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A Linear Programming Approach to Maximize Savings by Stretching Noncritical Activities

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Abstract: In this research, some concepts of linear programming and critical path method are reviewed to describe recent modeling structures that have been of great value in analyzing project time-cost trade-offs problems. This paper mainly provides a framework for the approach of stretching noncritical activities to complete the project in shortest possible duration at least cost within available maximum budgeting. This is achieved by crashing all activities simultaneously in the project network then using Linear Programming (LP) technique to build a model to maximize the savings that will yield from stretching noncritical activities. The noncritical activities can be stretched to their normal time until all slack in the different noncritical paths network is used up. The resultant savings from using of linear programming model must be subtracted from the initial cost of crashing all activities to obtain the final cost of project.

Key words: Linear programming; CPM; Time-cost trade-off approach; Crashing; Stretching; least cost-scheduling

INTRODUCTION

The success of CPM is that it utilizes the planner's knowledge, experience, and instincts in a logical way first to plan and then to schedule. CPM can save money through better planning. Construction managers are continually facing a situation in which they must take a decision whether to complete the project sooner than originally specified in the contract because of the clients request and /or to optimize the cost of expediting. Fortunately, two closely related operations research techniques, Linear Programming (LP) and Critical Path Method (CPM), are available to assist the project manager in carrying out these responsibilities (Nicholas, 2004; O'Brien, and Plotnick, 2006).

Several studies conducted on CPM and LP techniques for time-cost trade-off.

Priluck (1965) illustrates in his study the utilization of CPM at the time since it was still relatively new. Howard (1965) claims CPM as a complete project management tool. He illustrates this notion in his article. The use of CPM in time/cost trade-offs, resource leveling, and scheduling across multiple projects was becoming more popular. Using CPM in a time/cost analysis is beneficial because if a project needs expediting, knowing which activities control the project duration is helpful. Schumacher (1965) discusses the different evaluation techniques using CPM. The article briefly describes the different delay types, assigning responsibility of delays, and the special case of concurrent delays. Babu and Suresh (1996) suggest that the project quality may be affected by project crashing and develop linear programming models to study the tradeoffs among time, cost, and quality. Islam *et al.* (2004) claim that managers must become more rational in decision making by using more effective tools and techniques. This study provides a framework for reducing total project time at the least total cost by crashing the project network using Linear Programming (LP). Sakellaropoulos and Chassiakos (2004) aiming of their study to develop a solution method considering additional realistic project characteristics such as generalized activity precedence relations and external time constraints for particular activities. The proposed method is formulated as a linear/integer program and provides the optimal project time–cost curve and the minimum cost schedule. Ipsilandis (2006) supposes the Critical Path Method (CPM) and the Repetitive Scheduling Method (RSM) are the most often used tools for the planning, scheduling and

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control Linear Repetitive Projects (LRPs). In this study he presents a linear programming approach to address the multi objective nature of decisions construction managers face in scheduling LRPs. Bidhandi (2006) develops a mathematical model for project time compression problems in CPM/PERT type networks. The kind of this model is Mixed Integer Linear Program (MILP) with zero-one variables, and the Benders' decomposition procedure for analyzing this model has been developed. Bagherpour *et al*. (2006) suppose many traditional cost– time trades off models are computationally expensive to use due to the complexity of algorithms especially for large scale problems. They present a new approach to adapt linear programming to solve cost time trade off problems. Ke *et al*. (2009) claimed in their study that in real-life projects, both the trade-off between the project cost and the project completion time, and the uncertainty of the environment are considerable aspects for decision-makers.

Algorithm of Stretching Noncritical Activities by Using Linear Programming Technique:

Step 1: Determine the normal time, normal cost, crash time and crash cost for each activity to compute cost slope.

Step 2: Crash all activities in the project simultaneously to complete the project in shortest possible duration within available maximum budgeting, considering this point at which management's desired deadline has been reached.

Step 3: Draw the network project after crashing all activities.

Step 4: Determine the critical path and noncritical paths. Also, identify the critical activities.

Step 5: Compute the new total cost by adding the cost of the crashing to the current total cost.

