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Numerical Simulation of Vertical Penetration into Granular Beds

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Abstract: In this paper, we report the numerical results of simulations that capture the dynamics of projectiles penetrating vertically into a two dimensional granular bed. The simulations are conducted by using the two dimensional discrete element method based on soft particle contact model. We study the scaling relationship of the penetration distance of a vertical projectile with its impact velocity. The simulation results are in agreement with previous experimental results and suggest that the relation between the penetration distance of a projectile into a granular bed and its impact velocity follows the power law.

Key words: Discrete element method; granular bed; projectile; penetration; power law.

INTRODUCTION

There are still no clear explanations for several phenomena about the penetration of projectiles into granular beds. For instance, meteorites generally hit at non normal angles but nevertheless produce circular craters (Parisi D.R, 2004). Moreover, even though the dynamics of penetration problem has long been of interest and remains an active research area attracting the attention of mathematicians and physicists, no consensus has emerged regarding the equation that describes the velocity of the projectile during the period of penetrating into a granular bed. Only a few studies have been conducted over the last three decades to better characterize the dynamics of penetration problems. Those studies, in general, mainly focused on experimental investigations. In 1957, an equation was constructed to express the negative acceleration of projectile penetrating randomly packed sand (Allen, 1957). The equation has the form

$$-\frac{dv}{dt}=\alpha v^2+\beta v+\gamma \tag{1}$$

From figure 4, it can be concluded that the relationship between the penetration distance and the impact velocity approximately follows a power law of the form. where *α, β,* and γ are positive constants, *v* denotes the projectile penetration velocity at time *t*. Other studies proposed that the centrifuge modeling is an appropriate and powerful tool for investigating the penetration of projectiles into granular soil (Taylor, 1991). For different soils, power relationships between projectile penetration depth and projectile mass-to-area ratio are derived. The depth of penetration was derived as a function of initial velocity and material properties of the granular bed (Boguslavskii, 1996). A review of scaling laws for impact process and a brief for the current approaches that are used to study the processes of impacts was presented (Holsapple, 1993). The functional dependence of the crater diameter on the kinetic energy of falling ball into a sand filled container was investigated (Joseph, 1998). The maximum height of jet was found to depend on the impact velocity, gravity, and the effective viscosity of granular medium when a solid sphere impacts on a deep layer of a granular medium (Xu, Y., 2002). Walsh (2003) performed experiments on impact craters formed by dropping a steel ball vertically into a container of small glass beads. By explicit variation of ball density, diameter, and drop height, the crater diameter was confirmed to scale as the fourth power of the energy of the ball at impact. Lohse (2004) presented a simple continuum theory together with experiments and discrete element simulations to account for the void collapse leading to the formation of the upward and downward jets when a steel ball

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is dropped on a very fine sand bed in a well-defined and fully decompactified state. Bruyn & Walsh (2004) found that the penetration depth of a steel sphere, dropped vertically into a container of loosely packed small beads, increases linearly with the incident momentum of the projectile.

Attempts regarding the equation that describes the penetration velocity of a projectile penetrating into granular bed have been conducted by many researchers such as (*Nishida*, 2004; *Stone, 2004; McDonald,* 2006). Several scaling laws that govern the penetration process of a projectile into a granular bed were presented by a number of researchers (*Newhall*, 2003; Ambroso, 2005; Goldman, 2008). The effect of closed lateral walls on penetration depth of a projectile has experimentally been studied (Seguin, 2008; Ogale, 2006). A proposed model for the propagation of energy due to the impact of a projectile on a dense granular medium was established by Crassous *et al* (2007). Further results can be found in (Royer, 2008; Crassous, 2007).

II. Simulation Method and Initial Conditions:

Two dimensional discrete element simulations, based on soft particle contact model, were carried out to investigate the penetration process of projectiles into granular beds. The soft particle contact model was originally proposed by Cundall and Strack (1979) and improved by many others to study granular systems. For any two particles in contact, the normal contact force is modeled via a damped linear spring element, while the tangential contact force is determined by a linear spring in series with a frictional sliding element. The granular system is subjected to the standard gravity field and assumed to be dry, massive particles in a blanked space. Therefore, the friction effect, on the projectile motion, caused by the air before and after impact, can be neglected. The mechanical simulation parameters of the projectile and the bed granular particles are listed together with the environmental simulation parameters in Table 1. All parameters are non-dimensionalized using particle diameter, particle density and gravitational acceleration.

