

Comparison of Modified Shuffled Frog Leaping Algorithm and Other Heuristic Methods for Optimal Placement of Unified Power Flow Controllers in Electrical Power Systems

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Abstract: This paper presents a Modified Shuffled Frog Leaping Algorithm to obtain the optimal number and location of FACTS devices in a power system. Unified Power Flow Controller (UPFC) has great flexibility that can control the active and reactive power flow and bus voltages, simultaneously. Decoupled model of the UPFC is applied to maximize the system loadability subject to the transmission line capacity limits and specified voltage level. Shuffled frog-leaping algorithm (SFLA) is a new memetic meta heuristic algorithm with efficient mathematical function and global search capability. The objective is to maximize the transmission system loadability subject to the transmission line capacity limits and specified bus voltage levels. Using the proposed method, the location of UPFCs and their parameters are optimized simultaneously. The proposed approach is examined and tested on IEEE 14-bus system and IEEE 118-bus system. The results show that the steady state performance of power system can be effectively enhanced due to the optimal location and parameters of the UPFC.

Key words: Unified Power Flow Controller (UPFC); Maximize the Loadability of Transmission Lines; Modified Shuffled Frog Leaping Algorithm (MSFLA).

INTRODUCTION

Recently, the steady state performance of power system has become a matter of grave concern in system operation and planning. As the power system becomes more complex and more heavily loaded, it can be operated in unstable or insecure situations like the cascading thermal overloads, the frequency and voltage collapse. For a secure operation of the power system, it is essential to maintain the required level of security margin (Hao, J., 2004; Hingorani, N.G. and L. Gyugyi, 1999). Then, power system controllability is required in order to utilize the available network capacitance adequately. The development of FACTS devices based on the advance of semiconductor technology opens up new opportunities for controlling the load flow and extending the loadability of the available transmission network. The UPFC is one of the family members of FACTS devices for load flow control, since it can either simultaneously or selectively control the active and reactive power flow along the lines (Nabavi-Niaki, A. and M.R. Iravani, 1996; Noroozian, M., 1997). Several papers have been published about finding the optimal location of the UPFC with respect to different purposes and methods (Fang, W.L. and H.W. Ngan, 1999; Gerbex, S., 2001). In (Fang, W.L. and H.W. Ngan, 1999), augmented Lagrange multiplier method is applied to determine the optimal location of the UPFC to be installed. Although multi operating conditions can simultaneously be taken into consideration, the operating condition must be preassigned. Gerbex *et al* (2001) provides the genetic algorithm to optimize three parameters of the multi-type FACTS devices including TCSC, TCPST, TCVR and SVC: the location of the devices, their types and their values, but another kind of FACTS device -UPFC has not been considered.

Shuffled frog leaping algorithm (SFLA) is a memetic meta-heuristic that is based on evolution of memes carried by interactive individuals and a global exchange of information among the frog population. It combines the advantages of the genetic-based memetic algorithm (MA) and the social behavior-based PSO algorithm with such characteristics as simple concept, fewer parameters adjustment, prompt formation, great capability in global search and easy implementation. While proposed primarily for solving the multi-objective engineering problems such as water resource distribution (Eusuff, M.M. and K.E. Lansey, 2003), bridge deck repairs

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(Hatem Elbehairy, 2006) and job-shop scheduling arrangement (Alireza Rahimi-Vahed and Ali Hossein Mirzaei, 2007), the successful application of SFLA in solving TSP and the subsequent detailed analysis on its evolutionary mechanism may help for its widespread use in multi-objective optimization problem as well as the in-depth research in its theoretical aspect. The main objective of this paper is to develop an algorithm for finding and choosing the optimal location of the UPFC in order to maximizing the system loadability while simultaneously satisfying system operating constraints including transmission line capacity and voltage level limits. The optimal location problem of a given number of FACTS is converted to an optimization problem which is solved by the Modified Shuffled Frog Leaping Algorithm that has a strong ability to find the most optimistic results. In the following, the main results of tests on the IEEE 14-bus and 118-bus power systems for the proposed MSFLA method are shown to demonstrate the effectiveness of the proposed method.