Step 6: Use LP technique to build a model to maximize savings via maximize the durations of stretching noncritical activities that multiplied by their associated costs slope**.** The noncritical activities can be stretched to their normal time until all the slack in the different noncritical paths network is used up. The resultant savings must be subtracted from the initial cost of crashing all activities to obtain the final cost of project.

Linear Programming Technique:

Now let's define the variables and formulations of the problem.

A_i :Project's activities, where i =(1,2,3,,, n).

B : Maximum budgeting available.

T : Shortest possible duration to complete the project at least cost within available maximum budgeting.

$C_{N,\,i}$: Normal cost for activity i

$T_{N,\,i}$: Normal Time for activity i

$C_{C\,,\,i}$: Crash Cost for activity i

$T_{C\,,\,i}$: Crash Time for activity I

$$(Cost\ slope\ ,Ui\ ) = \frac{C_{c,i} - C_{N,i}}{T_{N,i} - T_{c,i}} \tag{1}$$

$T.C_a$: Total cost to complete the project considering crash all activities.

$$T.C_a = \sum_{i=1}^{n} C_{C,i} \tag{2}$$

$D_{C,\,q}$: Crash time of critical activity q, where q = (1,2,3…L).

$C.P._C\,(T)$: The critical path for the project network after all activities are crashing.

$$C.P._C\,(T) = \sum_{q=1}^{L} D_{c,q} \tag{3}$$

Y_i = the time when an event i will occur, measured since the beginning of the project.

S_j = Amount of times (measured in terms of days, weeks, months or some other units) that each noncritical activities j will be stretched, where j = (1, 2, 3…m)

U_j = (Cost slope) Crash cost per unit of time for noncritical j.

The objective is to maximize the durations of stretching noncritical activities that multiplied by their associated costs slope, then subtracting the resultant cost from the cost of crashing all activities, our LP objective function will be:

$$Maximize\ Z = \sum_{j=1}^{m} U_j S_j \quad (4)$$

This objective function is subject to some constraints. These constraints can be classified in to three categories.

1. Stretch time constraints: Just as reducing an activity's time from the normal time increases its cost, so stretching time from the crash time reduces its cost. Find the total savings cost by stretching all noncritical activity without extending the project. Start with those noncritical activities in noncritical paths that will yield the greatest savings—those with the greatest cost slope. The noncritical activities can be stretched until all the slack in the different noncritical paths network is used up.
2. Constraints unfolding the network: These set of constraints describe the structure of the network. As we mention earlier that the activities of a project are interrelated, the starting of some activities is dependent upon the completion of some other activities; we must have to establish work sequence of the activities through constraints.
3. Project completion constraints: This constraint will recognize that the last event (completion of last activities) must take place before the project deadline date.

So, the constraints are:

- Stretch time constraints:

S_j ≤ Allowable stretching time for activity j measured in terms of days, weeks, months or some other units.

- Constraints unfolding the Network: There will be one or more constraints for each event depending on the predecessor activities of that event.

As the event 1 will start at the beginning of the project, we begin by setting the occurrence time for event 1 equals to zero. Thus, $Y_1 = 0$.

The other events will be expressed as follows:

Start time of this activity (Yi) ≥ (start time + crashing duration+ stretch duration) for this immediate predecessor

- Project completion constraints:

$Y_m \leq$ Project deadline date after being stretched, where m indicate the last event of that project.

Application in Real Life:

The construction projects company accepted to construct a new house, there are 11 general activities where $A_i = (A_1, A_2, \ldots, A_{11})$ and the maximum budgeting that available for the client ($\$B$) is \$78,500, he wants to complete the house in shortest possible duration at least cost within available maximum budgeting (T) within 17 weeks.

So the management decides to use LP technique to build a model to maximize the savings that will yield from stretching noncritical activities. The noncritical activities can be stretched to their normal time until all the slack in the different noncritical paths network is used up. The resultant savings must subtract from the initial cost of crashing all activities to obtain the final cost of project. Table 1 summarizes a project with the numerical real life project with real data.

