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Economic Design of Retaining Wall Using Particle Swarm Optimization with Passive Congregation

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Abstract: This paper presents an effective optimization method for nonlinear constrained optimization of retaining structures. The proposed algorithm is based on the particle swarm optimization with passive congregation. The optimization procedure controls all geotechnical and structural design constraints while reducing the overall cost of the retaining wall. To applying the constraints, the algorithm employs penalty function method. To verify the efficiency of the proposed method, two design examples of retaining structures are illustrated. Comparison analysis between the results of the presented methodology, standard particle swarm optimization and nonlinear programming optimization method show the ability of the proposed algorithm to find better optimal solutions for retaining wall tasks than the others.

Key words: Economic design, Retaining wall, Particle swarm optimization, Passive congregation

INTRODUCTION

A retaining wall is defined as a structure whose primary purpose is to provide lateral support for soil or rock. Retaining walls have traditionally been constructed with plain or reinforced concrete, with the purpose of sustaining the soil pressure arising from the backfill. This study is concerned with reinforced concrete cantilever (RCC) retaining walls.

Several studies have been done to develop methodologies for the analysis and design of reinforced earth walls. However, limited work has been undertaken to develop methods for their optimum cost design. Saribas and Erbatur (1996) applied constrained nonlinear programming to optimum design of the retaining wall with seven design variables. This method is very efficient for a few design variables, but it's very time consuming for large numbers of variables. Ceranic *et al*. (2001) applied simulated annealing to a problem with only geometrical design variables. Hence, such an attempt has been made here, but in this study a problem with eight design variables include geometric and reinforcement variables is considered. The solution is carried out by a specially prepared computer program and the method has been used, is particle swarm optimization with passive congregation (PSOPC).

Particle swarm optimization (PSO) is a kind of random search algorithm that simulates nature evolutionary process and performs a good characteristic in solving some difficulty optimization problems (Kennedy and Eberhart, 1995). Compared with genetic algorithm and other similar evolutionary techniques, PSO has some attractive characteristics. On the one hand, PSO has very few parameters to be adjusted thus it is convenient to make the parameters obtain to the optimum values and large amount of calculation work and much time can be saved. On the other hand, PSO can find the optimal solutions or near the optimal solutions with a fast convergent speed, because it only has two computation formulas for iteration. In spite of these advantages, PSO algorithm is easy to relapse into local optimum in solving complex optimization problems. Recently, many investigations have been undertaken to improve the performance of the standard PSO (Van den Bergh and Engelbrecht, 2002; Xie *et al*., 2002; Mendes *et al*. 2004; He and Han, 2006; etc). However, one of the best improvements on this method has been done by He *et al*. (2004). They improved the PSO with passive congregation (PSOPC), which can improve the convergence rate and accuracy of the SPSO efficiently. Hence in the current study particle swarm optimizer with passive congregation was used for optimization of retaining wall. The algorithm handles the constraints using penalty function method.

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In this paper, the optimization framework by PSO and PSOPC is developed first, followed by description of design variables, constraints and objective function for optimization of retaining structures. Then, two numerical examples are described that apply the framework for economic design of retaining wall.

MATERIALS AND METHODS

1. Particle Swarm Optimization (PSO):

The original particle swarm optimization algorithm introduced by Kennedy and Eberhart in 1995 (Kennedy and Eberhart, 1995). The PSO is derived from a simplified version of the flock simulation. It also has features that are based upon human social behavior.

PSO contains a number of particles which called the swarm. The particles are initialized randomly in the multi dimensional search space of an objective function. Each particle represents a potential solution of the optimization problem. The particles fly through the search space and their positions are updated based on each particle's personal best position as well as the best position found by the swarm. The objective function is evaluated for each particle during iterations, and the fitness value is used to determine which position in the search space is better than the others.