2. UPFC Equivalent Circuit:

In this paper, a simplified equivalent circuit of UPFC given in is used and is shown in Figure 1.

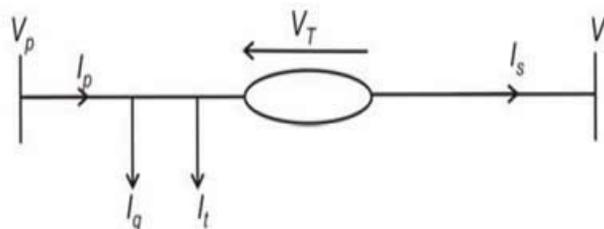


Fig. 1: Equivalent circuit of UPFC.

The three controllable parameters of UPFC are V_T , ϕ_T and I_q . V_T denotes the magnitude of the voltage injected in series with the transmission line through the series transformer. ϕ_T is the phase angle of this voltage. I_q is the shunt reactive current of UPFC. The UPFC parameters V_T , ϕ_T and I_q are chosen within a range due to physical and economic limitations.

$$V_T \in [V_{T\min}, V_{T\max}], \phi_T \in [0, 2\pi]$$

$$I_q \in [-I_{q\max}, I_{q\max}]$$

The limits of UPFC parameters are taken from (Fang, W.L. and H.W. Ngan, 2005).

UPFC Power Equations:

The equivalent circuit of UPFC embedded in transmission line i-j is shown in Figure 2. The two power injections $(P_{i(inj)}, Q_{i(inj)})$ and $(P_{j(inj)}, Q_{j(inj)})$ of the UPFC are calculated according to the following expressions (Fang, W.L. and H.W. Ngan, 2005):

$$P_{i(inj)} = -G_{ij}(e_i e_T + f_j f_T) + B_{ij}(f_j e_T - e_j f_T) + 2G'(e_i e_T + f_i f_T) \tag{1}$$

$$Q_{i(inj)} = G'(f_i e_T - e_i f_T) - B'(e_i e_T + f_i f_T) - V_i I_q \tag{2}$$

$$P_{j(inj)} = -G_{ij}(e_i e_T + f_j f_T) - B_{ij}(f_j e_T - e_j f_T) \tag{3}$$

$$Q_{j(inj)} = G_{ij}(e_i f_T - f_j e_T) + B_{ij}(e_j e_T + f_j f_T) \tag{4}$$

Where:

$P_{i(inj)}, P_{j(inj)}$: the active power injections at bus i and j , respectively;

$Q_{i(inj)}, Q_{j(inj)}$: the reactive power injections at bus i and j , respectively;

e_i, f_i : real part and imaginary part of voltage at bus i ;

e_j, f_j : real part and imaginary part of voltage at bus j ;

e_T, f_T : real part and imaginary part of voltage of series voltage source, respectively and

$$e_T = V_T \cos(\varphi T) \quad , \quad f_T = V_T \sin(\varphi T)$$

V_i : the voltage magnitude of bus i ;

$G_{ij}, B_{ij}, g_{ij}, b_{ij}$: the parameters of line $i - j$

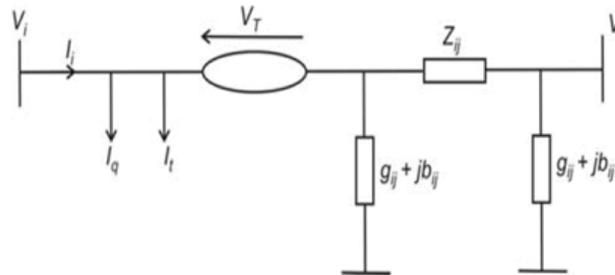


Fig. 2: Equivalent circuit of UPFC embedded branch.