RESULTS AND DISCUSSIONS

Project Completion by Crashing All Activities Simultaneously (Crashing Conditions):

We can find the total cost of the project and the shortest possible duration for project completion from equations (2) and (3) respectively, (see Table 2 and Figure 2).

$$\mathbf{T.C_a} = \sum_{i=}^{W} C_{C,i} = C_{C,A} + C_{C,B} + C_{C,C} + \cdots + C_{C,W} = \$78{,}500$$

$$C.P._{c}\,(T) = \sum_{i=}^{w} C_{C,i} = C_{C,A} + C_{C,B} + C_{C,C} + \cdots + C_{C,W} = \text{ 17 weeks}$$

Table 1: Activity data in normal and crash conditions

Activities	Predecessors (precedence)	Normal time	Normal cost	Crash time	Crash cost	Max Crashing in time	Max stretching in time	Cost slope
A_1	----	3	5000	2	7000	1	1	2000
A_2	A_1	4	4000	2	5000	2	2	500
A_3	A_2	4	7000	4	7000	0	0	----
A_4	A_2	3	3000	1	5000	2	2	1000
A_5	A_2	5	6000	2	10500	3	3	1500
A_6	A_5 , A_3	4	8000	3	10000	1	1	2000
A_7	A_4	3	4000	1	5500	2	2	750
A_8	A_7	6	6000	4	9000	2	2	1500
A_9	A_6	7	5000	4	8000	3	3	1000
A_{10}	A_8, A_9	4	6000	2	7500	2	2	750
A_{11}	A_3, A_5	9	3000	7	4000	2	2	500

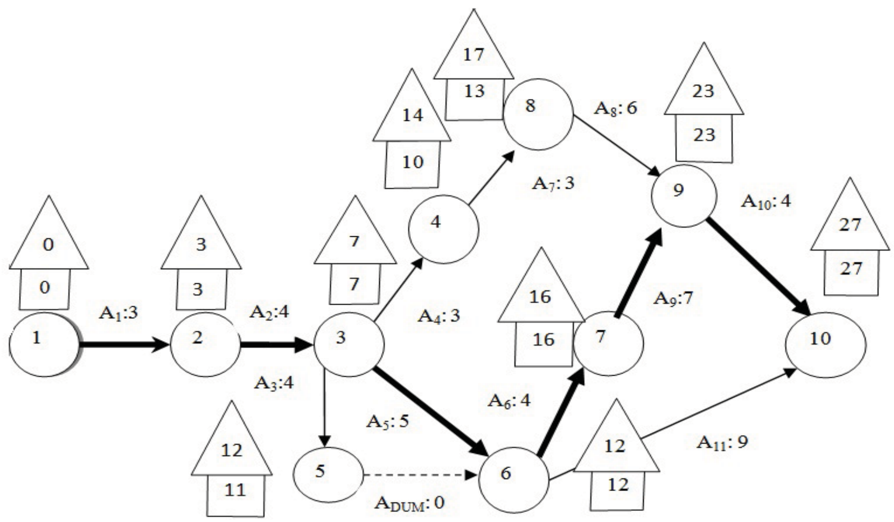

Fig. 1: Critical path, earlist start and finifh, latest start and finish in normal conditions