At every iteration, the update moves a particle by adding a change velocity Vi^{k+1} to the current position X_i^k as illustrated in the following equation (Kennedy and Eberhart, 1995):

$$X_i^{k+1} = X_i^k + V_i^{k+1} \tag{1}$$

The velocity is a combination of three contributing factors: (1) previous velocity V_i^k, (2) movement in the direction of the local best P_i^k, and (3) movement in the direction of the global best P_g^k. The mathematical formulation is expressed as (Shi and Eberhart, 1998):

$$V_i^{k+1} = w \times V_i^k + c_1 \times r_1 \times \left(P_i^k - X_i^k\right) + c_2 \times r_2 \times \left(P_g^k - X_i^k\right) \qquad i = 1, 2, 3, \ldots, N \tag{2}$$

where w is an inertia weight to control the influence of the previous velocity; r_1 and r_2 are two random numbers uniformly distributed in the range of (0, 1); c_1 and c_2 are two acceleration constants usually considered equal 2; P_i^k is the best position of the i^{th} particle up to iteration k and P_g^k is the best position among all particles in the swarm up to iteration k. The inertia weighting function in Eq. (2) is usually calculated using following equation:

$$w = w_{max} - (w_{max} - w_{min}) \times k / G \tag{3}$$

where w_{max} and w_{min} are maximum and minimum values of w, G is the maximum number of iterations and k is the current iteration number.

2. PSO with Passive Congregation (PSOPC):

POS algorithm roots in simulating the predatory behavior of bird populations. This congregation action has two kinds: active congregation and passive congregation. The latter is an attraction of an individual to the other group members but no display of social behavior. Fish schooling is one of the representative types of passive congregation and the PSO is inspired by it. Adding the passive congregation model to the SPSO may increase its performance. He *et al*. (2004) proposed a hybrid PSO with passive congregation (PSOPC). The update velocity equation in PSOPC is defined as follows (He *et al*., 2004):

$$V_i^{k+1} = w \times V_i^k + c_1 \times r_1 \times \left(P_i^k - X_i^k\right) + c_2 \times r_2 \times \left(P_g^k - X_i^k\right) + c_3 \times r_3 \times \left(R_i^k - X_i^k\right) \tag{4}$$

where R_i^k is a particle selected randomly from the swarm, c_3 is the passive congregation coefficient, and r_3 is a uniform random sequence in the range of 0 to 1.

Several benchmark functions have been tested by this approach in (He *et al*., 2004). Experimental results indicate that the PSO with passive congregation improves the search performance on the benchmark functions significantly.

3. Constraint Optimization of Retaining Wall

In optimal design problem of retaining wall the aim is to minimize the construction cost of the wall under constraints. This optimization problem can be expressed as follows:

$$\begin{aligned} &minimize && f(\boldsymbol{X}) && \\ &subject\,to && g_i(\boldsymbol{X}) \le 0 && i = 1,2,\ldots,p \\ & && h_j(\boldsymbol{X}) = 0 && j = 1,2,\ldots,m \\ & && L_k \le X_k \le U_k && k = 1,2,\ldots,n \end{aligned} \tag{5}$$

where $f(\boldsymbol{X})$ is the objective function $g_i(\boldsymbol{X})$, $h_j(\boldsymbol{X})$ are inequality and equality constraints respectively and L_k, U_k are lower and upper bound constraints. Similar to other stochastic optimization methods, the PSO algorithm is defined for unconstrained problems. A number of approaches have been taken in the evolutionary computing field to do constraint handling. The main approach is penalty method. Penalty methods add a penalty to the objective function to decrease the quality of infeasible solutions. In the current study, penalty function defined by the following equation is used:

$$F(\boldsymbol{X}) = f(\boldsymbol{X}) + r\sum_{i=1}^{p+m} q_i(\boldsymbol{X})^l \tag{6}$$

where $f(X)$ is the original objective function of the problem in Eq. (5), r and l are the penalty factor and the power of the penalty function which considered equal to 1×10^3 and 2, respectively.

$$q_i(\boldsymbol{X}) = \begin{cases} max\{0, g_i(\boldsymbol{X})\} & 1 \le i \le p \\ \left|h_{i-p}(\boldsymbol{X})\right| & p+1 \le i \le p+m \end{cases}$$

To economic design of retaining wall, the objective function, design variables and design constraints should be defined explicitly. In the following sections a description of these parameters for optimization of RCC retaining wall are presented.

3.1. Design Variables:

Figure 1 shows the cross-section of a reinforced earth retaining wall. As it is shown in this figure, the eight design variables are listed as follows:

X_1: Width of heel
X_2: Stem thickness at the top
X_3: Stem thickness at the bottom
X_4: Width of toe
X_5: Thickness of base slab
X_6: Vertical steel area of the stem per unit length of wall
X_7: Horizontal steel area of the toe per unit length of wall
X_8: Horizontal steel area of the heel per unit length of wall

3.2. Design Constraints:

According to Bowles (1982) and ACI (2005) the design constraints may be classified as geotechnical and structural requirements that summarized in the following sections. These requirements represent the failure modes as a function of the design variables. More details about calculation of each failure mode could be found in (Sivakumar Babu and Munwar Basha, 2008).

