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Numerical Model for Studying Cloud Formation Processes in the Tropics

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Abstract: A simple numerical model for demonstrating local cloud formation processes in the tropics is developed. This paper describes some tests of the model in dry air before the inclusion of moisture. The model equations are: the density equation, the wind equation, the vertical velocity equation, and the temperature equation. The model domain is two-dimensional with length 100 km and height 17.5 km. Plans for experiments to investigate the error and stability effects of the deep convection approximation, the treatment of velocities at the boundaries, smoothing in time, and smoothing in space are discussed. Equations for the simplified treatment of moisture in the atmosphere in future numerical experiments are also given.

Key words: Numerical weather prediction; Finite differences; Cloud formation

INTRODUCTION

Numerical weather prediction (NWP) models are techniques used to predict the future state of the weather by solving a set of equations which govern the behavior of the atmosphere. A simplified numerical model for studying the behavior of air in the troposphere in a tropical climate is described in this paper. The model equations are derived from the fundamental system of partial differential equations of computational of fluid dynamics Pilasluck Sornkaew. They show students how NWP models are constructed without involving the complicated transformations in actual operational models.

Model Description:

There is one horizontal dimension *x,* the vertical dimension *z* and the time dimension *t* The variables are located on a staggered grid with stretched grid spacing in the vertical dimension *z* and constant grid spacing in the horizontal dimension *x,*

Table 1: shows the list of symbols is used in this paper.

Symbol	*Units*	*Description*
x	*m*	Horizontal distance
z	*m*	Vertical height
t	*s*	Time
g	*m/s*2	Acceleration of gravity
R	*j/kgK*	Gas constant for air
c_v	*j/kgK*	Specific heat of air
ρ	*kg/m*3	Air density
u	*m/s*	Horizontal velocity
w	*m/s*	Vertical velocity
T	*K*	Temperature
z_0	*m*	Roughness length of the surface
q_s	*W/m*2	Surface heating rate per unit area
ρ_v	*kg/m*3	Water vapor density
m_c	*kg/m*3	Condensed cloud water per unit volume
L	*j/kg*	Latent heat of condensation of water

In this model the molecular viscosity terms are omitted, the body forces are friction at the Earth's surface in the horizontal momentum equation and gravity in the vertical momentum equation; the Coriolis force is omitted; heating and cooling of the air occur at the Earth's surface; kinetic energy and potential energy in the temperature equation are omitted, and the deep convection approximation is used.

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The deep convection approximation Roger, (2002) is

$$\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}=-\frac{w}{\rho^0}\frac{\partial \rho^0}{\partial z}, \tag{1}$$

where the initial values of density $\rho^0(z)$ are calculated from a steady background temperature profile $T^0(z)$ and the assumption of hydrostatic equilibrium in the undisturbed atmosphere.

The steady background temperature profile is an approximation to the annual mean upper air temperatures at Bangkok Thai Meteorological Department, (1951-1980) represented by the formula

$$T^0(z)=302-0.00675z, \tag{2}$$

where $T^0(z)$ is in Kelvin and z is in meter.

A. Governing equations:

1)The density equation:

The density forecast equation derived from the continuity equation John (1995) is

$$\frac{\partial \rho}{\partial t}=-u\frac{\partial \rho}{\partial x}-w\frac{\partial \rho}{\partial z}-\frac{\rho w}{T^0}\left(\frac{\partial T^0}{\partial z}+\frac{g}{R}\right). \tag{3}$$

2)The wind equation:

The wind forecast equation derived from the horizontal momentum equation John, (1995) is

$$\frac{\partial u}{\partial t}=-u\frac{\partial u}{\partial x}-w\frac{\partial u}{\partial z}-\frac{RT}{\rho}\frac{\partial \rho}{\partial x}-\frac{R\partial T}{\partial x}-\frac{0.16u|u|}{\left[\ln\left(\frac{\Delta z}{2z_0}\right)\right]^2\Delta z}. \tag{4}$$

The last term is horizontal friction, which is applied only in the layer of air of thickness Δz at the Earth's surface where the roughness length is z_0.

The vertcal velocity equation:

The vertical velocity equation derived from the vertical momentum equation John, (1995) is

$$\frac{\partial w}{\partial t}=-u\frac{\partial w}{\partial x}-w\frac{\partial w}{\partial z}-\frac{RT}{\rho}\frac{\partial \rho}{\partial z}-R\frac{\partial T}{\partial z}-g. \tag{5}$$

The temperature equation:

The temperature forecast equation derived from the energy equation John, (1995) is

$$\frac{\partial T}{\partial t}=-u\frac{\partial T}{\partial x}-w\frac{\partial T}{\partial z}-\frac{RT}{c_v}\left(\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}\right)+\frac{q_s}{c_v\rho\Delta z}. \tag{6}$$

The surface heating term q_s is applied only in the layer of air at the Earth's surface.

B. Finite difference set up:

The domain of the model is divided into a 25×100 array of cells. The horizontal width of each cell is Δx=1 kilometer. A vertical coordinate s is used in accordance with the transformation $z=75s+25s^2$ in order to give thin layers at the Earth's surface and thick layers at the top of the troposphere Exell, (2009).

The model variables are evaluated at points on an Arakawa-C grid. The model variables ρ and T are in the center of each cell, the horizontal velocity is on the left side of the cell, and the vertical velocity is on the bottom of the cell as shown in Fig. 1.