In a typical simulation, a projectile (two-dimensional disc) of diameter 3 *mm* and density 2500 kg/m^3 is released, vertically, with downward initial velocity and strikes the middle of the horizontal free surface of a granular bed. The random packing method is used in the present simulations to prepare mono-sized granular bed consisting of particles with diameter equal to 1 *mm*. The bed particles were allowed to fall down under the gravitational force into a rectangular two dimensional container with smooth walls. The dimensions of the granular bed are 50×40 *mm*. Figure 1 shows five sequential snapshots of a projectile impacting and penetrating a granular bed. During the entire process, one can identify three distinct regimes namely, impact, penetration, and collapse as found in [25]. As shown in figure 1, the projectile initially strikes the horizontal surface of the granular bed and then starts to penetrate the granular bed where a temporary crater forms. Due to countable collisions with the bed particles, there are several forces acting on the projectile during impact and penetration. These forces cause the projectile kinetic energy to dissipate during penetration and hence the projectile penetration velocity slows down till the projectile comes to rest after achieving the maximum penetration distance beneath the bed surface.

III. Scaling of Penetration Depth with Impact Velocity:

In order to describe a relationship between the penetration distance of a projectile and its impact velocity, a series of numerical simulations were conducted. The methodology was to vary the impact velocity of the projectile and keep the other simulation parameters alike as given in Table 1. The projectile impact velocities were changed from 5 to 65 *m/sec*. The particles bed are mono-sized with particle diameter 1.5 *mm*. Figure 2 shows the relationship between the impact velocities and the penetration distances under the bed surface for three different values of projectile diameter. At a specific impact velocity, the achieved penetration distances under the bed surface for projectiles with larger diameters are greater than that for projectiles with smaller diameters.

To extrapolate a power law that governs the relationship between the projectile impact velocity and its penetration distance, we convert the obtained data to log-log plots as shown in figure 3.
The results from the simulation compare well with previous experimental results such as those in [26]. Moreover, it is known that for accelerated linear theorem we have

$$v_{impact} = \sqrt{2\vec{g}\, d_{penetration}}$$

where $\vec{g}$ is the gravitational acceleration.

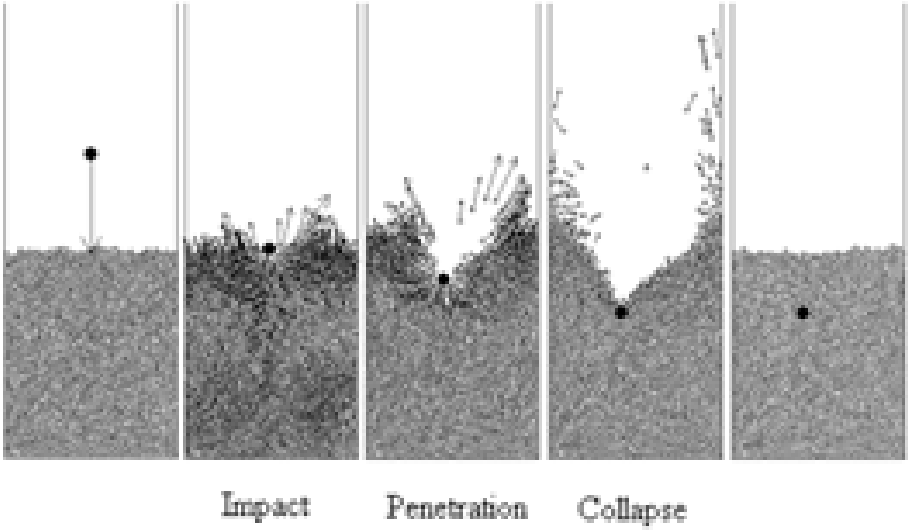

Fig. 1: Simulation snapshots showing the penetration process.