Sliding Failure Mode:

The net horizontal forces must be such that the wall is prevented from sliding along its foundation. The factor of safety against sliding is calculated from:

$$\Sigma Resisting\ forces/\Sigma Sliding\ forces \geq F.S_{slidin} \quad (7)$$

The minimum acceptable limit for $F.S_{sliding}$ is 1.5. The most significant sliding force component usually comes from the lateral earth pressure acting on the active (backfill) side of the wall. Such force may be intensified by the presence of vertical or horizontal loads on the backfill surface.

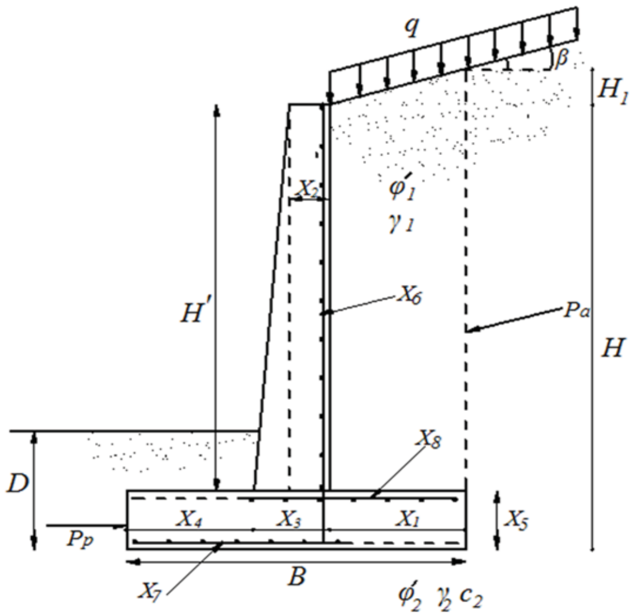

Fig. 1: Cross section of the RCC retaining wall. The figure is modified.

Overturning Failure Mode:
The stabilizing moments must be greater than the overturning moments to prevent rotation of the wall around its toe. The stabilizing moments result mainly from the self-weight of the structure, whereas the main source of overturning moments is the active earth pressure. The factor of safety against overturning is calculated from:

$$\Sigma Stabilizing\ moment\ /\ \Sigma Overturning\ moment \geq F.S_{overturning} \quad (8)$$

The factor of safety against overturning must be equal to or greater than 1.5.

Bearing Failure Mode:
The bearing capacity of the foundation must be large enough to resist the stresses acting along the base of the structure. The factor of safety against bearing capacity failure is calculated from:

$$q_{ult}\ /\ q_{\max} \geq F.S_{BC} \quad (9)$$

where q_{ult} is the ultimate bearing capacity of the foundation soil and q_{max} is the maximum contact pressure at the interface between the wall structure and the foundation soil. The minimum acceptable value for $F.S_{BC}$ is 3.0.

Eccentricity Failure Mode:
For stability, the line of action of the resultant force must lie within the middle third of the foundation base. For safety against eccentricity failure the following equation should be satisfied:

$$e \geq B\ /\ 6 \quad (10)$$

where e is eccentricity of the resultant force.

Toe Shear and Moment Failure Mode:

Toe slab of the wall has to be designed as a cantilever slab to resist moments and shear forces. The net loading acts upwards and flexural reinforcement has to be provided at the bottom of the toe slab. To prevent toe shear failure, Nominal shear stress at the junction of stem with toe slab should be less than shear strength of concrete:

$$\tau_c \geq \tau_{toe} \tag{11}$$

A critical section for the moment is considered at the junction of stem with toe slab. So Maximum bending moment at a vertical section at the junction of the stem with toe slab should be less than the moment of resistance of toe slab:

$$M_{R\,toe} \geq M_{toe} \tag{12}$$

Heel Shear and Moment Failure Mode:

Heel slab of the wall has to be designed as a cantilever slab to resist moments and shear forces. The net loading acts downwards and flexural reinforcement has to be provided at the top of the heel slab. Critical section for the shear is considered at the junction of stem with heel slab. Thus Nominal shear stress at this section should be less than shear strength of concrete:

$$\tau_c \geq \tau_{heel} \tag{13}$$

A critical section for the moment is considered at the junction of stem with heel slab. So Maximum bending moment at a vertical section at the junction of the stem with heel slab should be less than the resistance moment of the heel slab:

$$M_{R\,heel} \geq M_{heel} \tag{14}$$

Stem Shear and Moment Failure Mode:

The stem of the wall has to be designed as cantilever slab to resist moments and shear forces. Nominal shear stress of stem should be less than shear strength of concrete:

$$\tau_c \geq \tau_{stem} \tag{15}$$

Moreover, maximum bending moment at the face of the Support should be less than the resistance moment of stem:

$$M_{R\,stem} \geq M_{stem} \tag{16}$$

Upper Bound and Lower Bound Constraints:

As recommended in Bowles (1982) and ACI (2005) all design variables have practical minimum and maximum value. Hence these upper and lower bond constraints are presented in Table1.

Table 1: Upper bound and lower bound for design variables of retaining wall

Description	Lower bound	Upper bound
Thickness of base slab	$X_5 \geq H/12$	$X_5 \leq H/10$
Width of toe	$X_4 \geq 0.4*H/3$	$X_4 \leq 0.7*H/3$
Width of footing	$B \geq 0.4*H$	$B \leq 0.7*H$
Stem thickness at the top	$X_3 \geq 20$ cm	-
Vertical steel area of the stem	$X_6 \geq 0.0035(X_2+X_3-0.07)$	$X_6 \leq 0.016(X_2+X_3-0.07)$
Horizontal steel area of the toe	$X_7 \geq 0.0035(X_5-0.07)$	$X_7 \leq 0.016(X_5-0.07)$
Horizontal steel area of the heel	$X_8 \geq 0.0035(X_5-0.07)$	$X_8 \leq 0.016(X_5-0.07)$

3.3. Objective Function:

The total cost of the retaining wall is considered as the objective function for the analysis to be carried out. Thus the cost function may be expressed in the form of below:

$$f(\boldsymbol{X}) = C_c V_c + C_e V_e + C_b V_b + C_f A_f + C_s W_s \tag{17}$$

where, P_c, P_e, P_b, p_f and P_s show the unit price of concrete, excavation, backfill, formwork, and reinforcement respectively. In addition, V_c ,V_e and V_b denote the volume of concrete, excavation and backfill, A_f shows the area of formwork and W_s indicates the weight of steel. In current study, unit price for these items are given according to Table 2 (Yepes *et al.,* 2008).

Table 2: RCC retaining walls assembly unit price (Yepes *et al.,* 2008)

Item	Unit	Unit price (USD/m)
Earth removal	m^3	3.01
Foundation formwork	m^2	18.03
Foundation formwork	m^2	18.63
Reinforcement	kg	0.56
Concrete in foundations	m^3	50.65
Concrete in stem	m^3	56.66
Earth fill-in	m^3	4.81

Finally, to economic design of retaining wall the fitness function will be obtained by substituting Eq. (17) as well as inequality constraints presented in section 3.2, into Eq. (6). Therefore, the final objective function can be formulated as follows:

$$F(\boldsymbol{X}) = f(\boldsymbol{X}) + r\sum max\{0, g_i(\boldsymbol{X})\}^l \tag{18}$$

The implementation procedure of the economic design of the RCC retaining wall using PSOPC and based on above explanation is constructed as follows:

1. Initialize a set of particles positions and velocities randomly distributed throughout the design space bounded by specified limits.
2. Evaluate the objective function values using Eq. (18) for each particle in the swarm.
3. Update the optimum particle position at current iteration and global optimum particle position.
4. Update the velocity vector as specified in Eq. (4) and update the position of each particle according to Eq. (1).
5. Repeat steps 2–4 until the stopping criteria is met.

RESULTS AND DISCUSSION

To verify the good performance of the proposed algorithm, two numerical examples of RCC retaining walls will be solved by the present method. In order to optimum design of these cases, the optimal design procedure is coded in MATLAB. An appropriate choice of the parameters for PSOPC is selected sensitively by solving various design problems. Therefore, population of 40 individuals was used. The value of acceleration constants c_1 and c_2 were both 2. w_{max} and w_{min} were chosen as 0.95 and 0.45 respectively. The passive congregation coefficient c_3 was set to 0.4. In the present study, the optimization procedure was terminated when one of the following stopping criteria was met: (i) the maximum number of generations is reached; (ii) after a given number of iterations, there is no significant improvement of the solution.