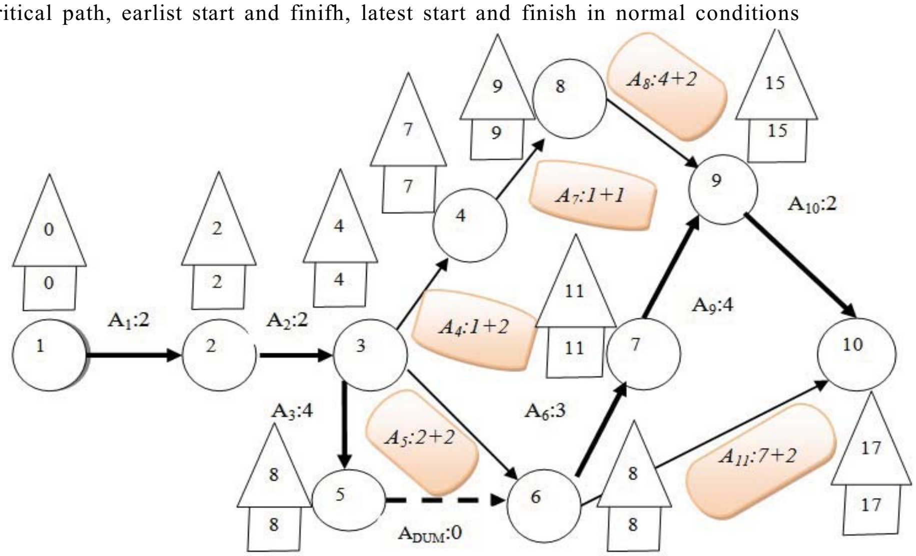

Fig. 2: Critical path, earlist start and finifh, latest start and finish and mechanism of stretching noncritical activities

We can identify the critical path of the project to determine the shortest project's completion time. Based on the calculation, the sample project will be completed within 17 weeks. The calculation is shown in Table 3.

Table 2: Project completion in crash duration and crash cost

Activity Name	On Critical Path	Activity Time	Earliest Start	Earliest Finish	Latest Start	Latest Finish	Total float (LS)i-(ES)I
A_1	Yes	2	0	2	0	2	0
A_2	Yes	2	2	4	2	4	0
A_3	Yes	4	4	8	4	8	0
A_4	no	1	4	5	9	10	5
A_{DUM}	Yes	0	8	8	8	8	0
A_5	no	2	4	6	6	8	2
A_6	Yes	3	8	11	8	11	0
A_7	no	1	5	6	10	11	5
A_8	no	4	6	10	11	15	5
A_9	Yes	4	11	15	11	15	0
A_{10}	Yes	2	15	17	15	17	0
A_{11}	no	7	8	15	10	17	2
Project Completion Time = 17 weeks			Total Cost of Project = $78,500,			Number of Critical Path(s) = 1	

Linear Programming Model to Stretch Noncritical Activities:

For the purpose of modeling the problem, it is necessary to define the activities in terms of starting and ending event. The total number of events in this project is 10.

Table 3: Head and Tail Events of Activities

Activities	Tail Event (Starting Event)	Head Event (Ending Event)
A_1	1	2
A_2	2	3
A_3	3	5
A_4	3	4
A_{DUM}	5	6
A_5	3	6
A_6	6	7
A_7	4	8
A_8	8	9
A_9	7	9
A_{10}	9	10
A_{11}	6	10

Defining the variables:

Let, Y_1 = Time when event 1 will occur.
Y_2 = Time when event 2 will occur.
Y_3 = Time when event 3 will occur.
Y_4 = Time when event 4 will occur.
Y_5 = Time when event 5 will occur.
Y_6 = Time when event 6 will occur.
Y_7 = Time when event 7 will occur.
Y_8 = Time when event 8 will occur.
Y_9 = Time when event 9 will occur.
Y_{10} = Time when event 10 will occur.
S_{A4}= Number of days activity D will be stretched.
S_{A5}= Number of days activity E will be stretched.
S_{A7}= Number of days activity G will be stretched.
S_{A8}= Number of days activity H will be stretched.
S_{A11}= Number of days activity I will be stretched.

From equation (4) we obtain equation (5)

$$Maximize\ Z = 1000\ S_{A4} + 1500\ S_{A5} + 750\ S_{A7} + 1500\ S_{A8} + 500\ S_{A11} \quad (5)$$

Constraints of the Model:
Stretch time constraints:

$S_{A4} \leq 2$ (6)
$S_{A5} \leq 2$ (7)
$S_{A7} \leq 2$ (8)
$S_{A8} \leq 3$ (9)
$S_{A11} \leq 2$

Constraints Unfolding the Network:

We begin by setting the event occurrence time for event 1 to be $Y_1 = 0$. The constraints describe the structure of the network are as follows:

$Y_1 = 0.$
$Y_2 - Y_1 - S_{A1} \geq 2$ (10)
$Y_3 - Y_2 - S_{A2} \geq 2$ (11)
$Y_5 - Y_3 - S_{A3} \geq 4$ (12)
$Y_6 - Y_5 - S_{Adum} \geq 0$ (13)
$Y_6 - Y_3 - S_{A5} \geq 2$ (14)
$Y_4 - Y_3 - S_{A4} \geq 1$ (15)
$Y_8 - Y_4 - S_{A7} \geq 1$ (16)
$Y_7 - Y_6 - S_{A6} \geq 3$ (17)
$Y_9 - Y_7 - S_{A9} \geq 4$ (18)
$Y_9 - Y_8 - S_{A8} \geq 4$ (19)
$Y_{10} - Y_9 - S_{A10} \geq 2$ (20)
$Y_{10} - Y_6 - S_{A11} \geq 7$ (21)