4. Modified Shuffled Frog Leaping Algorithm (MSFLA):

4.1 Shuffled Frog Leaping Algorithm (SFLA):

Shuffled Frog Leaping Algorithm (SFLA) is a heuristic search algorithm presented for the first time by Eusuff and Lansey 2006. The main purpose of this algorithm was achieving a method to solve complicated optimization problems without any use of traditional mathematical optimization tools (Eusuff, M., 2006; Eusuff, M. and K.E. Lansey, 2003; Amiri, M. Fathian, A. Maroosi, 2007; Elbeltagi, E., 2005). In fact, the SFL algorithm is combination of “meme-based genetic algorithm or Memetic Algorithm” and “Particle Swarm Optimization (PSO)”. This algorithm has been inspired from memetic evolution of a group of frogs when seeking for food. In this method, a solution to a given problem is presented in the form of a string, called “frog” which has been considered as a control vector in this paper as follows in (5). The initial population of frogs is partitioned into groups or subsets called “memeplexes” and the number of frogs in each subset is equal. The SFL algorithm is based on two search techniques: local search and global information exchange techniques. Based on local search, the frogs in each subset improve their positions to have more foods (to reach the best solution). In second technique, obtained information between subsets is compared to each other (after each local search in subsets). The procedure of SFL algorithm will be as follows:

- 1) An initial population of “ P ” frogs (P solutions) created randomly which considered in this paper as follows:

$$\text{Population} = \begin{bmatrix} X_1 \\ \cdot \\ \cdot \\ X_P \end{bmatrix}_{(P) \times (2 \times N_{tie})} \tag{5}$$

$$X = [Tie_1, Tie_2, \dots, Tie_{N_{tie}}, Sw_1, Sw_2, \dots, Sw_{N_{tie}}]$$

- 2) The entire population is divided into m subsets (m memeplexes), each containing n frogs (i.e., $P = m \times n$), in such a way that the first frog of sorted population goes to the first memeplex, the second frog goes to the second memeplex, frog m goes to m memeplex, and frog $m+1$ goes to the first memeplex again, etc. therefore, in each memeplex, there will be n frogs.
- 3) This step is based on local search. Within each local memeplex, the frogs with the best and the worst fitness are identified as and, respectively. Also, the frog with the global best fitness (the best solution) is identified as. Then, the position of the worst frog is updated (based on frog leaping rule) as follows:

$$D_i = rand \times (X_b - X_w)$$

$$X_w(new) = X_w(old) + D_i \tag{6}$$

$$(-D_{min} \leq D_i \leq D_{max})$$

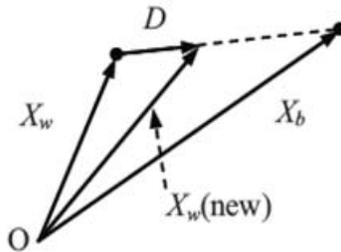


Fig. 3: The original frog leaping rule

Where $rand$ is a random number between 0 and 1; D_{max} is the maximum allowed change in frog's position. If this process produces a better solution ($X_w(new)$), new position of the worst frog, it replaces the worst frog's position ($X_w(old)$). Otherwise, the calculations in equations 1 and 2 are repeated with respect to the global best frog (i.e. replaces). If no improvement becomes possible in this case, then a new solution is randomly generated to replace the worst frog (X_w). Because of all arrays in X are integers, obtained solutions from equations 1 and 2 must be rounded after each iteration.

- 4) Continue of previous step for a number of predefined iterations.
- 5) After improvement in frog's positions, new population is sorted in a descending order according to their fitness.
- 6) If the convergence criteria are satisfied, stop. Otherwise, go to step 2 and repeat again.