Example 1:

In this example, the optimum design of RCC retaining wall with height of 4m by PSOPC is presented. Other input parameters for this problem are given in Table 3. Results of the optimization are tabulated in Table 4.

This problem previously solved by Saribas and Erbatur (1996) utilizing nonlinear programming method by considering only seven design variables. They applied different unit prices and did not measure the cost of excavation, formwork and backfill. The optimum price obtained by Saribas and Erbatur (1996) was 82.47 USD/m. For the sake of comparison, this example was solved again using presented methodology and in the same situations. The optimum price evaluated using PSOPC was 75.30 USD/m, which is lower than that evaluated by Saribas and Erbatur (1996).

Table 3: Input parameters for optimum design of retaining wall

Input parameter	Unit	Value for Example 1	Value for Example 2
Height of stem	m	3.0	5.5
Internal friction angle of retained soil	degree	36	36
Internal friction angle of base soil	degree	0	34
Unit weight of retained soil	kN/m^3	17.5	17.5
Unit weight of base soil	kN/m^3	18.5	18.5
Unit weight of concrete	kN/m^3	23.5	23.5
Cohesion of base soil	KPa	125	100
Depth of soil in front of wall	m	0.5	0.75
Surcharge load	KPa	20	30
Backfill slop	degre	10	10
Concrete cover	cm	7.0	7.0
Yield strength of reinforcing steel	MPa	400	400
Compressive strength of concrete	MPa	21	21
Shrinkage and temporary reinforcement percent	-	0.002	0.002
Factor of safety for overturning stability	-	1.5	1.5
Factor of safety against sliding	-	1.5	1.5
Factor of safety for bearing capacity	-	3.0	3.0

Table 4: Optimization result for retaining wall

Design variable	Unit	Optimum values for example 1	Optimum values for example 2
Width of heel (X_1)	m	0.77	0.91
Stem thickness at the top (X_2)	m	0.20	0.34
Stem thickness at the bottom (X_3)	m	0.35	0.64
Width of toe (X_4)	m	0.62	1.42
Thickness of base slab (X_5)	m	0.27	0.5
Vertical steel area of the stem (X_6)	cm^2	14	31
Horizontal steel area of the toe (X_7)	cm^2	6.8	20
Horizontal steel area of the heel (X_8)	cm^2	6.3	13
Best price (USD/m)		257	665

Table 5 presents a statistical comparison between the results evaluated by PSOPC and PSO. In order to make a fair comparison among the algorithms, a maximum number of iteration is considered as stopping criteria. The best price evaluated by each algorithm is a measure of the strength of the algorithm. Each algorithm was run 50 times and the average elapsed time is considered as a measure of the computational time. From the results of Table 5, it is obvious that, the best price obtained by PSOPC is 257 USD/m and is slightly lower than those obtained by PSO (261 USD/m). Moreover, the number of iterations and elapse time for PSOPC is lower than those required by PSO.

Table 5: Comparisons of the results of different methods

Problem		Best result		Average result		Worst result	
		PSO	PSOPC	PSO	PSOPC	PSO	PSOPC
Example 1	Iteration	1820	1210	2100	1400	1520	1010
	Elapse time (s)	92	61	103	71	75	46
	Best price	261	257	265	261	274	279
Example 2	Iteration	2810	1860	2950	1790	2620	1980
	Elapse time (s)	69	38	65	34	57	36
	Best price	684	665	698	676	721	724

Example 2:

Optimization of RCC retaining wall with height of 5.5m is investigated in this example. Other input parameters for this example are given in the last column of Table 3. Results of the economic design are summarized in the last column of Table 4. Moreover, Table 5 shows the comparison between the results obtained by PSO and PSOPC. As derived from Table 5, the best price evaluated for this case by PSOPC is 665 USD/m after 1860 iterations. However, the best price achieved by PSO is 684 USD/m after 2810 iterations.

Conclusion:

An effective optimization method proposed to economic design of retaining structure. The algorithm is based on particle swarm optimization with passive congregation. For optimization of retaining wall, the objective function was considered as total cost design of retaining walls. Furthermore, both geotechnical and structural constraints were used for optimization procedure. A computer program is developed in MATLAB and the user is only required to feed the input parameters like soil and material properties and safety factors.

For the studies carried out here, it is found that PSOPC is a suitable technique for optimization of RCC retaining structures. The method is able to find a better optimal solution compared with PSO and nonlinear programming. Moreover, the number of iterations and computation time for PSOPC are lower than those required by PSO.

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