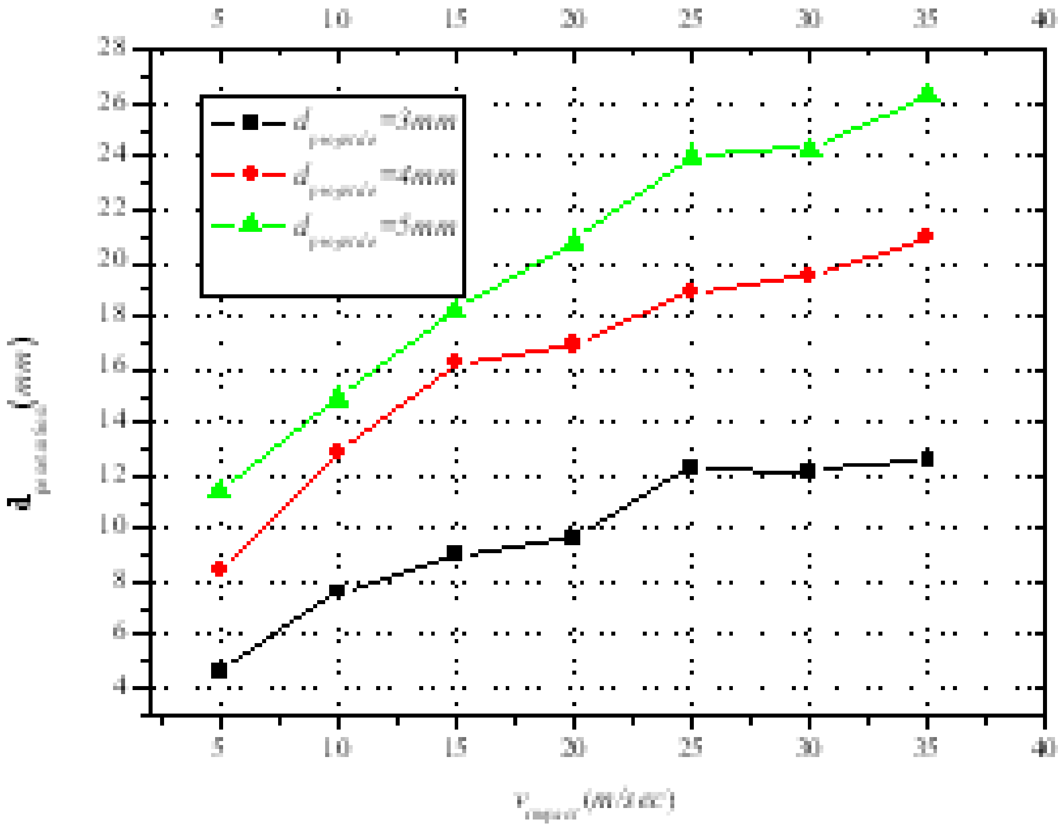

Fig. 2: Impact velocity versus penetration distance.

