters  $(\alpha, \beta, \mu, P_0^{max}, \gamma_0^{min})$  are positive constants, then the second order derivative could be obtained as follow:

$$\frac{\partial^2 u_0}{\partial p_0^2} = -\alpha^2 e^{-\alpha(p_0^{max} - p_0)} - \mu \beta^2 S^2 e^{-\beta(p_0 * S_0 - \gamma_0^{min})} < 0 \tag{7}$$

Thus,  $u_0$  is concave in  $p_0$ . Moreover, the concavity of PU utility function can be seen in Fig. 2 Therefore, the Nash equilibrium exists in this game.

Saraydar *et al.* (2002) in theorem (7) proved that the terminal algorithm converges to Nash equilibrium of non-cooperative power game with pricing. Similarly, all SU terminals in our game converged to the Nash equilibrium, in which the best response is computed as:

$$\tilde{p}_k(\tau_i) = \arg \max_{p_k \in P_k} u'_k(p_k, p_{-k}(\tau_{i-1})) \tag{8}$$

where  $\tau_i \in T$  is the time instance update. Then the best response of  $k$ -th SU transmit power is:

$$r_k^*(\tau_i) = \min(\tilde{p}_k(\tau_i), p_k^{max}) \tag{9}$$

Here, we noted that each SU optimize the net utility over the modified power strategy space  $P_k$ , which is bounded by  $\gamma_k \leq 2 \ln M$ .

On the other hands, the optimal power level for PU is obtained when PU try to find the power level  $\tilde{p}_0$  from the PU power strategy space  $P_0$  that maximize the utility function  $u_0$ . Optimal power of PU can be achieved at a point for which the first partial derivative of  $u_0$  with respect to  $p_0$  is equal to zero ( $\partial u_0 / \partial p_0 = 0$ ).

$$\frac{\partial u_0}{\partial p_0} = -\alpha e^{-\alpha(p_0^{max} - p_0)} + \mu\beta S_0 e^{-\beta(S_0 * p_0 - \gamma_0^{min})} = 0 \quad (10)$$

Finally, the following equation is obtained.

$$\tilde{p}_0(\tau_i) = \frac{\alpha p_0^{max} + \beta \gamma_0^{min} + \ln \frac{\mu\beta}{\alpha} + \ln S_0(\tau_{i-1})}{\alpha + \beta S_0(\tau_{i-1})} \quad (11)$$

And the best response is found to be

$$r_0^*(\tau_i) = \min(\tilde{p}_0, p_0^{max}) \quad (12)$$

From (11), it can be seen that PU adjusts its power to maximize its utility function depend on the SUs transmit power that includes in the parameter  $S_0$ .

If the best response correspond to the PU is standard function, then the Nash equilibrium in the game is unique.

Yates R.D. 1996 proved that the interference function is a standard function, and then the best response of PU utility function is a standard function, which satisfies:

Positivity:  $r_0^*(p_{-0}) > 0, \forall p \in P$

Monotonicity: if  $p > p'$ , then  $r_0^*(p_{-0}) > r_0^*(p'_{-0})$

Scalability: for all  $\delta > 1$ ,  $\delta r_0^*(p_{-0}) = r_0^*(\delta p_{-0})$

where  $p_{-0} = I_0$ . In other words, when the SUs increase their transmit power, PU may increase or decrease its transmit power. Therefore, the best response of PU is a standard function and the Nash equilibrium is unique.

**System Performance and Numerical Results:**

The system parameters value which were used in this simulations are as follows:  $K=20$  is the number of SUs,  $P_0^{max} = 5$ ,  $P_k^{max} = 20$ ,  $h_{pk} = h_{sk} = 1 \forall k \in K$ ,  $h_{s0} = h_{p0} = 1$ ,  $c_{sp} = c_{jk} = c_{ps} = 0.12$ ,  $\gamma_0^{min} = 5$ ,  $\alpha = 0.2$ ,  $\beta = 2$ ,  $\mu = 0.2$ ,  $M = 80$ ,  $\rho_k = 4.08 \forall k \in K$ ,  $\rho_0 = 2.72$  and  $v = 0.025$

In this simulation, we compared between the pricing-based and no-pricing power control in the cognitive radio networks considering the PU as a decision maker.

In Fig. 3, the total amount of interference reduced 65% by using pricing-based model. Reducing the amount of interference in cognitive radio networks is the key to increase the system capacity and assist PU to achieve its QoS requirement. SIR degradation of the PU is shown in Fig. 4, PU could not achieve its target SIR in no-pricing mode when  $K>14$ , and SIR reach to 2.05 when  $k=20$ . Whereas, in pricing-based model PU could achieve its SIR until  $k=17$ . Greater than 17 SIR of PU is near the target value  $\gamma_0^{min}$  (4.94, 483, 4.58). PU and SUs should achieve their target QoS requirements, otherwise the existence of the user in the networks just represent a source of interference. Although the utility of each PU and each SU decreased when the other SU terminal

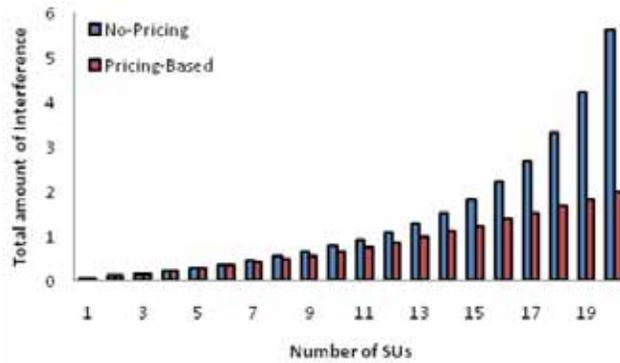
enters the system, there is a number of terminals maximized the system sum of utilities. This suggests that a novel-admission control algorithm would do well to limit the number of SUs to the number that maximizes system utilities.