4.2. Modified Shuffled Frog Leaping Algorithm (MSFLA):

According to previous section, the worst frog in each memeplex improves its position toward the best frog's position or the global best position in the same memeplex. But according to equations 12 and 13 and Fig. (1), the possible new position of the worst frog is restricted in the line segment between its current position (X_w) and the best frog's position (X_b), and the worst frog will never jump over the best one

(see Fig. (2)). these limitations not only slow down the convergence speed, but also cause premature convergence. Hence, the equations 1 and 2 must be replaced by new equations as follows:

$$D_i = rand \times C(X_b - X_w) + W$$

$$W = [r_1 w_{1,max}, r_2 w_{2,max}, \dots, r_{N_{ie}} w_{N_{ie},max}]^T \tag{7}$$

$$X_w(new) = \begin{cases} X_w + D & \text{if } |D| \leq D_{max} \\ X_w + \frac{D}{\sqrt{D^T D}} D_{max} & \text{if } |D| \geq D_{max} \end{cases} \quad (8)$$

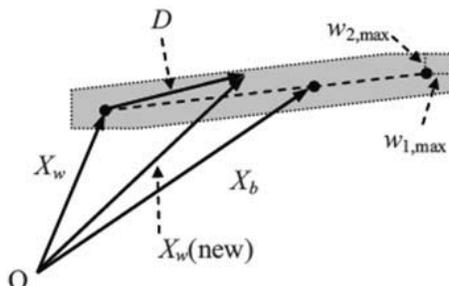


Fig. 4: The new frog leaping rule

Where rand is a random number between 0 and 1; C is a constant in the range between 1 and 2; r_i are random numbers between -1 and 1; $w_{i,max}$ are the maximum allowed perception and action uncertainties in the i th dimension of the search space; D_{max} is the maximum allowed change in frog's position. Because

of all arrays in X are integers, obtained solutions from equations 7 and 8 must be rounded after each iteration. By applying equations 7-8 local search space in each memplex increases. Therefore, the convergence speed increases and convergence probability to achieve the best solution will increase. For applying MSFL algorithm to a reconfiguration problem, following steps must be taken:

- 1) In this step, required parameters and information such as branch impedance, switch positions, number of memplexes, number of frogs and etc, are defined and determined.
- 2) The constrained objective function is converted to an unconstrained objective function according to:

$$F(x) = f(x) - k_1 \left(\sum_{j=1}^{N_{eq}} (h_j(x))^2 \right) - k_2 \left(\sum_{j=1}^{N_{ueq}} (\text{Max}[0 - g_i(x)])^2 \right) \quad (9)$$

Above formula is objective function of optimal placement of UPFC problem where N_{eq} and N_{ueq} are the number of equal and unequal constraints, respectively. Also, $g_i(x)$ and $h_i(x)$ are equal and unequal constraints, respectively. $k_1, k_2 (k_1, k_2) > 0$ are penalty factors which must have a large value.

Problem Formulation:

The aim of the optimization is to perform a best utilization of the existing transmission lines. In this respect, UPFC device is located in order to maximize the system loadability while observing thermal and voltage constraints. In other words, it was tried to increase the power transmitted by power system as much as possible to the costumers with holding power system in security state in terms of branch loading and voltage levels. The objective function is made in order to penalize configurations of the UPFC which lead to overload transmissions lines and over or under voltage at busses. The objective function is defined as the sum of two terms. The first one is related to the branch loading which penalizes overloads in lines. This term is called LF and is computed for all lines of the power system, if branch loading is less than 100% its value is equal to 1; otherwise, it decreases exponentially with respect to the overload. To accelerate the convergence, product

of values for all objective functions is calculated. The second part of the objective function is for voltage levels that are named BF. This function is calculated for all buses of power system. For voltage levels between 0.95 and 1.05, values of the objective functions is equal to 1. Outside this range, value decreases exponentially with the voltage deviations. Therefore, for a configuration of UPFCs, objective function is given as:

$$LF = \begin{cases} 1 & , BL < 100 \\ \exp[0.0461(100 - BL)] & , BL \geq 100 \end{cases} \quad (10)$$

$$BF = \begin{cases} 1 & , 0 \leq V_L \leq 100 \\ \exp[-23.0259 |1 - V_L| - 0.05] & , 1.05 \leq V_L \leq 1.25 \\ & \text{or } 0.75 \leq V_L \leq 0.95 \end{cases} \quad (11)$$

$$\text{Objective Function } OF = \prod_{i=line} LF_i + \prod_{j=buses} BF_j \quad (12)$$

Where, LF is the line flow index and BL is the Branch Loading (Percentage of the line flow with respect to the line capacity rate). BF is bus voltage index and V_L is per unit value of the bus voltages. In this paper, all loads are increased in the same proportion and it is assumed that the increase in real power generation due to this increase in load is met by the generator connected to slack bus.