Project Completion Constraints:

$Y_{10} \leq 17$ (22)

By analyzing the project on crashing all activities simultaneously basis of the shortest possible duration of completion of the project is 17 weeks.

As the manager wants to complete the project in shortest possible duration to complete project at least cost within available maximum budgeting (17 weeks), so the last event should be completed before or on 17^{th} week. So, the maximum extent of crashing all activities is 17 weeks. Beyond this time period, the project cannot be crashed.

All Variables Are Greater than or Equal Zero:

$S_{A1}, S_{A2}, S_{A3}, S_{A4}, S_{A5}, S_{A7}, S_{A8}, S_{A9}, S_{A10}, S_{A11}, Y_1, Y_2, Y_3, Y_4, Y_5, Y_6, Y_7, Y_8, Y_9, Y_{10}$

Solution and Analyses for Linear Programming Model:
Solution and Analyses Table for Linear Programming Model Variables:

Solution of the model is presented in table 4, which shows the solution of the problem. It includes the decision variable value, contribution to the objective, and reduced cost of each decision variable. This also indicates the status of whether the decision variable is on the final basis. This table is available when the optimal solution is achieved.

Solution summary in table 4 indicates that an activity A_4 should be stretched at 2 weeks, A_5 at 2 weeks, A_7 at 1week, A_8 at 2weeks and A_{11} at 2 weeks. Adding the stretch-time amounts to the crash completion, the result indicates that activity A_4 should be completed in 3 weeks, A_5 in 4 weeks, A_7 in 2 weeks, A_8 in 6 weeks and A_{11} in 9 weeks.

The total saving cost by stretching all noncritical activity without extending the project is $9,750 and the duration to complete the project in shortest possible duration at least cost within available maximum budgeting is 17 weeks.

So the final project cost is computed by subtracting the saving cost $9,750 from the cost of crashing all activities $78,500. So, the final project cost is .

In our project reduced cost of a current non-basic variable S_{A1} is -2,250. It means the current coefficient of this variable which is now 0 must increased 2,250 (that means the coefficient would be 2,250 or up) to get a basic (positive) value of this variable in the optimum solution (see Table 4).

This Analysis shows also the ranges of the objective function coefficients, the final value of variable S_{A4} in the objective function is 2. The current coefficient of the variable is 1000, allowable decrease is 750 and allowable increase is infinity or M (see Table 4). It indicates our current solution would remain optimum if crash cost per unit of time for activity A_4 varies from $750 to M.

Table 4: Solution summary by using Win.QSB program.

No	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	S_{A1}	0	0	0	-2,250	at bound	-M	2,250
2	S_{A2}	0	0	0	-2,250	at bound	- M	2,250
3	S_{A3}	0	0	0	0	basic	0	M
4	S_{A4}	2	1,000	2,000	0	basic	750	M
5	S_{A5}	2	1,500	3,000	0	basic	500	M
6	S_{A6}	0	0	0	-1,000	at bound	- M	1,000
7	S_{A7}	1	750	750	0	basic	0	1,000
8	S_{A8}	2	1,500	3,000	0	basic	750	M
9	S_{A9}	0	0	0	-1,000	at bound	-M	1,000
10	S_{A10}	0	0	0	-1,750	at bound	- M	1,750
11	S_{A11}	2	500	1,000	0	basic	0	1,500
12	S_{dummy}	0	0	0	0	at bound	-M	0
13	Y_1	0	0	0	0	basic	- M	M
14	Y_2	2	0	0	0	basic	- M	2,250
15	Y_3	4	0	0	0	basic	- M	2,250
16	Y_4	7	0	0	0	basic	-250	750
17	Y_5	8	0	0	0	basic	0	M
18	Y_6	8	0	0	0	basic	-1,000	M
19	Y_7	11	0	0	0	basic	-1,000	M
20	Y_8	9	0	0	0	basic	-750	750
21	Y_9	15	0	0	0	basic	-1,750	M
22	Y_{10}	17	0	0	0	basic	-2,250	M
	Objective	Function	(Max.) =	9,750				