In our system, PU allowed SUs to share available spectrum but when the SIR of PU started to degrade below minimum target SIR, the system punished PU and utility function fell down (Fig. 5).

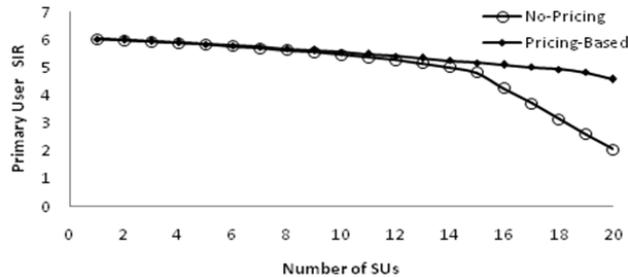
In Fig. 6, we have shown that the power used by PU in pricing-base model reached to the maximum power when  $k > 18$ , and reached to maximum power in no-pricing model when  $k > 14$ .

In other hand, the main objective of each SU is to maximize its utility function. Fig. 7 explained the average utility of SUs, which the maximum value of average utility occurs when  $k=4$  for pricing-based mode and No-pricing mode. Average utilities decrease due to the increase of interference when the new SU enter the system.

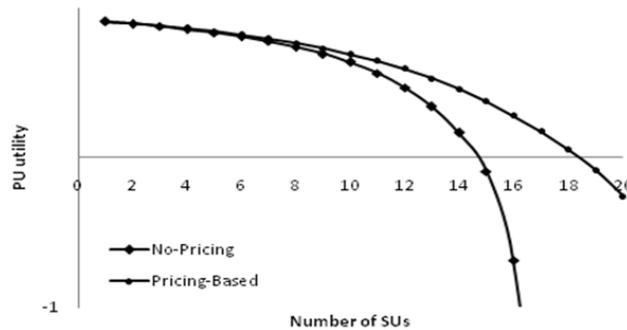
In Fig. 8, the maximum value of sum of SUs utilities occurs when  $k=13$  but there is a difference between pricing and no pricing based models.



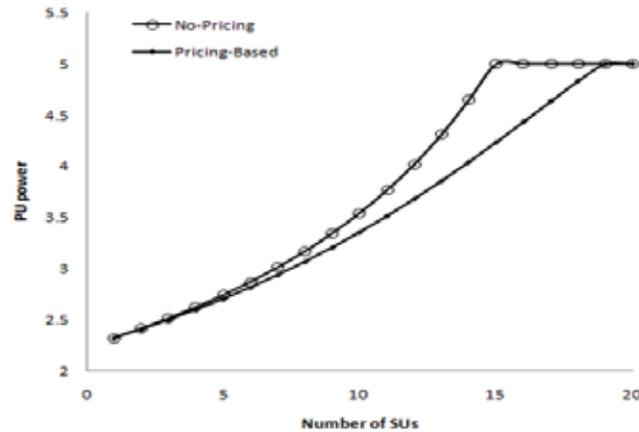
**Fig. 3:** Total amount of interference from all Sus



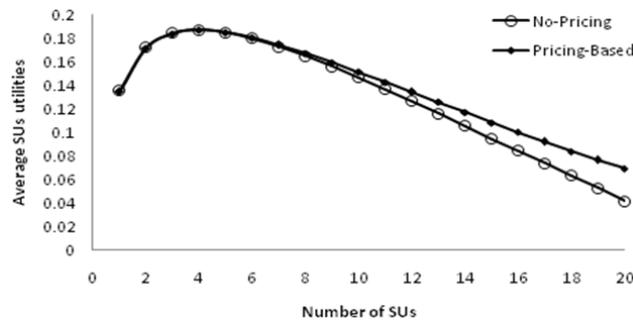
**Fig. 4:** PU Signal to Interference Ratio (SIR)



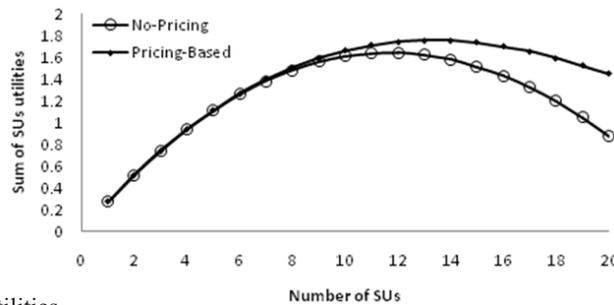
**Fig. 5:** PU utility vs. Number of Sus



**Fig. 6:** PU transmit power



**Fig. 7:** Average SUs utilities



**Fig. 8:** Sum of SUs utilities

**Conclusion:**

We proposed a new approach for primary secondary power control game that can be used in the dynamic exclusive use model. In this model, PU can sell and trade its own spectrum to SUs, but it has to ensure that its QoS requirement can be achieved. We introduced a new economic-based utility function for PU as a function of SIR and transmit power, which PU adjusts its power to maximize its utility and avoid degradation in its SIR. PU was punished if it cannot achieve its QoS requirements by using its maximum power. We also introduced a linear pricing function to the SUs utility function, where each SU seeks to maximize its net utility instead of utility function. The pricing function was used to reduce the system harmful (Interference) effect. A non-cooperative power control game for our system was formulated, in which PU acted as a decision maker in the game like SUs. Existence and uniqueness of Nash equilibrium were also proved for our proposed game. The numerical results showed that the pricing-based power control algorithm improved the system utilities, reduced PU transmission power, and reduced amount of interference.

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