6. MSFLA Optimization Strategy:

The MSFLA-based approach for solving the optimal placement of UPFC problem to minimized consists of three objectives takes the following steps:

Step 1: Input The number of UPFCs to be placed and the initial load factor and line and bus data, and bus voltage limits.

Step 2: Calculate the power flow based on OPF Method.

Step 3: Create an initial population of k frogs generated randomly. The initial population of individuals is created satisfying the UPFC constraints and also it is verified that only one device is placed in each line.

Step 4: Sort the population increasingly and divide the frogs into p memplexes each holding q frogs such that $k = p \times q$. The division is done with the first frog going to the first memplex, second one going to the second memplex, the p th frog to the p th memplex and the $p + l$ th frog back to the first memplex. The configuration of N number of UPFCs is defined with two parameters namely the location of UPFCs and their corresponding controllable parameters such as V_T , ϕ_T and I_q . The construction of memplex for MSFLA implementation is shown in Figure 5.

L_1	...	L_N	V_{T1}	...	V_{TN}	ϕ_{T1}	...	ϕ_{TN}	I_{q1}	...	I_{qN}
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Fig. 5: Construction of memplex.

Step 5: For each memplex if the bus voltage is within the limits, calculate the fitness function in equation (12). Otherwise, that memplex is infeasible.

Step 5-1: Set $p_1 = 0$ where p_1 counts the number of memplexes and will be compared with the total number of memplexes p . Set $y_1 = 0$ where $y_1 = 0$ counts the number of evolutionary steps and will be compared with the maximum number of steps (y_{max}), to be completed with in each memplex.

Step 5-2: Set $p_1 = p_1 + 1$.

Step 5-3: Set $y_1 = y_1 + 1$

Step 5-4: For each memplex, the frogs with the best fitness and worst fitness are identified as X_w and X_b , respectively. Also the frog with the global best fitness X_g is identified. Then the position of the worst frog X_w for the memplex is adjusted as follows:

$$B_i = rand(.) \times (X_b - X_w)$$

$$new X_w = old X_w + B_i \quad (-B_{max} \leq B_i \leq B_{max}) \tag{13}$$

Where rand (.) is a random number between 1 and 0 and B_{max} is the maximum allowed change in the frogs position. If the evolutions produce a better frog (solution), it replaces the older frog. Otherwise, X_b is replaced by X_g in (13) and the process is repeated. If non improvement becomes possible in this case a random frog is generated which replaces the old frog.

Step 5-5: If $P_1 \leq P$, return to step5-2. If $y_1 \leq y_{max}$, return to step 5-3. Other wise go to step 4.

Step 6: Check the convergence. If the convergence criteria are satisfied, stop. Otherwise, consider the new population as the initial population and return to the step4. The best solution found in the search process is considered as the output results of the algorithm.

Step 7: Print out the optimal solution to the target problem (the previous best memplex which contains the location and parameters of UPFCs with the corresponding load factor).

Step 8: Stop the procedure.

Numerical Results:

To verify the effectiveness and efficiency of the proposed MSFLA based loadability maximization approach, the IEEE 14-bus power system and the IEEE 118-bus power system are used as the test systems. The numerical data for IEEE 14-bus and IEEE 118-bus systems are taken from (The university of Washington Archive,). The simulation studies are carried out in MATLAB environment.