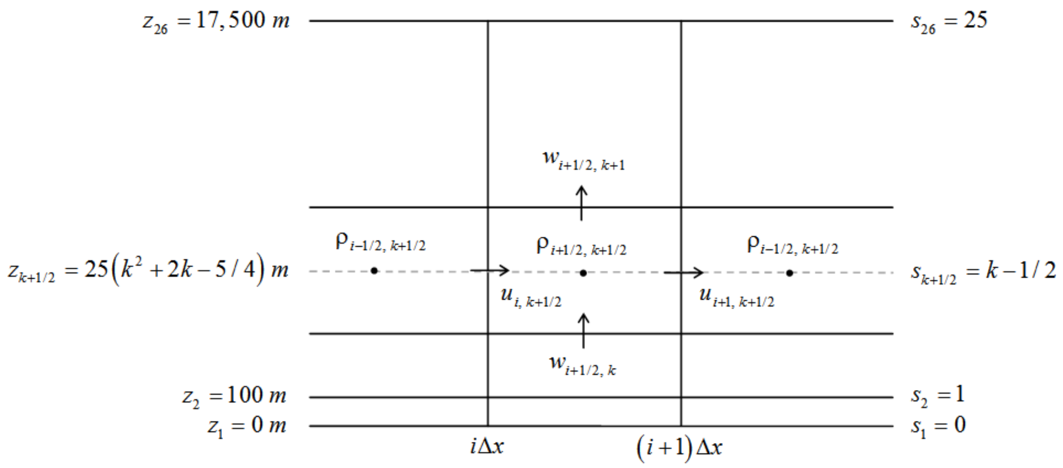

Fig. 1: The grid of the simple model with the stretched grid.

The leapfrog method Joe, (1993) is used to calculate the model variables at time *n*+1 from the values at time *n*; the Euler method is used for the first time step. First and second order finite difference approximations are used in the modeling of space derivatives.

The initial values of the model variables in each cell are functions of the height of the cell above the Earth's surface, but are constant along the horizontal rows of cells. The temperature equation Exell, (2009) is given by

$$T^0_{k+0.5} = 302 - 0.00675\left(25k^2 + 50k - 31.25\right), \tag{7}$$

where $k = 1, 2, ..., 25$.

It is assumed u and w are zero everywhere in the initial state, and that the initial values of ρ and T satisfy the hydrostatic equation.

Numerical Experiments:

Four different experiments are done to study the effects of the deep convection approximation and smoothing in time. The smoothing is done to remove the checkerboard pattern that develops when the leapfrog method is used.

A. Case 1:

- Without deep convection approximation.
- Boundary conditions

  Constant values of ρ and T in the cells i=1, i=100 and k=25.
  Constant value of u in the cells i=1, i=101 and k=25.
  Constant value of w in the cells i=1, i=100, k=1 and k=26.

B. Case 2:

- Deep convection approximation.
- Boundary conditions

  Constant values of ρ and T in the cells i=1, i=100 and k=25.
  Constant value of u in the cells i=1, i=101 and k=25.
  Constant value of w in the cells i=1, i=100, k=1 and k=25.

C. Case 3:

- Deep convection approximation.
- Boundary conditions

  Constant values of ρ and T in the cells i=1, i=100 and k=25.
  Constant value of u in the cells i=1, i=101 and k=25.
  Constant value of w in the cells i=1, i=100 k=1 and k=25.

- Smoothing in time.

RESULTS AND DISCUSSION

The object of these experiments is to find the largest value of Δt that gives stable results for model times of the order one hour, and to determine the errors in the results. Preliminary results have shown that Δt=0.3 seconds is too large. It also appears that using the deep convection approximation with smoothing in time by a Robert-Asselin filter Eugenia Kalnay, (2003) gives the best results for half-an-hour forecast, while not using the deep convection approximation gives the worst results. But using Δt=0.2 seconds without using the deep convection approximation gives the best results for one-hour forecast.

However, previous results have shown that this very simple model can give reasonable results in numerical experiments on vertical movement processes in the tropics (Pilasluck Sornkaew).

Future Work:

Future developments of this model will include water vapor and condensed cloud water. The equations to be use are based on the simplifying assumptions that the thermal properties of moist air are equal to those of dry air and the water vapor density is obtained as a function of temperature by integrating the Clausius-Clapeyron equation Rogers and Yau, (1989) with a constant latent heat of condensation.

A. Unsaturated air:

1)The temperature equation:

$$\frac{\partial T}{\partial t} = -u\frac{\partial T}{\partial x} - w\frac{\partial T}{\partial z} - \frac{RT}{c_v}\left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\right) + \frac{q_s}{c_v \rho \Delta z}. \tag{8}$$

2)The water vapor equation:

$$\frac{\partial \rho_v}{\partial t} = -u\frac{\partial \rho_v}{\partial x} - w\frac{\partial \rho_v}{\partial z} - \rho_v\left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\right). \tag{9}$$

3)The condensed cloud water equation:

$$m_c = 0. \tag{10}$$

B. Saturated air:

1)The temperature equation:

$$\frac{\partial T}{\partial t} = -u\frac{\partial T}{\partial x} - w\frac{\partial T}{\partial z} + \frac{W}{1+EQ}, \tag{11}$$

where $W = -\frac{RT}{c_v}\left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\right) + \frac{q_s}{c_v \rho \Delta z}$,

$$E = \frac{A(B-T)}{R_v T^3} e^{-B/T},$$

$$Q = \frac{L}{c_v \rho}.$$

2)The water vapor equation:

$$\frac{\partial \rho_v}{\partial t} = -u\frac{\partial \rho_v}{\partial x} - w\frac{\partial \rho_v}{\partial z} + \frac{EW}{1+EQ}. \tag{12}$$

3)The condensed cloud water equation:

$$\frac{\partial m_c}{\partial t} = -u\frac{\partial m_c}{\partial x} - w\frac{\partial m_c}{\partial z} - (\rho_v + m_c)\left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\right) - \frac{EW}{1+EQ}. \tag{13}$$

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