Solution and Analyses Table for Linear Programming Model Constraints:

This table shows the constraint status of the problem for the final solution. It includes the left-hand side, right-hand side, surplus or slack, and shadow price of each constraint. This also indicates the status of whether the constraint is Binding or not. This command is available when the optimal solution is achieved.

Right hand sensitivity of the constraints provides us information regarding the status of the constraints- which of these constraints are binding (fully utilized) or non-binding.

Binding constraints meaning the slack variable that added to the left-hand side of a less than or equal to ($\leq$) constraint to convert the constraint into an equality or the surplus variable that subtracted from the left-hand side of a greater than or equal to ($\geq$) constraint to convert the constraint into an equality is zero.

As well binding constraints having a value in the shadow price column (see Table 5) other than zero, means how much contribution these binding constraints will provide individually in the objective function, if the value of the right hand side of these constraints are increased by 1 unit. The allowable increase and allowable decrease columns of table 5 indicate the range of increase and decrease of the right hand side value of the binding resources within which the current shadow price would remain unchanged. For example the slack variable for C1is zero and the value in the shadow price column is 250, so the contribution of this binding constraint will provide individually in the objective function 250 if the value of the right hand side of this constraint is increased by 1 unit.

So, this model would help management to reach the optimality where sensitivity analysis would provide some flexibility in the model.

Conclusions:

This model will provide us a systematic and logical approach for decision making and ultimately increases the effectiveness of the decision. As the solution provide us the starting time of the activities, this can be used for monitoring the project.

Table 5: Solution of Constraint Summary by using Win.QSB program.

No. of constraint	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	C1	2	<=	2	0	250	1	3
2	C2	1	<=	2	1	0	1	M
3	C3	2	<=	2	0	750	1	3
4	C4	2	<=	3	1	0	2	M
5	C5	2	<=	2	0	0	2	M
6	C19	0	0	0	0	-2,250	0	0
7	C6	2	>=	2	0	-2,250	1	2
8	C7	2	>=	2	0	-2,250	1	2
9	C8	4	>=	4	0	0	-M	4
10	C9	0	>=	0	0	0	-M	0
11	C10	2	>=	2	0	-1,500	1	4
12	C11	1	>=	1	0	-750	0	2
13	C12	1	>=	1	0	-750	0	2
14	C13	3	>=	3	0	-1,000	2	3
15	C14	4	>=	4	0	-1,000	3	4
16	C15	4	>=	4	0	-750	3	5
17	C16	2	>=	2	0	-1,750	1	2
18	C17	7	>=	7	0	-500	7	9
19	C18	17	<=	17	0	2,250	17	18

In this work, we propose an algorithmic model based on linear programming to build a model to maximize savings via maximize the durations of stretching noncritical activities that multiplied by their associated costs slope. The format of the model lends itself to a wide range of variables and considerations.

The present modeling strategy has shown the resources of this interactive approach including a bulk of data to completely analyze the project is easily possible. It allows a great number of parameters to simulate project conditions and contractor's preference and provides potentially useful tool for decision making on project scheduling.

The cost of the network activities has been optimized for various overall duration. The optimum trade-off of time against cost has been made. This approach is an acceptable tool of management and proving to be not only superior method for planning, scheduling and controlling project progress, but also is very real and valuable assets to contractors in convincing the owner of their potentials and abilities. With the introduction of better and more rigorous methods of planning work, together with cost analysis, the construction control will become more systematic. In all of these, decisions must be made to carry out the operation in the best way possible in light of the restraints that are bound to exist.

This approach allows the user to easily manipulate different project networks of various difficulties representing real world applications, and to study the effectiveness of the model in the case of large projects. The implementation of the developed model is tested on a large number of linear optimization problems and shown to have more efficient and reliable results, generates a considerable computational savings, along with an increase in robustness.

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