7.1. IEEE 14-bus System:

Table I gives the optimal location and parameters of UPFCs for different loading factors for IEEE 14-bus system. From the results, it is observed that the loadability has been increased to 114.1% by installing an UPFC between buses 1 and 5. The maximum loadability with 2 UPFCs without violating the thermal and voltage constraints is 129.1 % . For this load factor, the UPFCs are embedded in lines connecting buses 2,4 and 2,5. From Figure 4, it is evident that, there is a maximum number of devices beyond which the efficiency of the network cannot be further improved. According to the used optimization criterion, for IEEE 14 bus system, the maximum number of UPFCs beyond which the loadability cannot be increased is 3. Table II summarizes the results as obtained by the three methods for the IEEE14-bus system using their proposed methodologies. The results show that the optimal solutions determined by MSFLA lead to increased loadability of the lines with less number of UPFCs, which confirms that MSFLA based present approach is capable of determining global optimal or near global optimal solution.

Table I: Optimal location and parameters of UPFCs for different load factors for IEEE 14-bus system

No. of UPFCs	Loading Factor	Location (Bus No -Bus No)	UPFC parameters		
			V_T (p. u.)	ϕ_T (deg)	I_n (p.u.)
1	1.141	1-5	0.135	76.130	0.0401
2	1.291	2-4	0.091	62.143	0.0590
		2-5	0.147	101.937	-0.0723
3	1.721	2-4	0.105	74.472	0.0149
		6-7	0.223	78.174	-0.0473
		2-5	0.184	145.760	0.0374

Table II: Comparison of simulation results of IEEE 14-bus system

Compared Algorithm	Maximum Possible Loadability
EP based Method	1.559
PSO based Method	1.671
MSFLA based Method	1.721

7.1. IEEE 118-bus System:

Table III gives the optimal location and parameters of UPFCs for different loading factors for IEEE-118 bus system. From the results, it is evident that the maximum possible loadability for the system is 121.4% with 5 UPFCs. Beyond this limit, the loadability cannot be improved with increase in UPFCs. Table IV compares the results as obtained by the two methods for the IEEE 118-bus system using their proposed methodologies. Results obtained by the proposed MSFLA based method are better than the one observed by the EP based method available in in terms of loadability.

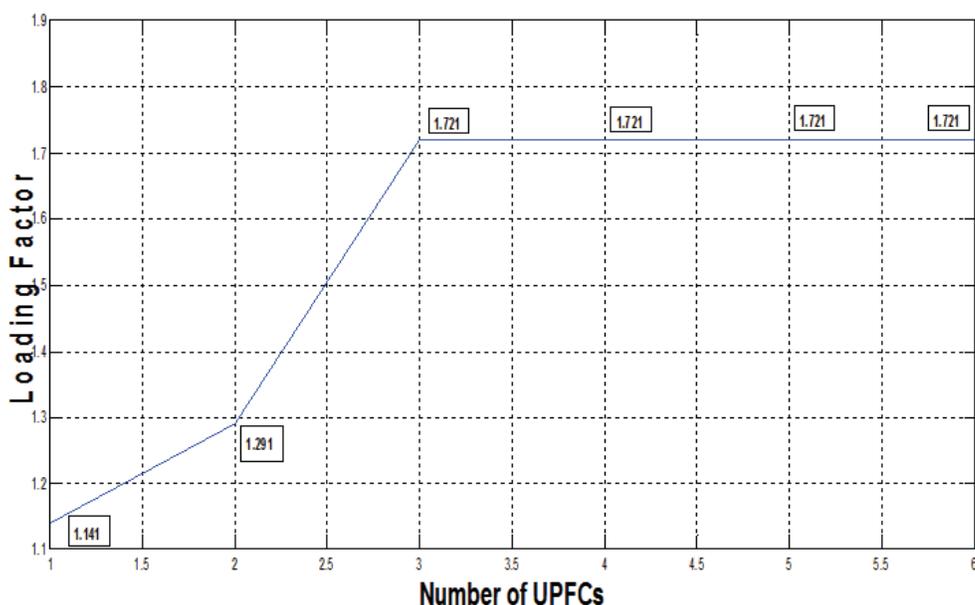


Fig. 6: Maximum loading factor with respect to given number of UPFCs for IEEE 14-bus system.