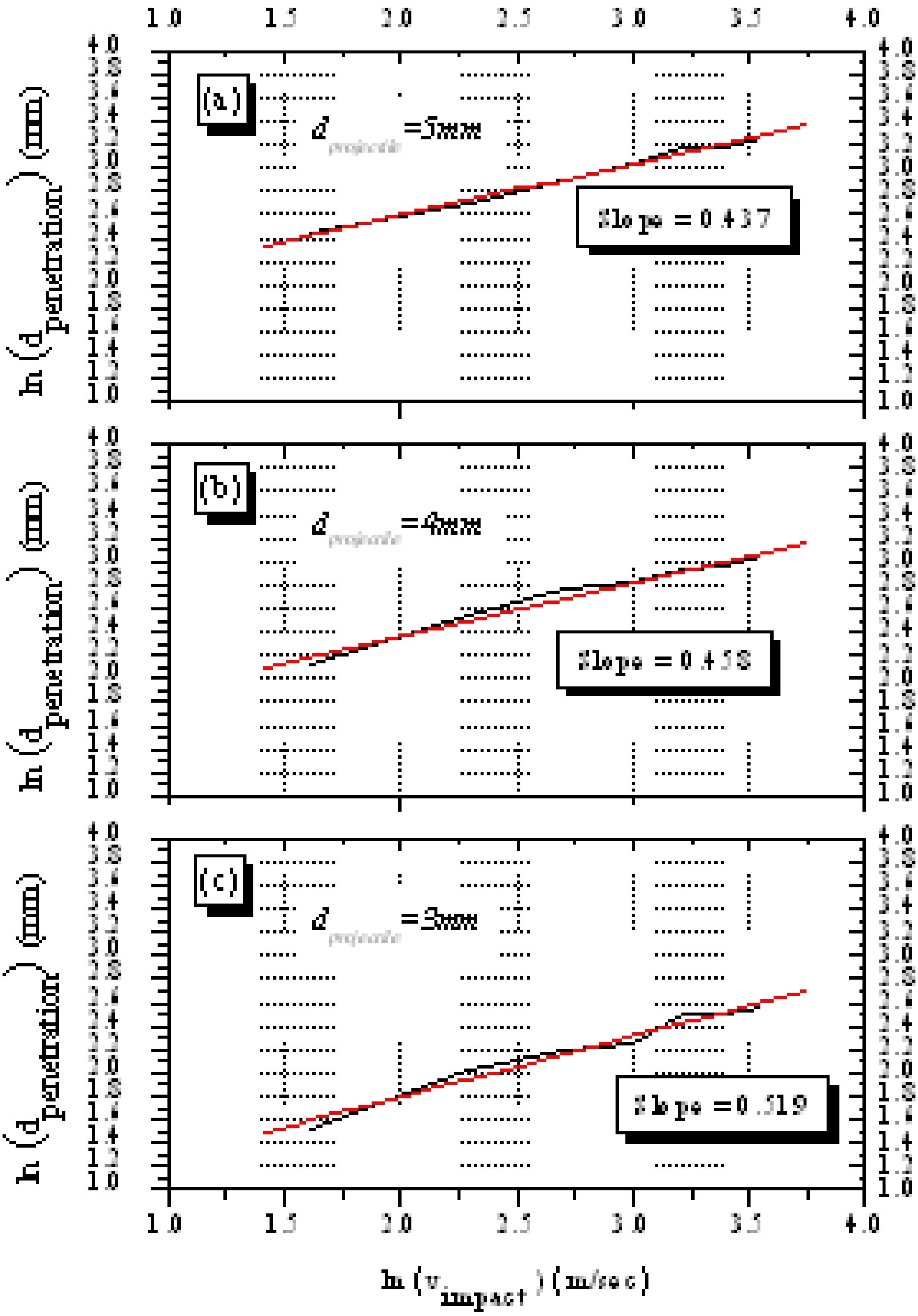

Fig. 3: Impact velocity versus penetration distance

Table 1: Parameters used in the simulation

Parameter	Value (dimensionless)
Number of bed particles, N_n	2050
Particle bed diameter, d/d_o (where d_o=1.0 mm)	1
Projectile diameter, d_p/d_o	3
Particle bed density, ρ/ρ_o (where $\rho_o = 1.0 \times 10^3 kg/m^3$)	1
Projectile density, ρ_p/ρ_o	2.5
Particle-particle normal spring stiffness, $K_{n,pp}/(\rho_o \left\Vert \vec{g} \right\Vert d_o^2)$	1.266×10^5
Particle-particle normal dashpot coefficient, $C_{n,pp}/(\rho_o \left\Vert \vec{g} \right\Vert^{0.5} d_o^{2.5})$	2.524×10^2
Particle-particle tangential spring stiffness, $K_{t,pp}/(\rho_o \left\Vert \vec{g} \right\Vert d_o^2)$	1.266×10^5
Particle-particle friction coefficient, μ_{pp}	0.1
Particle-wall normal spring stiffness, $K_{n,pw}$	1.566×10^5
Particle-wall normal dashpot coefficient, $C_{n,pw}/\rho_o \left\Vert \vec{g} \right\Vert^{0.5} d_o^{2.5}$	2.01×10^2
Particle-wall tangential spring stiffness, $K_{t,pw}/(\rho_o \left\Vert \vec{g} \right\Vert d_o^2)$	3.006×10^4
Particle-wall friction coefficient, μ_{pw}	0.1
Gravity in x-direction, $\left\Vert \vec{g} \right\Vert_x / \left\Vert \vec{g} \right\Vert$	0.0
Gravity in y-direction, $\left\Vert \vec{g} \right\Vert_y / \left\Vert \vec{g} \right\Vert$	9.8
Time step, $\Delta t \sqrt{\left\Vert \vec{g} \right\Vert / d_o}$	5.37×10^{-6}

IV. Concluding Remarks:

Two-dimensional soft particle discrete element simulations have been conducted to simulate the vertical penetration of a projectile into a bed of granular material. The penetration processes have been simulated for various different dynamical conditions. It has been found that the scaling law of the penetration distance of a projectile with its impact velocity follows a power law of the form

$$d_{penertation} \infty (v_{impact})^{\frac{1}{2}}$$

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