Table III: Optimal location and parameters of UPFCs for different load factors for IEEE 118-bus system

No. of UPFCs	Loading Factor	Location (Bus No -Bus No)	UPFC parameters		
			V_T (p. u.)	ϕ_T (deg)	I_0 (p.u.)
3	1.099	104-105	0.1213	108.392	-0.1163
		101-102	0.1248	101.337	0.0392
		80-96	0.0331	38.417	-0.0972
4	1.184	60-61	0.0862	96.356	-0.1092
		100-103	0.1423	229.390	0.0771
		88-89	0.2389	114.811	-0.1378
		80-97	0.1903	79.160	0.0266
5	1.214	70-75	0.1623	133.419	-0.0413
		54-55	0.0698	116.003	0.0199
		92-94	0.1273	96.118	-0.0828
		69-75	0.1502	232.422	-0.0183
		105-106	0.0599	273.749	-0.0681

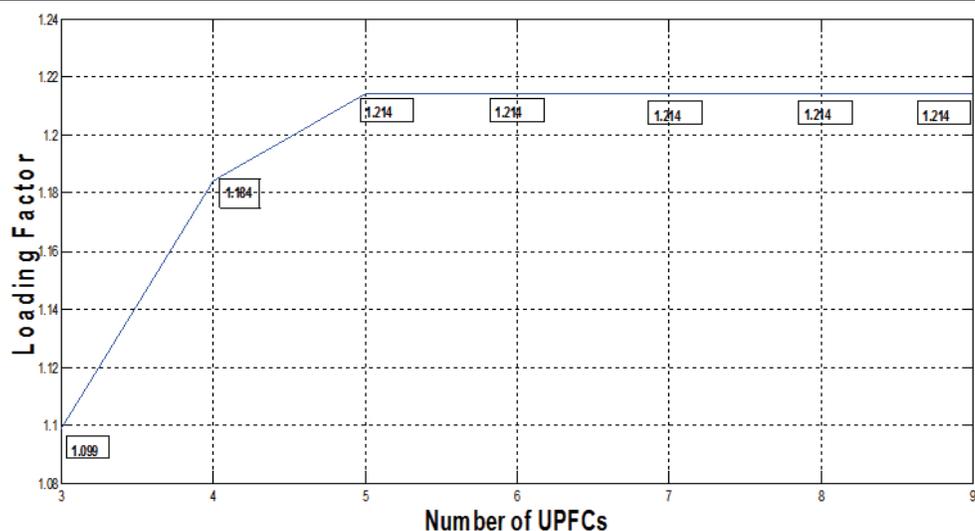


Fig. 7: Maximum loading factor with respect to given number of UPFCs for IEEE-118 bus system

Table IV: Comparison of simulation results of IEEE 118-bus system

Compared Algorithm	Maximum Possible Loadability
EP based Method	1.163
PSO based Method	1.191
MSFLA based Method	1.214

Conclusion:

In this paper, the optimal UPFC placement on an unstable power system because of load increasing has been investigated. A mathematical model for simultaneously optimizing location and parameters of the UPFCs is presented in this paper. A Modified Shuffled Frog Leaping Algorithm is used to solve this nonlinear programming problem. The case study of the IEEE 14-bus and 118-bus system has confirmed that the developed algorithm is correct and effective. The system loadability was employed as a measure of power system performance. Simulation results validate the efficiency of this new approach in maximizing the loadability of the system. Furthermore, the location of UPFCs and their parameters are optimized simultaneously. The performance of the proposed method demonstrated through its evaluation on the IEEE 14-bus power system and the IEEE 118-bus power system shows that MSFLA is able to undertake global search with a fast convergence rate and a feature of robust computation. The proposed algorithm is an effective and practical method for the allocation of UPFCs in large power systems.

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